Factorization for TMDs in SIDIS at Subleading Power

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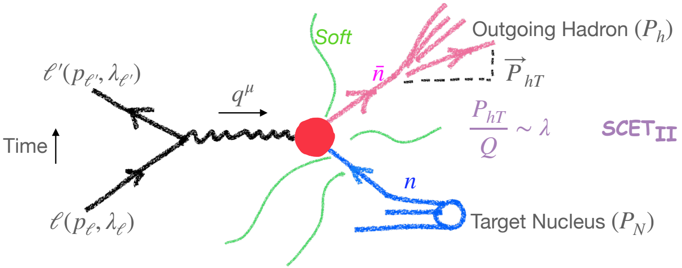
Outline

- Introduction to the Problem and Motivation
  - ▷ Semi-Inclusive DIS
  - ▷ Goal: factorize structure functions that first appear at subleading power
- Review of (Intro to) Soft-Collinear Effective Theory (SCET)
- Deriving Factorization: from Leading Power to Subleading Power

Semi-Inclusive DIS: Basics

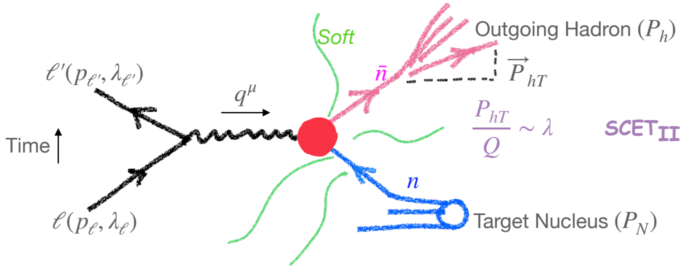


- Lorentz invariants $Q = \sqrt{-q^2}\,, \quad x = \frac{Q^2}{2P_N \cdot q}\,, \quad y = \frac{P_N \cdot q}{P_N \cdot p_\ell}\,, \quad z = \frac{P_N \cdot P_h}{P_N \cdot q}$
- Factorization (schematically)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} \overset{\lambda\to 0}{\sim} \int \mathrm{d}^2k_T\,\mathrm{d}^2p_T\,\mathrm{d}^2k_{sT}\,\mathcal{H}(Q)\,\hat{f}(x,\vec{k}_T)\,\hat{D}(z,\vec{p}_T)\,S(\vec{k}_{sT})$$
$$\times\,\delta^2(\vec{P}_{hT}/z - \vec{k}_T - \vec{p}_T + \vec{k}_{sT})$$
$$\sim \int \mathrm{d}^2b_T\,e^{\mathrm{i}\vec{b}_T\cdot\vec{P}_{hT}/z}\mathcal{H}(Q)\,\hat{f}(x,\vec{b}_T)\,\hat{D}(z,\vec{b}_T)\,S(\vec{b}_T)$$

- $\hat{f}/\hat{D}/S$, Transverse momentum dependent (TMD) beam/fragmentation/soft functions

Semi-Inclusive DIS: Basics

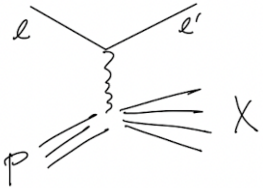


- Lorentz invariants $Q = \sqrt{-q^2}\,, \quad x = \frac{Q^2}{2P_N\cdot q}\,, \quad y = \frac{P_N\cdot q}{P_N\cdot p_\ell}\,, \quad z = \frac{P_N\cdot P_h}{P_N\cdot q}$
- $$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{2Q^4}\frac{y}{z}L_{\mu\nu}(p_\ell, p_{\ell'})W^{\mu\nu}(q, P_N, P_h)$$
- $$W^{\mu\nu}(q, P_N, P_h) = \sum_X\!\!\!\!\!\!\int \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{\mathrm{i}b\cdot q}\,\langle N|J^{\dagger\,\mu}(b)|h, X\rangle\,\langle h, X|J^\nu(0)|N\rangle$$

$$L^{\mu\nu}(p_\ell, p_{\ell'}) = \langle\ell|J_{\ell\ell}^{\dagger\mu}|\ell'\rangle\,\langle\ell'|J_{\ell\ell}^{\nu}|\ell\rangle$$
$$= 2\delta_{\lambda_\ell\lambda_{\ell'}}\left[\left(p_\ell^\mu p_{\ell'}^\nu + p_\ell^\nu p_{\ell'}^\mu - p_\ell\cdot p_{\ell'}\,g^{\mu\nu}\right) + \mathrm{i}\lambda_\ell\,\epsilon^{\mu\nu\rho\sigma}p_{\ell\rho}p_{\ell'\sigma}\right]$$

$$J^\mu = \sum_f \bar{q}_f\gamma^\mu q_f\,, \qquad J_{\ell\ell}^\mu = \bar{\ell}\gamma^\mu\ell$$

Tensor Decomposition for (Unporlarized) Inclusive DIS



- Summing over final states

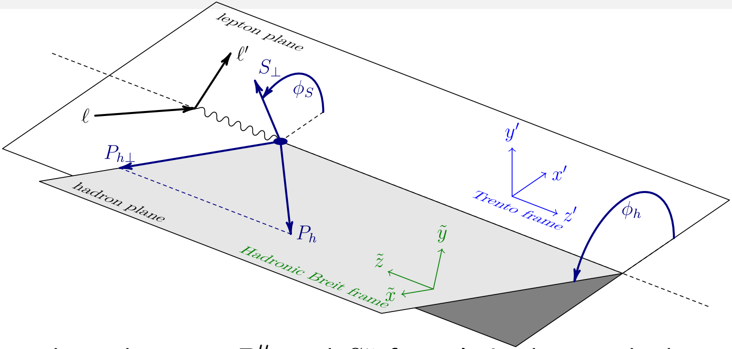
$$W^{\mu\nu}(q, P_N) = \sum_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4} \, e^{ib\cdot q} \, \langle N | J^{\dagger\,\mu}(b) | X \rangle \, \langle X | J^{\nu}(0) | N \rangle$$
$$= \int \frac{\mathrm{d}^4 b}{(2\pi)^4} \, e^{ib\cdot q} \, \langle N | J^{\dagger\,\mu}(b) J^{\nu}(0) | N \rangle$$

- $q_\mu W^{\mu\nu} = 0$, $W^{\mu\nu} = W^{\nu\mu}$, dependence on only two vectors q^μ and P_N^μ

$\Rightarrow$ Two structure functions

$$W^{\mu\nu}(q, P_N) = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \left(P_N^\mu - \frac{P_N \cdot q}{q^2} q^\mu \right) \left(P_N^\nu - \frac{P_N \cdot q}{q^2} q^\nu \right)$$

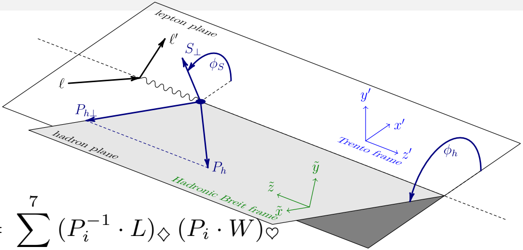
Kinematics and Tensor Decomposition for SIDIS



- Extra dependence on P_h^μ, and S^μ for polarized target hadron
- $S^\mu = \Big(0\,,\, S_T \cos\phi_S\,,\, S_T \sin\phi_S\,,\, -S_L\Big)_T$
- $W^{\mu\nu} = W_U^{\mu\nu} + S_L W_L^{\mu\nu} + S_T \cos(\phi_h - \phi_S) W_{T\tilde{x}}^{\mu\nu} + S_T \sin(\phi_h - \phi_S) W_{T\tilde{y}}^{\mu\nu}$
- Different polarization contributions of lepton/hadron $\left(\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}\right)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\kappa_\gamma}{1-\epsilon}\Big[(L\cdot W)_{UU} + \lambda_\ell (L\cdot W)_{LU} + S_L (L\cdot W)_{UL} + \lambda_\ell S_L (L\cdot W)_{LL} + S_T (L\cdot W)_{UT} + \lambda_\ell S_T (L\cdot W)_{LT}\Big]$$

Kinematics and Tensor Decomposition for SIDIS



- Projection

$$(L \cdot W)_{\diamondsuit\heartsuit} = \sum_{i=-1}^{7} (P_i^{-1} \cdot L)_{\diamondsuit} \, (P_i \cdot W)_{\heartsuit}$$

- Projectors defined in the hadronic Breit frame

$$P_{-1}^{\mu\nu} = (\tilde{x}^\mu \tilde{x}^\nu + \tilde{y}^\mu \tilde{y}^\nu)\,, \;\; P_0^{\mu\nu} = \tilde{t}^\mu \tilde{t}^\nu\,, \;\; P_1^{\mu\nu} = -(\tilde{t}^\mu \tilde{x}^\nu + \tilde{x}^\mu \tilde{t}^\nu)\,, \dots\,, \;\; P_7^{\mu\nu}$$

- $q \cdot L = q \cdot W = 0 \Rightarrow$ no $\tilde{z} \Rightarrow 3 \times 3 = 9$ projectors
- Parity and hermiticity constraints reduce # of structure functions

$\Rightarrow$ In total 18 structure functions [Bacchetta et al '06]

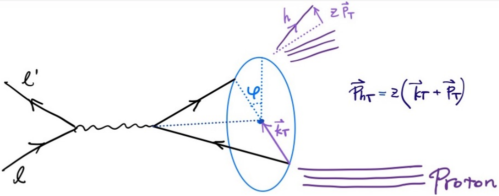
$$(L{\cdot}W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h) W_{UU}^{\cos(\phi_h)} + \epsilon\cos(2\phi_h) W_{UU}^{\cos(2\phi_h)}\,,$$

$$(L{\cdot}W)_{LU} = \sqrt{2\epsilon(1-\epsilon)}\sin(\phi_h) W_{LU}^{\sin(\phi_h)}\,,$$

$$(L{\cdot}W)_{LT} = \sqrt{1-\epsilon^2}\cos(\phi_h - \phi_S) W_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\left[\cos(\phi_S) W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h - \phi_S) W_{LT}^{\cos(2\phi_h-\phi_S)}\right],$$

……

Azimuthal Dependence: Cahn Effect $W_{UU}^{\cos\phi_h}$ [Cahn '78, '89]



- Partonic cross section $\ell q \to \ell q$ depends on φ,

$$\mathrm{d}\hat{\sigma} \sim \hat{s}^2 + \hat{u}^2 = \frac{Q^4}{y^2}\left[1 + (1-y)^2 - 4\frac{k_T}{Q}(2-y)\sqrt{1-y}\cos\varphi\right]$$

- Naive parton model calculation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\vec{P}_{hT}} \sim (\hat{s}^2 + \hat{u}^2) \otimes f_1(x, \vec{k}_T) \otimes D_1(z, \vec{p}_T)$$

$$\supset \mathcal{F}\left[\frac{k_{Tx}}{Q} f_1(x, \vec{k}_T) D_1(z, \vec{p}_T)\right] \cos\phi_h$$

- Intrinsic transverse momentum of partons inside hadrons $\Rightarrow W_{UU}^{\cos\phi_h}$

Cahn Effect $W_{UU}^{\cos\phi_h}$

- A more careful parton model calculation [Mulders, Tangerman '95]

$$\frac{x}{2z}W_{UU}^{\cos\phi_h} = \frac{2M_N}{Q}\mathcal{F}\Bigg\{\frac{-k_{Tx}}{M_N}\left[(f_1 + x\tilde{f}^\perp)D_1 + \frac{M_h}{M_N}x h_1^\perp \frac{\tilde{H}}{z}\right] - \frac{p_{Tx}}{M_h}\left[\left(x\tilde{h} - \frac{k_T^2}{M_N^2}h_1^\perp\right)H_1^\perp + \frac{M_h}{M_N}f_1\frac{\tilde{D}^\perp}{z}\right]\Bigg\}$$

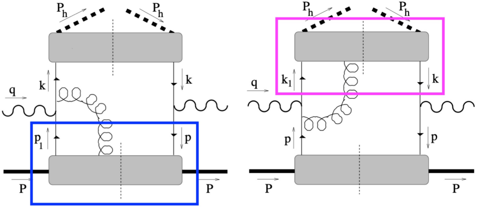
$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f\int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2\left(\vec{q}_T + \vec{p}_T - \vec{k}_T\right)\omega(\vec{p}_T,\vec{k}_T)\,g_f(x,p_T)\,D_f(z,k_T)$$

- Schematically (ignoring gauge invariance for now),

$f_1,\, h_1^\perp \in \left\langle N\middle|\bar{\psi}^\beta(b)\psi^{\beta'}(0)\middle|N\right\rangle$, $\quad D_1,\, H_1^\perp \in \sum_X\left\langle 0\middle|\psi^\alpha(b)\middle|h,X\right\rangle\left\langle h,X\middle|\bar{\psi}^{\alpha'}(0)\middle|0\right\rangle$

$\tilde{f}^\perp,\, \tilde{h} \in \left\langle N\middle|\bar{\psi}^{\beta'}(b)A^\rho(0)\psi^\beta(0)\middle|N\right\rangle$,

$\tilde{D}^\perp,\, \tilde{H}^\perp \in \sum_X\left\langle 0\middle|\psi^\alpha(b)\middle|h,X\right\rangle\left\langle h,X\middle|A^\rho(0)\bar{\psi}^{\alpha'}(0)\middle|0\right\rangle$



[Mulders, Tangerman '95]

Power Expansion in $\lambda = P_{hT}/Q \ll 1$ and Motivation

- Focus on the unporlarized hadron (different notation for labeling)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\delta_{\lambda_\ell\lambda_{\ell'}}}{1-\epsilon}\Big[\big(W_{-1}+\epsilon W_0\big)+\epsilon\cos(2\phi_h)W_3 + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h W_1 + \lambda_\ell\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h W_2\Big].$$

- $\epsilon = \frac{1-y}{1-y+\frac{1}{2}y^2}$
- $W_i = P_i^{\mu\nu}W_{\mu\nu}$ with projectors $P_i^{\mu\nu}$ (defined in the hadronic Breit frame)
- $P_{-1}^{\mu\nu} = (\tilde{x}^\mu\tilde{x}^\nu + \tilde{y}^\mu\tilde{y}^\nu)\,, \quad P_3^{\mu\nu} = \tilde{x}^\mu\tilde{x}^\nu - \tilde{y}^\mu\tilde{y}^\nu\,,$

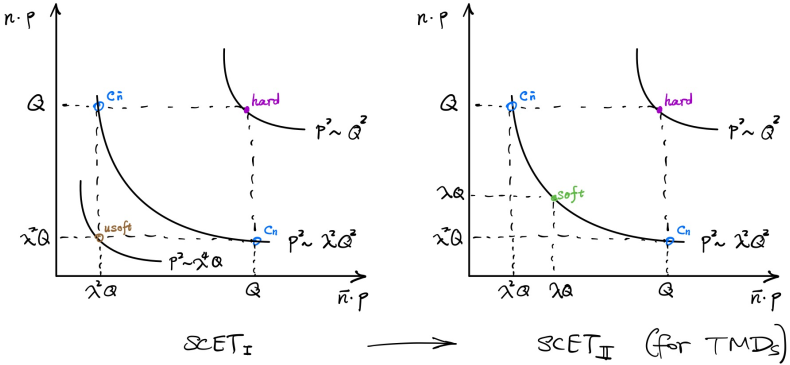
  $P_1^{\mu\nu} = -(\tilde{t}^\mu\tilde{x}^\nu + \tilde{x}^\mu\tilde{t}^\nu)\,, \quad P_2^{\mu\nu} = \mathrm{i}\big(\tilde{t}^\mu\tilde{x}^\nu - \tilde{x}^\mu\tilde{t}^\nu\big)\,, \quad P_0^{\mu\nu} = \tilde{t}^\mu\tilde{t}^\nu\,,$
- $W_{-1}\,, W_3 \sim \mathcal{O}\big(\lambda^0\big)$, standard factorization theorems (CSS, SCET)
- $W_1\,, W_2 \sim \mathcal{O}\big(\lambda\big)$
  - First treated in parton model (tree level matching) [Mulders, Tangerman '95]
  - Mismatch with perturbative results at tree level [Bacchetta et al '08]
  - Conjecture: Resolved by adding a LP soft function [Bacchetta et al '19]
- $W_0 \sim \mathcal{O}\big(\lambda^2\big)$, not considered in this work

$\Rightarrow$ *Use SCET to derive all-order factorization at subleading power*

Review of (Intro to) SCET

[Bauer, Fleming, Luke, Pirjol, Stewart '00, '01, '02]

- EFT for collinear/soft d.o.f.s with power counting parameter $\lambda \ll 1$
- Lightcone coordinate $p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p + \frac{\bar{n}^\mu}{2} n\cdot p + p_\perp$
- n_i-collinear particles: $(n_i\cdot p, \bar{n}_i\cdot p, p_{n_i\perp}) \sim Q(\lambda^2, 1, \lambda)$
- Ultrasoft $k^\mu \sim Q\lambda^2$ in SCET$_\text{I}$; Soft $k^\mu \sim Q\lambda$ in SCET$_\text{II}$ (for TMD)
- For SIDIS, take $n_1 = n /\!/ P_N$ and $n_2 = \bar{n} /\!/ P_h$



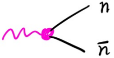
Review of (Intro to) SCET

[Bauer, Fleming, Luke, Pirjol, Stewart '00, '01, '02]

- SCET Lagrangian

$$\mathcal{L}_{\mathsf{SCET_{II}}} = \mathcal{L}_{\rm hard} + \mathcal{L}_{\rm dyn} = \Big(\sum_{i\geq 0}\mathcal{L}_{\rm hard}^{(i)}\Big) + \Big(\sum_{i\geq 0}\mathcal{L}_{\rm dyn}^{(i)} + \mathcal{L}_G^{(0)}\Big)\,,$$

▷ $\mathcal{L}_{\rm hard}^{(i)} = \sum_k C_k^{(i)}\mathcal{O}_k^{(i)} = \frac{\mathrm{i}e^2}{Q^2} J_{\ell\ell'\mu}\sum J_k^{(i)\mu}\,,$ Hard scattering operators



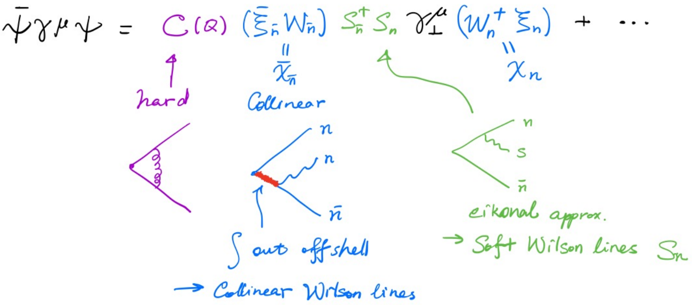
▷ $\mathcal{L}_{\rm dyn}^{(0)} = \mathcal{L}_n^{(0)} + \mathcal{L}_{\bar n}^{(0)} + \mathcal{L}_s^{(0)}\,,$ Collinear and soft dynamics factorize

▷ $\mathcal{L}_{\rm G}^{(0)}\,,$ Glauber: connect different sectors

Factorization = the Glauber contribution vanishes

Hard Operators for SIDIS

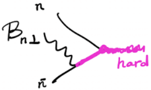
- Match QCD onto SCET $\Rightarrow$ Leading power current (operator)



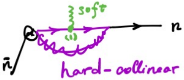
- $\xi_n(x)$: n-collinear quark field, which obeys $\frac{1}{4}\not{n}\not{\bar{n}}\xi_n = \xi_n$ and $\frac{1}{4}\not{\bar{n}}\not{n}\xi_n = 0$, "good components" of the quark field.
- Useful notation: $\chi_{n,\omega} = \delta(\omega - \bar{n}\cdot\mathcal{P})\chi_n$, $\mathcal{B}_{n\perp,\omega} = \delta(\omega + \bar{n}\cdot\mathcal{P})\mathcal{B}_{n\perp}$,

Hard Operators for SIDIS

- Leading power current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta} C_f^{(0)}(Q)\, \bar{\chi}^\alpha_{\bar{n},\omega_b} [S_{\bar{n}}^\dagger S_n]\, \chi^\beta_{n,\omega_a}$
- In general, operators are constructed using "building blocks"
  - ▷ Collinear quark and gluon χ_n, $\mathcal{B}^\mu_{n\perp} = \frac{1}{g}\Big[W_n^\dagger(x)\, \mathrm{i}D^\mu_{n\perp} W_n(x)\Big] \sim \lambda$
  - ▷ Soft quark and gluon $\psi_{s(n)} \sim \lambda^{3/2}$, $\mathcal{B}^\mu_{s(n)} \sim \lambda$
  - ▷ Momentum operators $\mathcal{P}_\perp$, $n \cdot \partial_s$, $\bar{n} \cdot \partial_s \sim \lambda$
- Operators get generated from two offshell scales
  - ▷ Hard (tree-level and beyond)



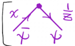
  - ▷ Hard-collinear (one-loop and beyond) [Bauer, Pirjol, Stewart '02]



$T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$, $T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(2)}\big]$, $T\big[J_{\mathrm{I}}^{(0)\mu}\mathcal{L}_{\mathrm{I}}^{(1)}\mathcal{L}_{\mathrm{I}}^{(1)}\big]$ in $\mathrm{SCET_I}$

$\rightarrow$ hard scattering operators in $\mathrm{SCET_{II}}$

Factorization for $W^{\mu\nu}$ at Leading Power

LP current $J^{(0)\mu} \sim \sum_f (\gamma_\perp^\mu)^{\alpha\beta}\, C_f^{(0)}(Q)\, \bar\chi^\alpha_{\bar n,\omega_b} [S_{\bar n}^\dagger S_n]\, \chi^\beta_{n,\omega_a} \sim C_f^{(0)}(Q) \Big[$ 

(+ Wilson lines)$\Big]$

- Plug it into $W^{(0)\mu\nu} \sim \langle N | J^{(0)\dagger\,\mu} | h, X\rangle \langle h, X | J^{(0)\nu} | N\rangle$
- Collinear fields yield quark correlators

$$\hat B_f^{\beta'\beta}(x, \vec b_T) = \Big\langle N \Big| \bar\chi_n^\beta(b_\perp)\, \delta(\omega_a - \overline{\mathcal{P}}_n)\, \chi_n^{\beta'}(0) \Big| N \Big\rangle$$

$$\hat{\mathcal{G}}_f^{\alpha\alpha'}(z, \vec b_T) = \frac{1}{2z} \sum\!\!\!\!\!\!\int_X \Big\langle 0 \Big| \delta(\omega_b - \overline{\mathcal{P}}_{\bar n})\, \chi_{\bar n}^\alpha(b_\perp) \Big| h, X \Big\rangle \Big\langle h, X \Big| \bar\chi_{\bar n}^{\alpha'}(0) \Big| 0 \Big\rangle$$

- Soft Wilson lines yield the TMD soft function

$$\mathcal{S}(b_T) = \frac{1}{N_c} \operatorname{tr} \Big\langle 0 \Big| \big[S_n^\dagger(b_\perp)\, S_{\bar n}(b_\perp)\big]\, \big[S_{\bar n}^\dagger(0)\, S_n(0)\big] \Big| 0 \Big\rangle\,.$$

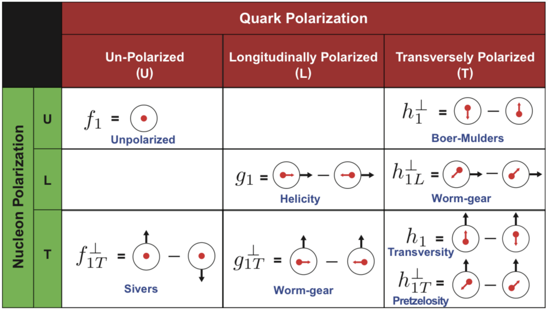
- Combine into the quark correctors

$$B_f^{\beta'\beta}(x, \vec b_T) = \hat B_f^{\beta'\beta}(x, \vec b_T) \sqrt{\mathcal{S}(b_T)}\,, \qquad \mathcal{G}_f^{\alpha'\alpha}(z, \vec b_T) = \hat{\mathcal{G}}_f^{\alpha'\alpha}(z, \vec b_T) \sqrt{\mathcal{S}(b_T)}$$

$\Rightarrow$ Factorized leading power hadronic tensor

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec q_T \cdot \vec b_T}\, \mathcal{H}_f^{(0)}(Q)\, \mathrm{Tr}\Big[B_f(x, \vec b_T)\, \gamma_\perp^\mu\, \mathcal{G}_f(z, \vec b_T)\, \gamma_\perp^\nu \Big]\,.$$

- Hard function: $\mathcal{H}_f^{(0)}(Q) = \big|C_f^{(0)}(Q)\big|^2$



$g_1 = g_{1L},\ g_{1T=g_{1T}^\perp}$

- In the momentum space, decompose into different Dirac structures

[Goeke, Metz, Schlegel '05]

$$B_f^{\beta'\beta}\left(x, \vec{k}_T\right) = \frac{1}{4}\Bigg\{ f_1 \slashed{n} - f_{1T}^\perp \frac{\epsilon_\perp^{\rho\sigma} k_{\perp\rho} S_{\perp\sigma}}{M_N} \slashed{n} + g_{1L}\, S_L \gamma_5 \slashed{n} - g_{1T} \frac{k_\perp \cdot S_\perp}{M_N} \gamma_5 \slashed{n}$$

$$+ h_1\, \gamma_5 \slashed{S}_\perp \slashed{n} + h_{1L}^\perp\, S_L \frac{\gamma_5\, \slashed{k}_\perp}{M_N} \slashed{n}$$

$$- h_{1T}^\perp \frac{k_\perp^2}{M_N^2} \left(\frac{1}{2} g_\perp^{\rho\sigma} \slashed{n} - \frac{k_\perp^\rho k_\perp^\sigma}{k_\perp^2}\right) S_{\perp\,\rho} \gamma_\sigma \gamma_5 \slashed{n} + \mathrm{i} h_1^\perp \frac{\slashed{k}_\perp \slashed{n}}{M_N} \Bigg\}^{\beta'\beta}$$

$$\mathcal{G}_f^{\alpha'\alpha}(z, \vec{p}_T) = \frac{1}{4}\left\{ D_1 \slashed{\bar{n}} + \mathrm{i} H_1^\perp \frac{[\slashed{p}_\perp, \slashed{\bar{n}}]}{2M_h} \right\}^{\alpha'\alpha}$$

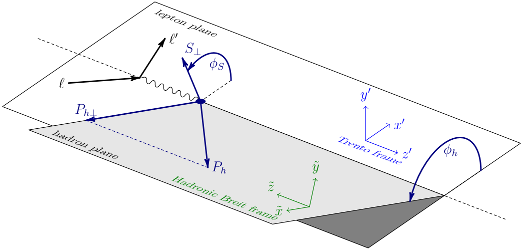
Factorization for Structure Functions: General Procedure

$$W^{\mu\nu}(q, P_N, P_h) = \sum_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{\mathrm{i}b\cdot q}\, \langle N|J^{\dagger\,\mu}(b)|h, X\rangle\, \langle h, X|J^\nu(0)|N\rangle$$

Goal: factorize $W_i = P_i^{\mu\nu} W_{\mu\nu}$ at next-to-leading power

- Match SCET currents (operators) with QCD: $J^\mu = J^{(0)\mu} + \sum_k J_k^{(1)\mu} + \dots$
  (in the factorization frame: $P_N^\mu = P_N^- \frac{n^\mu}{2}$, $P_h^\mu = P_h^+ \frac{\bar{n}^\mu}{2}$, $g_{F\perp}^{\mu\nu} = g_{B\perp}^{\mu\nu} + \mathcal{O}(P_{hT}/Q)$)
- $W^{(0)\mu\nu} \sim J^{(0)\dagger\mu} J^{(0)\nu}$, $\qquad W^{(1)\mu\nu} \sim \sum_k J^{(0)\dagger\mu} J_k^{(1)\nu} + J_k^{(1)\dagger\mu} J^{(0)\nu}$
- Expand projectors in the factorization frame: $P_i^{\mu\nu} = P_i^{(0)\mu\nu} + P_i^{(1)\mu\nu} + \dots$

$$P_1^{\mu\nu} = \frac{1}{2}(t^\mu x^\nu + x^\mu t^\nu) - \frac{q_T}{Q} x^\mu x^\nu + \dots, \quad P_2^{\mu\nu} = \frac{1}{2}(t^\mu x^\nu - x^\mu t^\nu) + \dots$$



Structure Functions at Leading Power

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T \, e^{\mathrm{i}\vec{q}_T \cdot \vec{b}_T} \, \mathcal{H}_f^{(0)}(Q) \, \mathrm{Tr}\left[B_f(x, \vec{b}_T) \, \gamma_\perp^\mu \, \mathcal{G}_f(z, \vec{b}_T) \, \gamma_\perp^\nu \right] .$$

- In the momentum space, decompose into different Dirac structures

[Goeke, Metz, Schlegel '05]

$$B_f^{\beta'\beta}(x, \vec{p}_T) = \frac{1}{4} \left\{ f_1 \not{n} + \mathrm{i} h_1^\perp \frac{[\not{p}_\perp, \not{n}]}{2M_N} \right\}^{\beta'\beta} + \dots ,$$

$$\mathcal{G}_f^{\alpha'\alpha}(z, \vec{k}_T) = \frac{1}{4} \left\{ D_1 \not{\bar{n}} + \mathrm{i} H_1^\perp \frac{[\not{k}_\perp, \not{\bar{n}}]}{2M_h} \right\}^{\alpha'\alpha}$$

- $h_1^\perp$ Boer-Mulders function, $H_1^\perp$ Collins function
- Contract $W^{(0)\mu\nu}$ with $P_{-1}^{(0)\mu\nu} = x^\mu x^\nu + y^\mu y^\nu$, $P_3^{(0)\mu\nu} = x^\mu x^\nu - y^\mu y^\nu$,

$$W_{-1}^{(0)} = \mathcal{F}\left[\mathcal{H}^{(0)} \, f_1 D_1 \right] ,$$

$$W_3^{(0)} = \mathcal{F}\left[-\frac{2\, p_{Tx}\, k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} \mathcal{H}^{(0)} \, h_1^\perp H_1^\perp \right] ,$$

[Collins, SCET, ...]

$$\mathcal{F}[\omega \, \mathcal{H} \, g \, D] = 2z \sum_f \int \mathrm{d}^2 p_T \, \mathrm{d}^2 k_T \, \delta^2\left(\vec{q}_T + \vec{p}_T - \vec{k}_T \right) \omega(\vec{p}_T, \vec{k}_T) \, \mathcal{H}_f(Q) \, g_f(x, p_T) \, D_f(z, k_T)$$

Conjecture in Literature for W_1: Adding a LP Soft Function

- Parton model calculation [Mulders, Tangerman '95]

$$\frac{x}{2z} W_{UU}^{\cos\phi_h} = \frac{2M_N}{Q} \mathcal{F}\left\{ \frac{-k_{Tx}}{M_N} \left[(f_1 + x\tilde{f}^{\perp}) D_1 + \frac{M_h}{M_N} x h_1^{\perp} \frac{\tilde{H}}{z} \right] - \frac{p_{Tx}}{M_h} \left[\left(x\tilde{h} - \frac{k_T^2}{M_N^2} h_1^{\perp} \right) H_1^{\perp} + \frac{M_h}{M_N} f_1 \frac{\tilde{D}^{\perp}}{z} \right] \right\}$$

$$\mathcal{F}[\omega\, \mathcal{H}\, g\, D] = 2z \sum_f \int \mathrm{d}^2 p_T\, \mathrm{d}^2 k_T\, \delta^2\!\left(\vec{q}_T + \vec{p}_T - \vec{k}_T \right) \omega(\vec{p}_T, \vec{k}_T)\, g_f(x, p_T)\, D_f(z, k_T)$$

- Mismatch with the direct tree level calculation was resolved by adding a LP soft function [Bacchetta et al '19]

Factorization for Structure Functions: General Procedure

$$W^{\mu\nu}(q, P_N, P_h) = \sum\!\!\!\!\!\!\int_X \int \frac{\mathrm{d}^4 b}{(2\pi)^4}\, e^{\mathrm{i}b\cdot q} \langle N|J^{\dagger\,\mu}(b)|h, X\rangle \langle h, X|J^{\nu}(0)|N\rangle$$

Goal: factorize $W_i = P_i^{\mu\nu} W_{\mu\nu}$ at next-to-leading power

- Match SCET currents (operators) with QCD: $J^\mu = J^{(0)\mu} + \sum_k J_k^{(1)\mu} + \dots$
- $W^{(0)\mu\nu} \sim J^{(0)\dagger\mu} J^{(0)\nu}$, $\qquad W^{(1)\mu\nu} \sim \sum_k J^{(0)\dagger\mu} J_k^{(1)\nu} + J_k^{(1)\dagger\mu} J^{(0)\nu}$
- Expand projectors in the factorization frame: $P_i^{\mu\nu} = P_i^{(0)\mu\nu} + P_i^{(1)\mu\nu} + \dots$

Categories of power corrections

1) Subleading current contributions, $P_i^{(0)} \cdot W^{(1)}$
2) Kinematic correction, $P_i^{(1)} \cdot W^{(0)}$
3) Subleading soft contributions including $\mathrm{SCET_{II}}$ subleading Lagrangians $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1/2)} + \mathcal{L}^{(1)} + \dots$

- Assumption: Glauber Lagrangian $\mathcal{L}_G^{(0)}$ doesn't spoil factorization

Kinematic Correction for W_1

- Taking

$$W^{(0)\mu\nu} = \frac{2z}{N_c} \sum_f \int \mathrm{d}^2 b_T \, e^{\mathrm{i}\vec{q}_T \cdot \vec{b}_T} \, \mathcal{H}_f^{(0)}(Q) \, \mathrm{Tr}\left[B_f(x, \vec{b}_T) \, \gamma_\perp^\mu \, \mathcal{G}_f(z, \vec{b}_T) \, \gamma_\perp^\nu \right],$$

contract with $P_1^{(1)\mu\nu} = -\frac{q_T}{Q} x^\mu x^\nu$

$\Rightarrow$ kinematic corrections for W_1

$$\mathcal{F}\left\{ -\frac{P_{hT}}{zQ} \, \mathcal{H}^{(0)} \left[f_1 D_1 - \frac{2 \, p_{Tx} \, k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h} \, h_1^\perp H_1^\perp \right] \right\} \in W_1$$

Subleading Current: $\mathcal{P}_\perp$ Acting on the Collinear Fields

Unique hard operator to all orders [Feige et al '17]

$$J_{\mathcal{P}}^{(1)\mu} \sim \frac{C_f^{(0)}}{2\omega_a}\bar{\chi}_{\bar{n},\omega_b}\left[S_{\bar{n}}^\dagger S_n\right]\gamma^\mu \not{\mathcal{P}}_\perp \not{\bar{n}}\chi_{n,\omega_a} + \text{h.c.} \sim$$

- Reparameterization $\left(n^\mu \to n'^\mu = n^\mu + \Delta_\perp^\mu\right)$ relates it with the LP one

$\Rightarrow$ The Wilson coefficient is identical to the leading power one, $C_f^{(0)}(Q)$

- Plug these currents into $J_{\mathcal{P}}^{(1)\dagger\mu} J^{(0)\nu} + J^{(0)\dagger\mu} J_{\mathcal{P}}^{(1)\nu}$

$$\hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c}\sum_f \int \mathrm{d}^2\vec{b}_T\, \mathcal{H}_f^{(0)}(Q)\, \mathcal{S}(b_T)$$
$$\times \left\{ \mathrm{Tr}\left[\hat{B}_{\mathcal{P}f}(x,\vec{b}_T)\,\gamma^\mu\, \hat{\mathcal{G}}_f(z,\vec{b}_T)\,\gamma^\nu\right] + \mathrm{Tr}\left[\hat{B}_f(x,\vec{b}_T)\,\gamma^\mu\, \hat{\mathcal{G}}_{\mathcal{P}f}(z,\vec{b}_T)\,\gamma^\nu\right]\right\}.$$

where $\hat{B}_{\mathcal{P}f}$ and $\hat{\mathcal{G}}_{\mathcal{P}f}$ are related to the LP corrector as

$$\hat{B}_{\mathcal{P}f}^{\beta'\beta}(x,\vec{b}_T)$$
$$\equiv \frac{1}{2Q}\theta(\omega_a)\left\{\left\langle N\middle|\bar{\chi}_n^\beta(b_\perp^\mu)\ \left[\not{\mathcal{P}}_\perp \not{\bar{n}}\,\chi_{n,\omega_a}(0)\right]^{\beta'}\middle|N\right\rangle + \left\langle N\middle|\ \left[\bar{\chi}_n(b_\perp^\mu)\,\not{\bar{n}}\,\not{\mathcal{P}}_\perp^\dagger\right]^\beta \chi_{n,\omega_a}^{\beta'}(0)\middle|N\right\rangle\right\}$$
$$= \mathrm{i}\frac{1}{2Q}\frac{\partial}{\partial b_\perp^\rho}\left[\gamma_\perp^\rho\,\not{\bar{n}}\,,\ \hat{B}_f(x,\vec{b}_T)\right]^{\beta'\beta},$$

Subleading Current: $\mathcal{P}_\perp$ Acting on the Collinear Fields

- Define $B_{\mathcal{P}f}$, $\mathcal{G}_{\mathcal{P}f}$ and $W_{\mathcal{P}}^{(1)\mu\nu}$

$$B_{\mathcal{P}f}^{\beta'\beta}(x,\vec{b}_T) \equiv \mathrm{i}\frac{1}{2Q}\frac{\partial}{\partial b_\perp^\rho}\Big[\gamma_\perp^\rho\,\bar{n}\!\!\!/\,,\,B_f(x,\vec{b}_T)\Big]^{\beta'\beta},\ \text{where } B_f^{\beta'\beta}(x,\vec{b}_T) = \hat{B}_f^{\beta'\beta}(x,\vec{b}_T)\sqrt{\mathcal{S}(b_T)}$$

$$W_{\mathcal{P}}^{(1)\mu\nu} \equiv \frac{2z}{N_c}\sum_f\int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q)$$
$$\times\Big\{\mathrm{Tr}\Big[B_{\mathcal{P}f}(x,\vec{b}_T)\,\gamma^\mu\,\mathcal{G}_f(z,\vec{b}_T)\,\gamma^\nu\Big] + \mathrm{Tr}\Big[B_f(x,\vec{b}_T)\,\gamma^\mu\,\mathcal{G}_{\mathcal{P}f}(z,\vec{b}_T)\,\gamma^\nu\Big]\Big\}\,.$$

- Equivalent to $\hat{W}_{\mathcal{P}}^{(1)\mu\nu}$ (noticing that $(n_\mu - \bar{n}_\mu)P_i^{\mu\nu} = \mathcal{O}(P_{hT}/Q)$)

$$W_{\mathcal{P}}^{(1)\mu\nu} - \hat{W}_{\mathcal{P}}^{(1)\mu\nu} = \frac{2z}{N_c}\sum_f\int \mathrm{d}^2\vec{b}_T\,\mathcal{H}_f^{(0)}(Q)\frac{\mathrm{i}}{Q}\left(\frac{\partial}{\partial b_\perp^\rho}\sqrt{\mathcal{S}(b_T)}\right)\sqrt{\mathcal{S}(b_T)}$$
$$\times\Big\{(\bar{n}^\nu - n^\nu)\,\mathrm{Tr}\Big[\gamma_\perp^\rho\,\hat{B}_f(x,\vec{b}_T)\,\gamma^\mu\,\hat{\mathcal{G}}_f(z,\vec{b}_T)\Big] + (n^\mu - \bar{n}^\mu)\,\mathrm{Tr}\Big[\hat{B}_f(x,\vec{b}_T)\,\gamma_\perp^\rho\,\hat{\mathcal{G}}_f(z,\vec{b}_T)\,\gamma^\nu\Big]\Big\}$$

- Same leading power functions appear, in momentum space

$$B_{\mathcal{P}f}(x,\vec{p}_T) = \frac{1}{2Q}\Big[p\!\!\!/_\perp\bar{n}\!\!\!/\,,\,B_f\Big] = \frac{1}{2Q}\Big\{f_1 p\!\!\!/_\perp - ih_1^\perp\frac{p_T^2\,[n\!\!\!/,\bar{n}\!\!\!/]}{2M_N}\Big\} + \dots$$

Subleading Operators: with $\mathcal{B}_\perp$ Insertion

- Fields and currents of definite helicity [Moult et al '15]

$$\mathcal{B}^a_{n\pm} = -\varepsilon_{\mp\mu}(n,\bar{n})\,\mathcal{B}^{a\mu}_{n\perp,\omega_c}\,,\ \chi^\alpha_{n\pm} = \frac{1\pm\gamma_5}{2}\chi^\alpha_{n,\omega_a}\,,\ J^{\bar{\alpha}\beta}_{\bar{n}n\pm} = \mp\sqrt{\frac{2}{\omega_a\,\omega_b}}\,\frac{\varepsilon^\mu_\mp(\bar{n},n)}{\langle n\mp|\bar{n}\pm\rangle}\,\bar{\chi}^{\bar{\alpha}}_{\bar{n}\pm}\,\gamma_\mu\chi^\beta_{n\pm}$$

$$\varepsilon^\mu_+(p,r) = \frac{\langle p+|\gamma^\mu|r+\rangle}{\sqrt{2}\langle rp\rangle}\,, \qquad \varepsilon^\mu_-(p,r) = -\frac{\langle p-|\gamma^\mu|r-\rangle}{\sqrt{2}[rp]}\,,$$

- The complete set of operators in the helicity basis [Feige et al '17]

$$O^{(1)a\,\bar{\alpha}\beta}_{1+-} = \mathcal{B}^a_{n+}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,-}\,, \qquad O^{(1)a\,\bar{\alpha}\beta}_{1-+} = \mathcal{B}^a_{n-}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,+}\,,$$
$$O^{(1)a\,\bar{\alpha}\beta}_{2--} = \mathcal{B}^a_{\bar{n}-}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,-}\,, \qquad O^{(1)a\,\bar{\alpha}\beta}_{2++} = \mathcal{B}^a_{\bar{n}+}\,J^{\bar{\alpha}\beta}_{\bar{n}n\,+}\,.$$

- Parity and charge conjugation invariance $\Rightarrow C^{(1)}_{\lambda_3\lambda_{12}} = C^{(1)}_{-\lambda_3-\lambda_{12}}$

$\Rightarrow$ Combination of helicity operators appear as

$$\mathcal{B}_{n+}J_{\bar{n}n-} + \mathcal{B}_{n-}J_{\bar{n}n+} = \frac{1}{\sqrt{\omega_a\omega_b}}\,\bar{\chi}_{\bar{n},\omega_b}\,[S^\dagger_{\bar{n}}S_n]\,\not{\mathcal{B}}_{\perp n,-\omega_c}\chi_{n,\omega_a}$$
$$\mathcal{B}_{\bar{n}-}J_{\bar{n}n-} + \mathcal{B}_{\bar{n}+}J_{\bar{n}n+} = \frac{1}{\sqrt{\omega_a\omega_b}}\,\bar{\chi}_{\bar{n},\omega_b}\not{\mathcal{B}}_{\perp\bar{n},\omega_c}\,[S^\dagger_{\bar{n}}S_n]\,\chi_{n,\omega_a}$$

- Same soft Wilson lines as LP since fields always appears as $S_n\chi_n$, $S_n\mathcal{B}_nS^\dagger_n$

Subleading Current: with $\mathcal{B}_{n\perp}$ Insertion

$$\frac{1}{\sqrt{\omega_a\omega_b}}\,\bar{\chi}_{\bar{n},\omega_b}[S_{\bar{n}}^\dagger S_n]\,\mathcal{B}\!\!\!/_{\perp n,-\omega_c}\chi_{n,\omega_a}\,, \qquad \frac{1}{\sqrt{\omega_a\omega_b}}\,\bar{\chi}_{\bar{n},\omega_b}\mathcal{B}\!\!\!/_{\perp\bar{n},\omega_c}\,[S_{\bar{n}}^\dagger S_n]\,\chi_{n,\omega_a}$$

- Hermiticity + $n \leftrightarrow \bar{n}$ symmetry: only one Wilson coefficient $C_f^{(1)}$
- Summing over helicities gives

$$\sum_{\lambda_e,\lambda_{12},\lambda_3} C_f^{(1)}\,\mathcal{B}_{\lambda_3}\,J_{\bar{n}n\lambda_{12}}J_{\lambda_e} \sim J_{\mathcal{B}}^{(1)\mu}\,J_{e\mu}$$

where $J_{e\mu} = \bar{e}\gamma_\mu e$ and (denoting $\xi = \omega_c/Q$),

$$\begin{aligned} J_{\mathcal{B}}^{(1)\mu} \sim (n^\mu + \bar{n}^\mu)\int & \mathrm{d}\omega_a \mathrm{d}\omega_b \mathrm{d}\omega_c\, C_f^{(1)}(Q,\xi) \\ & \times \Big[\delta(\omega_a+\omega_c-Q)\,\delta(\omega_b-Q)\,\bar{\chi}_{\bar{n},\omega_b}[S_{\bar{n}}^\dagger S_n]\mathcal{B}\!\!\!/_{\perp n,-\omega_c}\chi_{n,\omega_a} \\ & \quad + \delta(\omega_a-Q)\,\delta(\omega_b+\omega_c-Q)\,\bar{\chi}_{\bar{n},\omega_b}\mathcal{B}\!\!\!/_{\perp\bar{n},\omega_c}[S_{\bar{n}}^\dagger S_n]\chi_{n,\omega_a}\Big] \end{aligned}$$

Subleading Current: with $\mathcal{B}_{n\perp}$ Insertion

Denoting $\xi = \omega_c/Q$, define the q-g-q correlators as

$$\hat{\tilde{B}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T) \equiv Q\;\langle N|\,\big[\bar{\chi}^{\beta}_{n,\omega_a}\,\mathcal{B}^{\rho}_{\perp n,-\omega_c}\big](b^\mu_\perp)\,\chi^{\beta'}_{n}(0)\,|N\rangle\;,$$

$$\hat{\tilde{\mathcal{G}}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T) \equiv \frac{Q}{2z}\sum\!\!\!\!\!\!\int_X \langle 0|\,\big[\bar{\chi}^{\beta}_{\bar{n},\omega_b}\,\mathcal{B}^{\rho}_{\perp\bar{n},\omega_c}\big](b^\mu_\perp)\,|h,X\rangle\,\langle h,X|\,\chi^{\beta'}_{\bar{n}}(0)\,|0\rangle$$

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T) = \hat{\tilde{B}}^{\rho\,\beta'\beta}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\sqrt{\mathcal{S}(b_T)},$$
$$\tilde{\mathcal{G}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T) = \hat{\tilde{\mathcal{G}}}^{\rho\,\beta\beta'}_{\mathcal{B}\,\bar{f}}(z,\xi,\vec{b}_T)\sqrt{\mathcal{S}(b_T)}$$

$$W^{(1)\mu\nu}_{\mathcal{B}} = \frac{2z}{Q}\sum_f \int \mathrm{d}^2 b_T\, e^{\mathrm{i}\vec{q}_T\cdot\vec{b}_T}\int_0^1 \mathrm{d}\xi\,\mathcal{H}^{(1)}(Q,\xi)(n^\mu+\bar{n}^\mu)$$
$$\times\,\mathrm{Tr}\Big[\tilde{B}^{\rho}_{\mathcal{B}\,f}(x,\xi,\vec{b}_T)\,\gamma_\rho\,\mathcal{G}_f(z,\vec{b}_T)\,\gamma^\nu_\perp + B_f(x,\vec{b}_T)\,\gamma^\nu_\perp\,\tilde{\mathcal{G}}^{\rho}_{\mathcal{B}\,f}(z,\xi,\vec{b}_T)\,\gamma_\rho\Big] + \text{h.c.}\,.$$

$$\mathcal{H}^{(1)}(Q,\xi) = C^{(1)}_f(Q,\xi)\,C^{(0)}_f(Q)$$

In momentum space, $\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}f}$ can be decomposed as [Boer, Mulders, Pijlman '03] [Bacchetta, Mulders, Pijlman '04]

$$\tilde{B}^{\rho\,\beta'\beta}_{\mathcal{B}f}(x,\xi,\vec{p}_T) = \frac{xM_N}{2}\,\left\{\Big[(\tilde{f}^\perp - \mathrm{i}\tilde{g}^\perp)\frac{p_{\perp\sigma}}{M_N}\big(g^{\rho\sigma}_\perp - \mathrm{i}\epsilon^{\rho\sigma}_\perp\gamma_5\big) + \mathrm{i}(\tilde{h}+\mathrm{i}\,\tilde{e})\gamma^\rho_\perp\Big]\frac{\not{n}}{2}\right\}^{\beta'\beta} + \ldots$$

Vanishing Soft Contributions

- Subleading soft contributions exist in general, and are important in other processes
- In subleading SIDIS, we showed that they all vanish:

(1) Operators involving $\mathcal{B}_{s\perp}^{(n_i)\mu}$ yield

$\hat{\mathcal{S}}_1^\rho(b_\perp) \sim \mathrm{tr}\left\langle 0\middle| \left[S_n^\dagger(b_\perp)\, S_{\bar{n}}(b_\perp)\right] \left[S_{\bar{n}}^\dagger(0)\, S_n(0) g \mathcal{B}_{s\perp}^{(n)\rho}(0)\right] \middle| 0\right\rangle$

Vanish due to charge conjugation & parity invariance & Lorentz invariance & translation invariance of the vacuum

(2) Operators involving a $n \cdot \partial_s$, $n \cdot \mathcal{B}_s^{(\bar{n})}, \ldots$ give $\frac{\partial}{\partial b_s^{\mp}} \mathcal{S}(b_T, b_s^+ b_s^-)\Big|_{b_s^\pm \to 0}$,

which scales linear in $\bar{n}$ or n under RPI-III ($n \to e^\alpha n$, $\bar{n} \to e^{-\alpha}\bar{n}$) of SCET , thus vanishes

Vanishing Soft Contributions

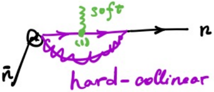
(3) SCET$_{\text{II}}$ Subleading Lagrangian insertions

$$W_{\mathcal{L}}^{(1)\mu\nu} \sim \langle N|J^{(0)\dagger\,\mu}(b)|h,X\rangle\,\langle h,X|\int \mathrm{d}^4x\mathrm{d}^4y\,T\big[J^{(0)\nu}(0)\mathcal{L}^{(1/2)}(x)\mathcal{L}^{(1/2)}(y)\big]|N\rangle$$
$$+\,\langle N|J^{(0)\dagger\,\mu}(b)|h,X\rangle\,\langle h,X|\int \mathrm{d}^4x\,T\big[J^{(0)\nu}(0)\mathcal{L}^{(1)}(x)\big]|N\rangle + \dots$$

Since μ, ν are transverse $\big(J^{(0)\mu} \sim (\gamma_\perp^\mu)^{\alpha\beta}\, C_f^{(0)}(Q)\, \bar\chi^\alpha_{\bar n,\omega_b}[S_{\bar n}^\dagger S_n]\,\chi^\beta_{n,\omega_a}\big)$, when contracting with $P_1^{\mu\nu} = -(\tilde t^\mu \tilde x^\nu + \tilde x^\mu \tilde t^\nu)$, $P_2^{\mu\nu} = \mathrm{i}\big(\tilde t^\mu \tilde x^\nu - \tilde x^\mu \tilde t^\nu\big)$, ... such contributions vanish

(4) $T\big[J_{\text{I}}^{(0)\mu}\mathcal{L}_{\text{I}}^{(1)}\big]$, $T\big[J_{\text{I}}^{(0)\mu}\mathcal{L}_{\text{I}}^{(2)}\big]$, $T\big[J_{\text{I}}^{(0)\mu}\mathcal{L}_{\text{I}}^{(1)}\mathcal{L}_{\text{I}}^{(1)}\big]$ in SCET$_{\text{I}}$

$\to$ hard scattering operators in SCET$_{\text{II}}$
Vanish since μ, ν in $J^{(0)}$ are (again) transverse



Results

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp \Big]$$ (Kinematic corrections)

$$- \mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp \Big]$$ (From the $\mathcal{P}_\perp$ operators)

$$+ \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp \Big) + \frac{2}{zQ}\Big(k_{px}\,f_1 \tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp \tilde{H} \Big)\Big]\Big\}$$ (From the $\mathcal{B}_\perp$ operators)

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f \int \mathrm{d}^2 p_T\,\mathrm{d}^2 k_T\,\delta^2\Big(\vec{q}_T + \vec{k}_T - \vec{p}_T\Big)\,\omega(\vec{k}_T,\vec{p}_T) \times \int_0^1 \mathrm{d}\xi\,\mathcal{H}_f(Q,(\xi))\,g_f(x,(\xi),k_T)\,D_f(z,(\xi),p_T)$$

- For example,

$$\mathcal{F}[k_{Tx}\mathcal{H}^{(1)}\tilde{f}^\perp D_1] = 2z\sum_f \int \mathrm{d}^2 p_T\,\mathrm{d}^2 k_T\,\delta^2\Big(\vec{q}_T + \vec{k}_T - \vec{p}_T\Big)\,k_{Tx} \times \int_0^1 \mathrm{d}\xi\,\mathcal{H}_f^{(1)}(Q,\xi)\,\tilde{f}_f^\perp(x,\xi,k_T)\,D_{1f}(z,p_T)$$

Results

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big]$$ (Kinematic corrections)

$$- \mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp\Big]$$ (From the $\mathcal{P}_\perp$ operators)

$$+ \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\tilde{D}^\perp + \frac{M_h}{M_N}k_{Tx}\,h_1^\perp\tilde{H}\Big)\Big]\Big\}$$

(From the $\mathcal{B}_\perp$ operators)

$$W_2 = \mathcal{F}\Big\{\mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(p_{Tx}\,\tilde{g}^\perp D_1 + \frac{M_N}{M_h}k_{Tx}\,\tilde{e}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{Tx}\,f_1\tilde{G}^\perp + \frac{M_h}{M_N}p_{Tx}\,h_1^\perp\tilde{E}\Big)\Big]\Big\}$$

$$\mathcal{F}[\omega\,\mathcal{H}\,g\,D] = 2z\sum_f\int \mathrm{d}^2p_T\,\mathrm{d}^2k_T\,\delta^2\big(\vec{q}_T + \vec{k}_T - \vec{p}_T\big)\,\omega(\vec{k}_T,\vec{p}_T) \times \int_0^1 \mathrm{d}\xi\,\mathcal{H}_f(Q,(\xi))\,g_f(x,(\xi),k_T)\,D_f(z,(\xi),p_T)$$

New in our results

- Vanishing of the subleading soft contributions
- Soft function, same as leading power (as conjectured in [Bacchetta et al '19])
- Two hard functions for all NLP structure functions, $\mathcal{H}^{(0)}(Q)$ and $\mathcal{H}^{(1)}(Q,\xi)$
- Dependence on ξ in $\mathcal{H}^{(1)}(Q,\xi)$ and the functions $\tilde{f}^\perp$, $\tilde{D}^\perp$, ...

Structure Functions with Full Spin Dependence

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\vec{P}_{hT}} = \frac{\pi\alpha^2}{Q^2}\frac{y}{z}\frac{\kappa_\gamma}{1-\epsilon}\Big[(L\cdot W)_{UU} + S_L(L\cdot W)_{UL} + \lambda_\ell(L\cdot W)_{LU} + \lambda_\ell S_L(L\cdot W)_{LL} + S_T(L\cdot W)_{UT} + \lambda_\ell S_T(L\cdot W)_{LT}\Big],$$

$$(L\cdot W)_{UU} = W_{UU,T} + \epsilon W_{UU,L} + \sqrt{2\epsilon(1+\epsilon)}\cos(\phi_h)W_{UU}^{\cos(\phi_h)} + \epsilon\cos(2\phi_h)W_{UU}^{\cos(2\phi_h)},$$

$$(L\cdot W)_{UL} = \sqrt{2\epsilon(1+\epsilon)}\sin(\phi_h)W_{UL}^{\sin(\phi_h)} + \epsilon\sin(2\phi_h)W_{UL}^{\sin(2\phi_h)},$$

$$(L\cdot W)_{LU} = \sqrt{2\epsilon(1-\epsilon)}\sin(\phi_h)W_{LU}^{\sin(\phi_h)},$$

$$(L\cdot W)_{LL} = \sqrt{1-\epsilon^2}W_{LL} + \sqrt{2\epsilon(1-\epsilon)}\cos(\phi_h)W_{LL}^{\cos(\phi_h)},$$

$$(L\cdot W)_{UT} = \sin(\phi_h-\phi_S)\left[W_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon W_{UT,L}^{\sin(\phi_h-\phi_S)}\right] + \epsilon\left[\sin(\phi_h+\phi_S)W_{UT}^{\sin(\phi_h+\phi_S)} + \sin(3\phi_h-\phi_S)W_{UT}^{\sin(3\phi_h-\phi_S)}\right] + \sqrt{2\epsilon(1+\epsilon)}\left[\sin(\phi_S)W_{UT}^{\sin(\phi_S)} + \sin(2\phi_h-\phi_S)W_{UT}^{\sin(2\phi_h-\phi_S)}\right],$$

$$(L\cdot W)_{LT} = \sqrt{1-\epsilon^2}\cos(\phi_h-\phi_S)W_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\left[\cos(\phi_S)W_{LT}^{\cos(\phi_S)} + \cos(2\phi_h-\phi_S)W_{LT}^{\cos(2\phi_h-\phi_S)}\right].$$

- We also have results for spin-dependent $\mathcal{O}(P_{hT}/Q)$ structure functions

Structure Function with Full Spin Dependence

- For example

$$W_{UT}^{\sin\phi_S} = \mathcal{F}\Big\{ -\frac{q_T}{2Q}\,\mathcal{H}^{(0)}\left(\frac{k_{Tx}}{M_N} f_{1T}^{\perp} D_1 - \frac{2p_{Tx}}{M_h} h_1 H_1^{\perp}\right) \text{(Kinematic corrections)}$$

$$+\,\mathcal{H}^{(0)}\left(-\frac{k_T^2 + \vec{k}_T\cdot\vec{p}_T}{2M_N Q} f_{1T}^{\perp} D_1 + \frac{p_T^2 + \vec{k}_T\cdot\vec{p}_T}{M_h Q} h_1 H_1^{\perp}\right) \text{(From the } \mathcal{P}_\perp \text{ operators)}$$

$$+\mathcal{H}^{(1)}\left[\frac{xM_N}{Q}\left(2\tilde{f}_T D_1 - \frac{\vec{k}_T\cdot\vec{p}_T}{M_N M_h}\left(\tilde{h}_T - \tilde{h}_T^{\perp}\right) H_1^{\perp}\right)\right.$$

$$\left.-\frac{M_h}{zQ}\left(2h_1\tilde{H} + \frac{\vec{k}_T\cdot\vec{p}_T}{M_N M_h}\left(g_{1T}\tilde{G}^{\perp} + f_{1T}^{\perp}\tilde{D}^{\perp}\right)\right)\right]\Big\} \text{(From the } \mathcal{B}_{n\perp} \text{ operators)}$$

Discussion

Anomalous dimensions

- Rapidity anomalous dimension is the same as at leading power

$$\tilde{B}_{\mathcal{B}\,f}^{\rho\,\beta'\,\beta}(x, \xi, \vec{b}_T, \mu, \zeta) = \hat{\tilde{B}}_{\mathcal{B}\,f}^{\rho\,\beta'\,\beta}(x, \xi, \vec{b}_T, \mu, \nu^2/\zeta)\sqrt{\mathcal{S}(b_T, \mu, \nu)},$$

$$\Rightarrow \frac{\mathrm{d}\log \tilde{B}_{\mathcal{B}\,f}^{\rho\,\beta'\,\beta}}{\mathrm{d}\log\zeta} = \frac{1}{4}\frac{\mathrm{d}\log\mathcal{S}}{\mathrm{d}\log\nu} = \frac{1}{4}\gamma_\nu(\mu, b_T)$$

- Anomalous dimension of $C_f^{(1)}$ have been calculated to one loop, with single log dependence on ξ [Beneke et al, '17 '18]

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}C_f^{(1)}(Q, \xi, \mu) = \int \frac{\mathrm{d}\xi'}{\xi'}\gamma_{f\,f'}^{(1)}(\xi, \xi', Q, \mu)C_{f'}^{(1)}(\xi', Q, \mu)$$

Discussion

- At leading order, $C_f^{(1)}$ is independent on ξ from tree level matching,
- ξ can be integrated in q-g-q correlators. $W_1, W_2, \cdots$ then fully agree with [Bacchetta et al '06] at leading order (after inclusion of the soft function, as conjectured in [Bacchetta et al '19])
- Anomalous dimension results for $C_f^{(1)}(Q, \xi, \mu)$ confirms the nontrivial ξ dependence

$\Rightarrow$ Disproves the simpler factorization theorem in [Bacchetta et al '19]

- Our results includes radiative corrections: fixed order + resumed logs
- LO $\xrightarrow{\text{Fact.}}$ NLO + LL + NLL + ...!
- Recent overlapping work on operator basis, perturbative corrections and anomalous dimensions [Vladimirov, Moos, Scimemi '21]. Complimentary to our results:
  - ▷ they did not present factorization formula for cross sections or prove absence of new soft effects
  - ▷ they calculated $C^{(1)}$ at $\mathcal{O}(\alpha_s)$, which is useful for improving precision

Summary & Outlook

$$W_1 = \mathcal{F}\Big\{-\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T\cdot\vec{k}_T}{M_N M_h}\,h_1^\perp H_1^\perp\Big] \qquad \text{(Kinematic corrections)}$$

$$-\,\mathcal{H}^{(0)}\Big[\frac{p_{Tx}+k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\,k_{Tx} + k_T^2\,p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp\Big] \qquad \text{(From the } \mathcal{P}_\perp \text{ operators)}$$

$$+\,\mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp\Big) + \frac{2}{zQ}\Big(k_{px}\,f_1\tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\,h_1^\perp\tilde{H}\Big)\Big]\Big\}$$

$$\text{(From the } \mathcal{B}_\perp \text{ operators)}$$

- Derived factorization of $W^{\mu\nu}$ at subleading power, including contribution from subleading operators with insertion of $\mathcal{P}_\perp$ and $\mathcal{B}_\perp$
- Showed the factorization formulae of subleading structure functions $W_{UU}^{\cos(\phi_h)}$, $W_{UL}^{\sin(\phi_h)}$, $W_{LU}^{\sin(\phi_h)}$, $W_{LL}^{\cos(\phi_h)}$, $W_{UT}^{\sin(\phi_S)}$, $W_{UT}^{\sin(2\phi_h-\phi_S)}$, $W_{LT}^{\cos(\phi_S)}$, $W_{LT}^{\cos(2\phi_h-\phi_S)}$, including contributions from the kinematic correction and the subleading operators
- Future Directions: phenomenology including perturbative and resummation effects for $W_{UU}^{\cos(\phi_h)}$, $W_{UL}^{\sin(\phi_h)}$, ...

Summary & Outlook

$$W_1 = \mathcal{F}\Big\{ -\frac{P_{hT}}{zQ}\,\mathcal{H}^{(0)}\Big[f_1 D_1 - \frac{2\,p_{Tx}\,k_{Tx} - \vec{p}_T \cdot \vec{k}_T}{M_N M_h}\, h_1^\perp H_1^\perp \Big]$$ (Kinematic corrections)

$$- \mathcal{H}^{(0)}\Big[\frac{p_{Tx} + k_{Tx}}{Q} f_1 D_1 + \frac{p_T^2\, k_{Tx} + k_T^2\, p_{Tx}}{Q M_N M_h} h_1^\perp H_1^\perp \Big]$$ (From the $\mathcal{P}_\perp$ operators)

$$+ \mathcal{H}^{(1)}\Big[\frac{2x}{Q}\Big(k_{Tx}\,\tilde{f}^\perp D_1 + \frac{M_N}{M_h} p_{Tx}\,\tilde{h}\,H_1^\perp \Big) + \frac{2}{zQ}\Big(k_{px}\, f_1 \tilde{D}^\perp + \frac{M_h}{M_N} k_{Tx}\, h_1^\perp \tilde{H} \Big)\Big]\Big\}$$ (From the $\mathcal{B}_\perp$ operators)

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Thanks for your attention!