A separable Bethe-Salpeter approach to deuteron structure

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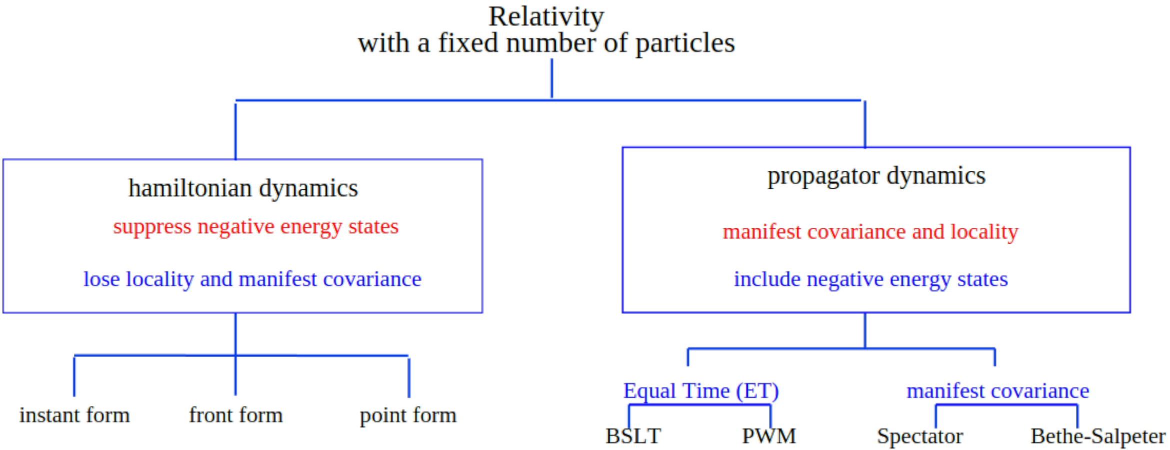
Introduction

- **Main idea**: deuteron structure in a separable Bethe-Salpeter approach.
  - **Separability** means the Bethe-Salpeter equation can be solved.
- Solving a Bethe-Salpeter equation has several benefits:
  - Ensures covariance.
  - Ensures correct normalization.
  - Allows two-body currents to be derived from Lagrangian.
- The true NN interaction isn't separable—need a model.
  - This talk is about such a model.
- I'm going to present some in-progress work here.
  - Details and results aren't all finalized.
  - Thoughts and suggestions are welcome!

The variety of approaches

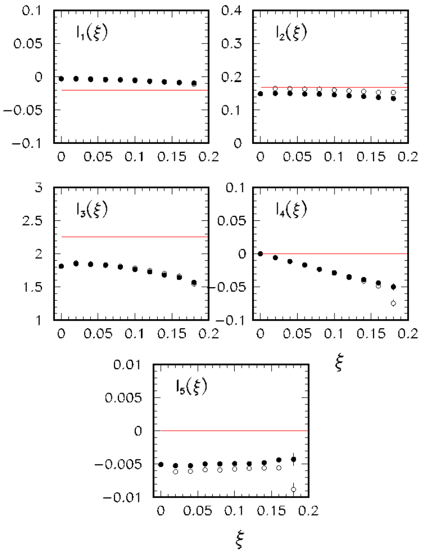


- Figure above from Gilman & Gross, AIP Conf. Proc. 603 (2001) 55
- This talk is about a **Bethe-Salpeter** approach. (but without locality)

- Generalized parton distributions exhibit **polynomiality**.

$$\int \mathrm{d}x\, x H_1(x,\xi,t) = \mathcal{G}_1(t) + \xi^2 \mathcal{G}_3(t) \qquad \text{etc.}$$

  - Required for unambiguous extraction of energy-momentum tensor from GPDs.
- Polynomiality requires covariance.
  - X. Ji, J. Phys. G24 (1998) 1181
- Finite Fock expansion (standard method) violates covariance.
  - Example: landmark calculation of Cano and Pire EPJA 19 (2004) 423



Non-local Lagrangian

- Adapted from **non-local NJL model**.
  - Bowler & Birse, Nucl. Phys. A582 (1995) 655
  - Modified to be a nucleon-nucleon interaction.
- V and T currents in *isosinglet* channel:

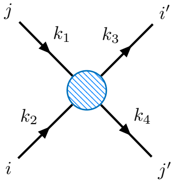
$$B^{\mu}_{Vn}(x) = \frac{1}{2}\int \mathrm{d}^4 z\, f_n(z)\psi^{\mathsf{T}}\left(x+\frac{z}{2}\right)C^{-1}\tau_2\gamma^{\mu}\psi\left(x-\frac{z}{2}\right)$$
$$B^{\mu\nu}_{Tn}(x) = \frac{1}{2}\int \mathrm{d}^4 z\, f_n(z)\psi^{\mathsf{T}}\left(x+\frac{z}{2}\right)C^{-1}\tau_2\,\mathrm{i}\sigma^{\mu\nu}\psi\left(x-\frac{z}{2}\right)$$

  - $f_n(z)$ a spacetime form-factor; regulates UV divergences.
  - $f_n(z) \to \delta^{(4)}(z)$ gives (local) four-point contact interaction.
- Interaction Lagrangian:

$$\mathscr{L}_I = \sum_{n=1}^{N}\left\{g_{Vn}B^{\mu}_{Vn}(B_{Vn\mu})^* + \frac{1}{2}g_{Tn}B^{\mu\nu}_{Tn}(B_{Tn\mu\nu})^*\right\}$$

Kernel

- Momentum-space Feynman rule for interactions:



$$= \sum_{n=1}^{N} \left\{ g_{Vn} \gamma^{\mu} C \otimes C^{-1} \gamma_{\mu} + \frac{g_{Tn}}{2} \sigma^{\mu\nu} C \otimes C^{-1} \sigma_{\mu\nu} \right\} \widetilde{f}_n(k_1 - k_2) \widetilde{f}_n(k_3 - k_4)$$

  - **Separable interaction**: initial & final momentum dependence factorize.
  - (isospin dependence suppressed to compactify formula)
- $\widetilde{f}_n(k)$ is Fourier transform of $f_n(z)$; I choose **Yukawa form**:

$$\widetilde{f}_n(k) \equiv \frac{\Lambda}{k^2 - \Lambda_n^2 + \mathrm{i}0}$$

  - Λ_n is the regulator scale (non-locality scale).
  - Each Λ_n can be different!

Quantum numbers in kernel

- Kernel encodes channels with multiple quantum numbers:

$$\gamma^\mu C \otimes C^{-1}\gamma_\mu = \left(\gamma^\mu - \frac{\not{p}p^\mu}{p^2}\right) C \otimes C^{-1} \left(\gamma_\mu - \frac{\not{p}p_\mu}{p^2}\right) + \frac{1}{p^2}\not{p}C \otimes C^{-1}\not{p}$$

spin-one

spin-zero

$$\sigma^{\mu\nu} C\otimes C^{-1}\sigma_{\mu\nu} = \frac{1}{p^2}\sigma^{\mu p} C \otimes C^{-1}\sigma_{\mu p} + \left(\sigma^{\mu\nu} - \frac{\sigma^{\mu p}p^\nu - \sigma^{\nu p}p^\mu}{p^2}\right) C \otimes C^{-1} \left(\sigma^{\mu\nu} - \frac{\sigma^{\mu p}p^\nu - \sigma^{\nu p}p^\mu}{p^2}\right)$$

even parity

odd parity

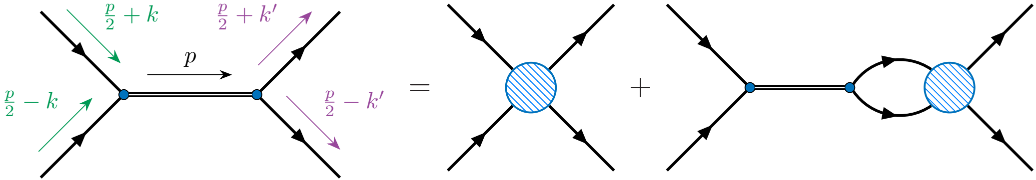
  - p is center-of-mass momentum (deuteron momentum)
- Need only structures with deuteron quantum numbers:

$$\gamma_V^\mu \equiv \gamma^\mu - \frac{\not{p}p^\mu}{p^2} \qquad\qquad \gamma_T^\mu \equiv \frac{\mathrm{i}\sigma_{\mu p}}{\sqrt{p^2}}$$

  - Other structures fully decouple in the T-matrix equation!

Bethe-Salpeter equation for the T-matrix

- Bethe-Salpeter equation (BSE) for T-matrix given by:



- **Separability** of interaction entails **separability** of T-matrix:

$$\mathcal{T}(p,k,k') = \sum_{\substack{n=1\\m=1}}^{N} \sum_{\substack{X=V,T\\Y=V,T}} \frac{\Lambda_n}{k'^2-\Lambda_n^2}\frac{\Lambda_m}{k^2-\Lambda_m^2}\left(\gamma_X^{\mu} C \otimes C^{-1}\gamma_{Y\mu}\right) T_{XY}^{nm}(p^2)$$

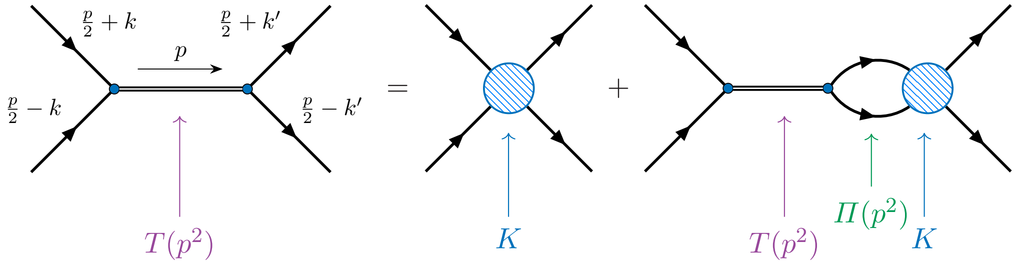
- Helpful to arrange $T_{XY}^{nm}(p^2)$ into a matrix.

Matrix form of the T-matrix

- Helpful to arrange $T^{nm}_{XY}(p^2)$ into a matrix.

$$T(p^2) = \begin{bmatrix} T^{11}_{VV}(p^2) & T^{11}_{VT}(p^2) & T^{12}_{VV}(p^2) & T^{12}_{VT}(p^2) & \cdots \\ T^{11}_{TV}(p^2) & T^{11}_{TT}(p^2) & T^{12}_{TV}(p^2) & T^{12}_{TT}(p^2) & \cdots \\ T^{21}_{VV}(p^2) & T^{21}_{VT}(p^2) & T^{22}_{VV}(p^2) & T^{22}_{VT}(p^2) & \cdots \\ T^{21}_{TV}(p^2) & T^{21}_{TT}(p^2) & T^{22}_{TV}(p^2) & T^{22}_{TT}(p^2) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

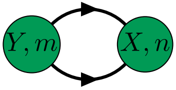
- $N \times N$ grid of 2×2 block matrices.
- With this, the Bethe-Salpeter equation will become an algebraic matrix equation!



Kernel

$$K = \begin{bmatrix} g_{V1} & 0 & 0 & \cdots \\ 0 & g_{T1} & 0 & \cdots \\ 0 & 0 & g_{V2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Bubble matrix

$\Pi(p^2) =$ 

$$T(p^2) = K - K\Pi(p^2)T(p^2)$$

- T-matrix solution given by:

$$T(p^2) = \left(1 + K\Pi(p^2)\right)^{-1} K$$

- Deuteron bound state pole exists where:

$$\det\big(1 + K\Pi(p^2 = M_{\mathrm{D}}^2)\big) = 0$$

- Deuteron vertex from residues at pole:

$$T(p^2 \approx M_{\mathrm{D}}^2) \approx -\frac{1}{p^2 - M_{\mathrm{D}}^2} \begin{bmatrix} \alpha_1^2 & \alpha_1\beta_1 & \alpha_1\alpha_2 & \cdots \\ \alpha_1\beta_1 & \beta_1^2 & \alpha_2\beta_1 & \cdots \\ \alpha_1\alpha_2 & \alpha_2\beta_1 & \alpha_2^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

  - α and β are coefficients in deuteron Bethe-Salpeter vertex.
  - Correct normalization automatically from solving Bethe-Salpeter equation.

Deuteron vertex

- The result of all this is a deuteron Bethe-Salpeter vertex:

$$\Gamma_{\mathrm{D}}^{\mu}(p,k) = \sum_{n=1}^{N} \frac{\Lambda_n}{k^2 - \Lambda_n^2 + \mathrm{i}0} \left\{ \alpha_n \gamma_V^{\mu} + \beta_n \gamma_T^{\mu} \right\} C \tau_2$$

  - Simple k dependence fixed by separable interaction.
  - Can be used to **covariantly** calculate all sorts of observables.
- Relationship to fundamental model parameters:

$$\{\Lambda_n, g_{Vn}, g_{Tn}\} \to \{M_{\mathrm{D}}, \alpha_n, \beta_n\}$$

  - One constraint from empirical M_{D} value.
  - More constraints from empirical properties, e.g. charge radius.
- *More constraints from good behavior of non-relativistic limit!*

Non-relativistic reduction

- Non-relativistic, momentum-space wave function:

$$\psi_{\mathrm{NR}}(\boldsymbol{k},\lambda) \sim \frac{-1}{\sqrt{8M_{\mathrm{D}}}} \frac{\bar{u}(\boldsymbol{k},s_1)(\Gamma_{\mathrm{D}}\cdot\varepsilon_\lambda)\bar{u}^{\mathsf{T}}(-\boldsymbol{k},s_2)}{\boldsymbol{k}^2 + m\epsilon_{\mathrm{D}}}$$

- Working out the Dirac matrix algebra and using the limit $\boldsymbol{k}^2 \ll m^2$ will give:

$$\psi_{\mathrm{NR}}(\boldsymbol{k},\lambda) = 4\pi\Big\{u(k)Y^{\lambda}_{101}(\hat{k}) + w(k)Y^{\lambda}_{121}(\hat{k})\Big\}$$

$$u(k) = \sum_{j=0}^{N} \frac{C_j}{\boldsymbol{k}^2 + B_j^2} \qquad w(k) = \sum_{j=0}^{N} \frac{D_j}{\boldsymbol{k}^2 + B_j^2}$$

$$C_j = C_j(\alpha_n, \beta_n, \Lambda_n) \qquad D_j = D_j(\alpha_n, \beta_n, \Lambda_n)$$

$$B_0 = \sqrt{m\epsilon_{\mathrm{D}}}$$

$$B_n = \Lambda_n$$

  - $u(k)$ is S-wave, $w(k)$ is D-wave.

The Sum-of-Yukawas parametrization

- This is a standard parametrization for deuteron wave functions:

$$u(k) = \sum_{j=0}^{N} \frac{C_j}{\boldsymbol{k}^2 + B_j^2} \qquad w(k) = \sum_{j=0}^{N} \frac{D_j}{\boldsymbol{k}^2 + B_j^2}$$

  - First used by Paris group, Lacombe *et al.*, PLB 101 (1981) 139
- Entails coordinate-space forms:

$$u(r) = \sum_{j=0}^{N} C_j \, \mathrm{e}^{-B_j r} \qquad w(r) = \sum_{j=0}^{N} D_j \, \mathrm{e}^{-B_j r} \left(1 + \frac{3}{B_j r} + \frac{3}{(B_j r)^2}\right)$$

- Requires $B_0 = \sqrt{\epsilon_{\mathrm{D}} m}$ to get the right large-r behavior. (**Check!**)
- Require the following for correct $r \to 0$ behavior:

$$\sum_{j=0}^{N} C_j = \sum_{j=0}^{N} D_j = \sum_{j=0}^{N} D_j B_j^{-2} = \sum_{j=0}^{N} D_j B_j^2 = 0$$

  - These impose *extra constraints* on separable kernel.
  - Need $N \geq 3$ for non-zero D-wave.

Two ideas

1. **Old idea**: use the $\{B_j, C_j, D_j\}$ from an existing wave function to build deuteron vertex
   - Numerically unstable; finely-tuned cancelations
   - Saw some success by refitting with fewer terms; less accuracy

2. **New idea**: build a *minimal consistent model*
   - Smallest N consistent with non-zero D-wave
   - Fit parameters to static deuteron properties
   - **I'll pursue this idea here**

Minimal separable model

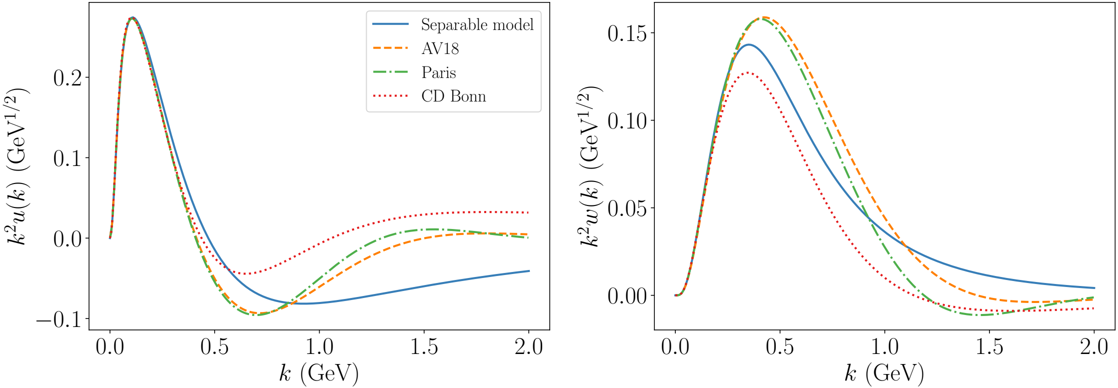
- One typically uses $B_j = B_0 + B'j$ (just 1 parameter for the B_j's).
- $\boldsymbol{N = 3}$ smallest model with non-zero D-wave and correct $r \to 0$ behavior.
  - $2N + 3 = 8$ parameters.
- 4 constraints from $r \to 0$ behavior, 1 from normalization.
  - $2N - 2 = 4$ *free* parameters.
- Empirical D/S asymptotic ratio can add another constraint:

$$\lim_{r\to\infty} \frac{w(r)}{u(r)} = \frac{D_0}{C_0} = \eta_{D/S} = 0.0256(4)$$

- Last 3 parameters can be set with 3 electromagnetic properties.
  - Charge radius (r_d), magnetic moment (μ_d), quadrupole moment (Q_d)
- One can obtain the kernel parameters from the Yukawa parametrization:

$$\{C_j\}, \{D_j\} \to \{\alpha_n\}, \{\beta_n\} \to \{g_{Vn}\}, \{g_{Tn}\}$$

Non-relativistic wave function



- Softer D-wave than AV18 & Paris
- Harder D-wave than CD-Bonn

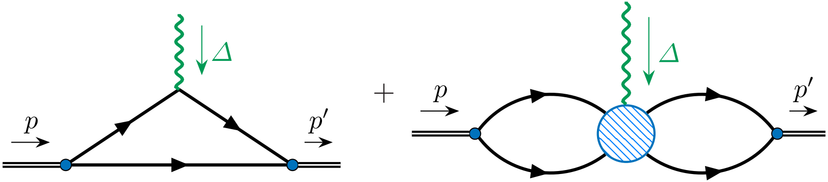
Things to do with this framework

- Electromagnetic form factors (**obtained!**)
- Collinear parton distributions (**obtained!**)
  - Includes b_1 structure function and EMC ratio.
- Gravitational form factors (in progress)
  - Manifest covariance helpful here.
  - Previous non-covariant work (AF & Cosyn, PRD) found inconsistencies in EMT components.
  - Currently hit a snag in separable model calculations (open to advice/suggestions!)
- Generalized parton distributions (in progress)
  - GPDs are the **main goal** of this project.
  - Existing deuteron GPDs violate polynomiality.
  - Manifest covariance of this framework *guarantees* polynomiality.

Electromagnetic current of deuteron

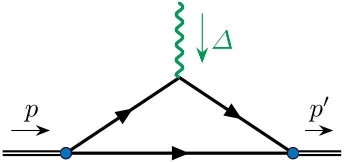
- Sum of nucleon impulse and two-body currents:

$$J_D^\mu(p, p') =$$

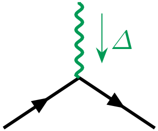


  - Two-body current uniquely determined by gauge invariance.
  - Non-local interaction requires Wilson lines.
- Diagrams can be evaluated *exactly* in the separable model!
  - Symbolic algebra program needed though—results are *long* (hundreds of lines of generated Fortran code)
  - Results are covariant too.

- Nucleon impulse given by **triangle diagram**



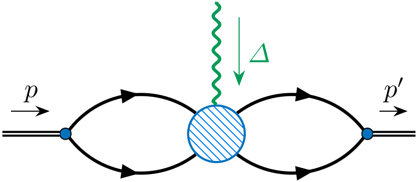
- Can use standard nucleon form factors in vertex;



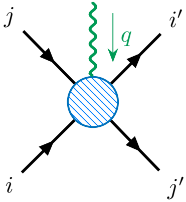
$$= \gamma^\mu F_{1N}(t) + \frac{\mathrm{i}\sigma^{\mu\Delta}}{2m_N} F_{2N}(t)$$

  - Using Kelly parametrization; meaning to try Ye *et al.*

- Two-body currents given by **bicycle diagram**



- Derived from Wilson lines in vertex:



$$= -e \sum_X g_X \left\{ \widetilde{h}^\mu_X(k,q) \widetilde{f}_X(k') - \widetilde{f}_X(k) \widetilde{h}^\mu_X(k',q) \right\} \left(\gamma^\nu_X C \tau_2 \right)_{i'j'} \left(C^{-1} \tau_2 \overline{\gamma}_{X\nu} \right)_{ij} F_{1N}(q^2)$$

$$\widetilde{h}^\mu_X(k,q) = \int_{-1}^{+1} \mathrm{d}\tau \, \frac{\mathrm{sgn}(\tau) \Lambda_X \left(k^\mu - \tau \frac{q^\mu}{2} \right)}{\left[\left(k - \tau \frac{q}{2} \right)^2 - \Lambda_X^2 \right]^2}$$

Why F_{1N}?

- A free Lagrangian for pointlike nucleons might look like:

$$\mathscr{L} = \overline{\psi}(x)\left(\frac{\mathrm{i}}{2}\overleftrightarrow{\not\partial} - e\not{A}(x)\right)\psi(x) - \frac{e\kappa}{4m_N}F_{\mu\nu}(x)\overline{\psi}(x)\sigma^{\mu\nu}\psi(x)$$

  - Effectively, $F_{1N}(t) = 1$ and $F_{2N}(t) = \kappa$.
  - Anomalous magnetic moment in non-minimal Puali coupling.
- Smearing this over spacetime might look like:

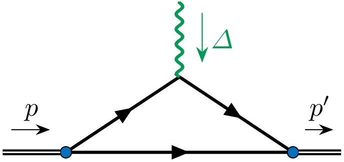
$$\mathscr{L} = \frac{\mathrm{i}}{2}\overline{\psi}(x)\overleftrightarrow{\not\partial}\psi(x) - e\int \mathrm{d}^4y\, \widetilde{F}_{1N}(y)\overline{\psi}(x)\not{A}(x+y)\psi(x)$$
$$- \frac{e}{4m_N}\int \mathrm{d}^4y\, \widetilde{F}_{2N}(y)F_{\mu\nu}(x+y)\overline{\psi}(x)\sigma^{\mu\nu}\psi(x)$$

  - Wilson lines should be smeared if minimal coupling is.
  - F_{1N} is what smears the non-minimal coupling.
- Foreshadowing: it's less clear what to do for gravitational structure of vertex.

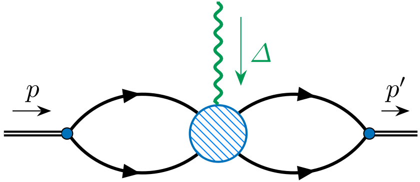
Electromagnetic form factors

- Standard form factor breakdown:

$$J_D^{\mu}(p,p') =$$



$$+$$



$$= -2p^{\mu}(\varepsilon \cdot \varepsilon'^{*})G_1(t) + \left[\varepsilon'^{*\mu}(\varepsilon \cdot \Delta) - \varepsilon^{\mu}(\varepsilon'^{*} \cdot \Delta)\right]G_2(t) + \frac{p^{\mu}}{M_{\mathrm{D}}^2}(\varepsilon \cdot \Delta)(\varepsilon'^{*} \cdot \Delta)G_3(t)$$

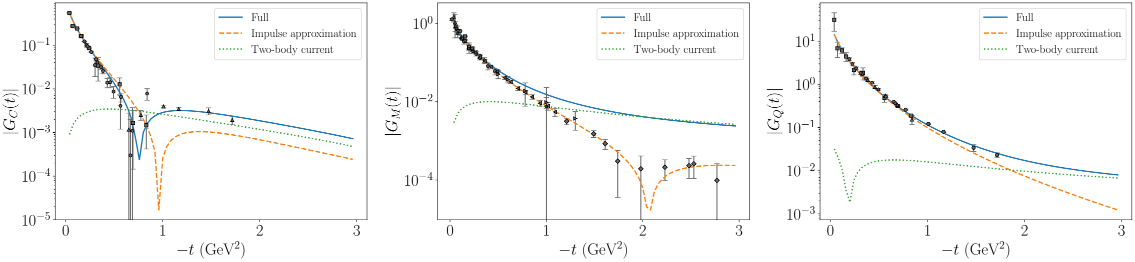
- Often use Sachs-like form factors:

$$G_C(t) = G_1(t) - \frac{t}{6M_{\mathrm{D}}^2}G_Q(t)$$ **Coulomb** form factor

$$G_M(t) = G_2(t)$$ **magnetic** form factor

$$G_Q(t) = G_1(t) - G_2(t) + \left(1 - \frac{t}{4M_{\mathrm{D}}^2}\right)G_3(t)$$ **quadrupole** form factor

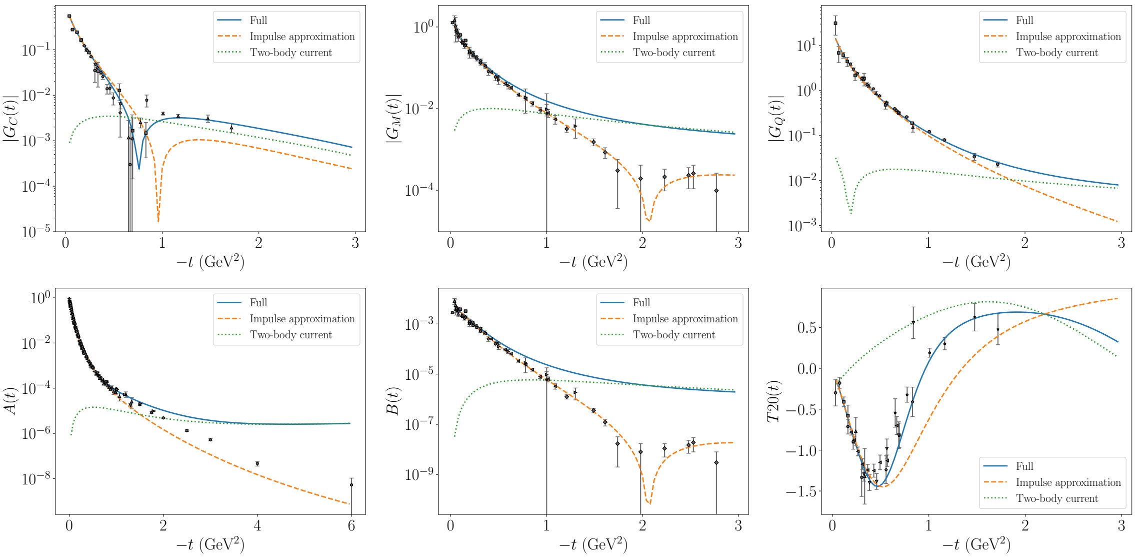
Model results for form factors



	Empirical	Model (total)	Impulse approx.	Two-body current
r_d (fm)	2.12799	2.126	2.127	-0.0776
μ_d (μ_N)	0.8574382284	0.876	0.876	0
Q_d (e-fm^2)	0.2859	0.296	0.286	0.010

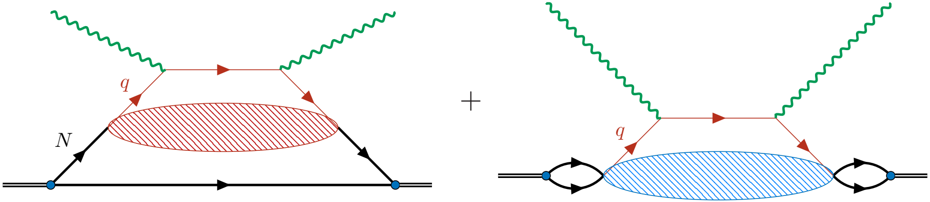
- Model has mixed success.
  - Better at smaller $-t$.
- Two-body currents are significant.

Electromagnetic structure

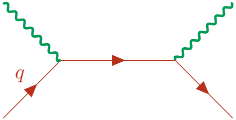


Virtual Compton scattering amplitude

- Sum of **convolution** and **two-body current** diagrams:



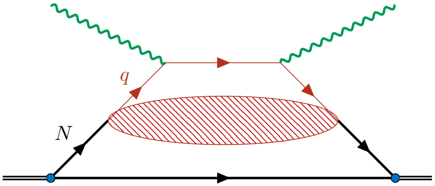
- Diagrams evaluated in the Bjorken limit:



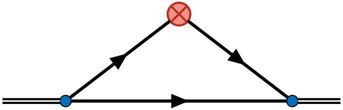
$$= \xrightarrow[Q^2\to\infty]{} \sum_q e_q^2 \frac{1}{2} \int \frac{\mathrm{d}z}{2\pi}\, \mathrm{e}^{\mathrm{i}x_D p^+ z^-}\, \bar{q}\left(-\frac{z^-}{2}\right) \gamma^+ q\left(\frac{z^-}{2}\right)$$

  - Here $0 < x_D < 1$, in contrast to usual normalization.
  - $0 < x_{\mathrm{Bj}} = \frac{M_d}{m_N} x_D < \frac{M_d}{m_N} \approx 2$ is the usual variable.
  - x_D is easier to use in calculations.
  - Compare empirical data in terms of x_{Bj}.

- Effective **triangle diagram** (Bjorken limit):



$$\xrightarrow[Q^2\to\infty]{}$$



- Convolution formula results:

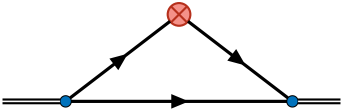
$$H_i(x_D,\xi,t,Q^2;\lambda) = \sum_{N=p,n}\int_{-1}^{1}\frac{\mathrm{d}y}{|y|}\left[h_i(y,\xi,t;\lambda)H_N\left(\frac{x_D}{y},\frac{\xi}{y},t,Q^2\right)\right.$$
$$\left.+\,e_i(y,\xi,t;\lambda)E_N\left(\frac{x_D}{y},\frac{\xi}{y},t,Q^2\right)\right]$$

  - Forward limit ($t\to 0$) gives standard PDF convolution:

$$q_d(x_D,Q^2;\lambda) = \sum_{N=p,n}\int_{x_D}^{1}\frac{\mathrm{d}y}{y}f(y;\lambda)q_N\left(\frac{x_D}{y},Q^2\right)$$

Light cone density (triangle diagram)

- **Light cone density**: a PDF assuming pointlike nucleons.



$$\xrightarrow[\text{pointlike nucleons}]{} f(y;\lambda)$$

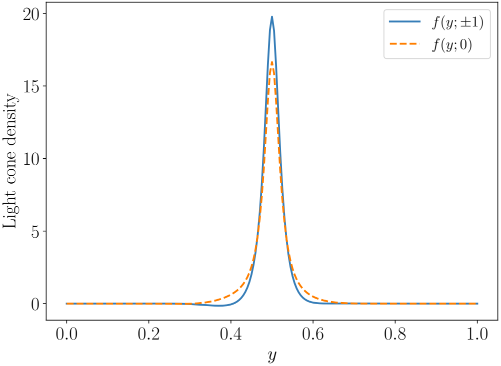
- Nucleon sum rule obeyed:

$$\sum_{N=p,n} \int_0^1 \mathrm{d}y\, f(y;\lambda) = 2$$

- Momentum sum rule *seemingly* violated!

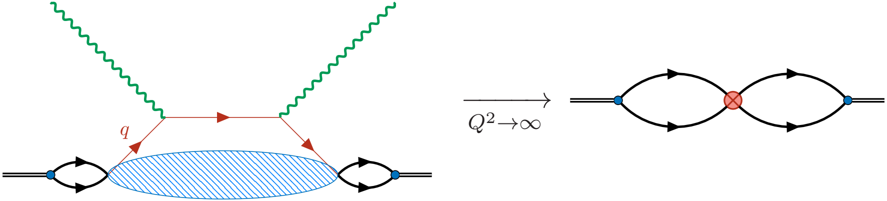
$$\sum_{N=p,n} \int_0^1 \mathrm{d}y\, y f(y;\lambda) = \begin{cases} 1.0042 & : \quad \lambda = \pm 1 \\ 0.9978 & : \quad \lambda = 0 \end{cases}$$

  - **Interaction carries momentum**

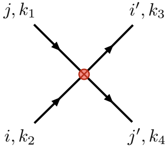


Bicycle diagram

- Effective **bicycle diagram** (Bjorken limit):



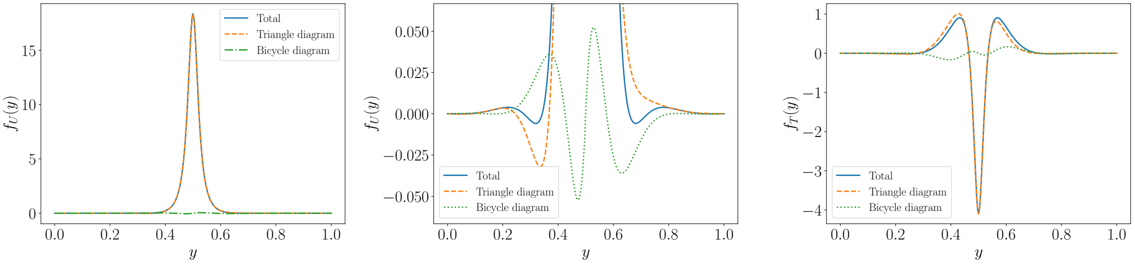
- New Feynman rule for operator insertion:



$$= -\frac{1}{2}\sum_X g_X \mathcal{S}\Big\{\widetilde{h}^+_X(k,0)\big(\delta(xp^+-k_1^+)+\delta(xp^+-k_2^+)\big)\widetilde{f}_X(k')$$

$$- \widetilde{f}_X(k)\widetilde{h}^+_X(k',0)\big(\delta(xp^+-k_3^+)+\delta(xp^+-k_4^+)\big)\Big\}\big(\gamma^\nu_X C\tau_2\big)_{i'j'}\big(C^{-1}\tau_2\overline{\gamma}_{X\nu}\big)_{ij}$$

  - …assuming pointlike nucleon.
  - Use convolution formula to fold in NN vertex structure???



- Momentum sum rule obeyed.
  - Saved by the bicycle diagram!
  - Also makes LCD symmetric.
- Slight negative support.
  - Either a flaw with the framework …
  - …or a feature of renormalization?
    cf. Collins, Rogers & Sato, PRD (2022)

$$f_U(y) = \frac{f(y,0) + f(y,+1) + f(y,-1)}{3}$$

$$f_T(y) = f(y,0) - \frac{f(y,+1) + f(y,-1)}{2}$$

Collinear parton distributions

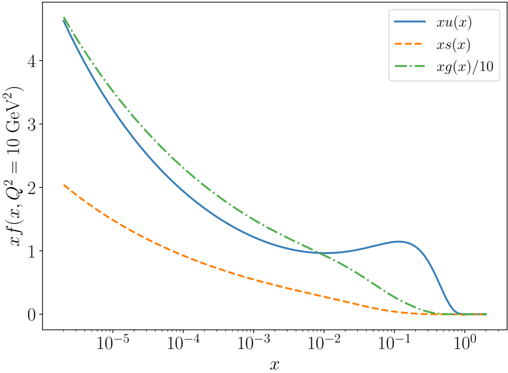
- Deuteron PDFs via convolution:

$$q_d(x_D, Q^2; \lambda) = \sum_{N=p,n} \int_{x_D}^{1} \frac{\mathrm{d}y}{y}\, q_N\left(\frac{x_D}{y}, Q^2\right) f(y;\lambda)$$

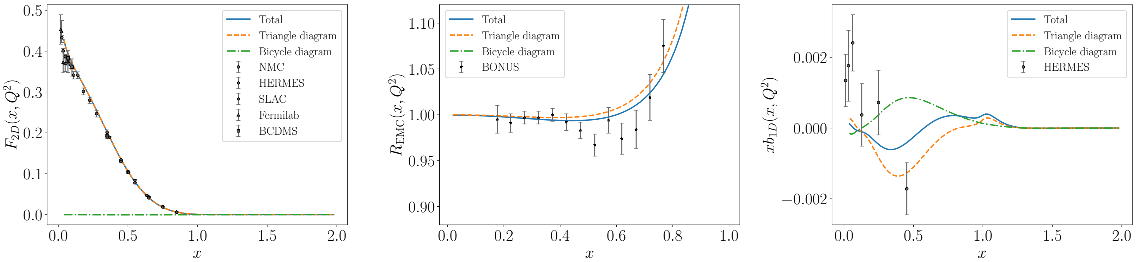
- Use JAM PDFs for nucleon.
  - C. Cocuzza *et al.*, PRD106 (2022) L031502
- Same PDF for triangle & bicycle diagrams.
  - Plot in terms of

$$x = x_{\mathrm{Bj}} \equiv \frac{Q^2}{2m_N\nu} = \frac{M_d}{m_N} x_D \approx 2x_D$$

    - This is the *standard* x variable.



DIS structure functions



- Use free nucleon PDFs inside both triangle & bicycle diagrams.
  - Use JAM22 PDFs.
- **Unpolarized** $F_{2D}(x_{\rm Bj}, Q^2)$ structure function.
  - Looks reasonable.
  - But no EMC effect when using free PDFs.
- **Tensor-polarized** $b_{1D}(x_{\rm Bj}, Q^2)$ structure function looks standard.
  - Total (blue curve) looks similar to Cosyn &al., PRD (2017).
  - Can't explain HERMES b_1 data.

- The energy-momentum tensor describes **density** and **flow** of energy & momentum.

Energy density

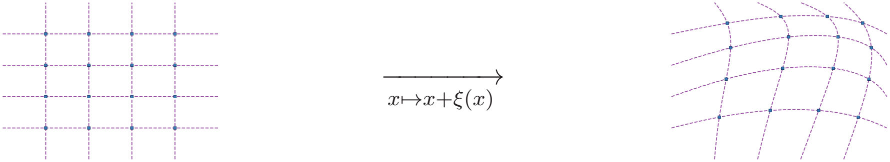
Momentum densities

$$T^{\tilde{\mu}\nu}(x) = \begin{bmatrix} T^{00}(x) & T^{01}(x) & T^{02}(x) & T^{03}(x) \\ T^{10}(x) & T^{11}(x) & T^{12}(x) & T^{13}(x) \\ T^{20}(x) & T^{21}(x) & T^{22}(x) & T^{23}(x) \\ T^{30}(x) & T^{31}(x) & T^{32}(x) & T^{33}(x) \end{bmatrix}$$

Energy fluxes

Stress tensor

- Conserved current from *local* spacetime translations (**Noether's second theorem**):



  - **Noether's theorems**: symmetries imply conservation laws
  - *Local* translation: move spacetime differently everywhere

- The **energy-momentum tensor** is a response to these deformations

$$\Delta S_{\mathrm{QCD}} = \int \mathrm{d}^4x \, T^{\mu\nu}_{\mathrm{QCD}}(x) \partial_\mu \xi_\nu(x)$$

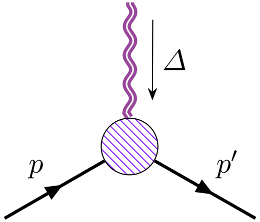
  - Conserved if the action is invariant
  - Basically, equivalent to doing a gravitational gauge transform.

AF, Phys. Rev. D 106 (2022) 125012

- The energy-momentum tensor is parametrized using **gravitational form factors**
  - It's just a name.
  - The energy-momentum tensor is the source of gravitation
  - But we don't really use gravitation to measure them
- Spin-zero example:

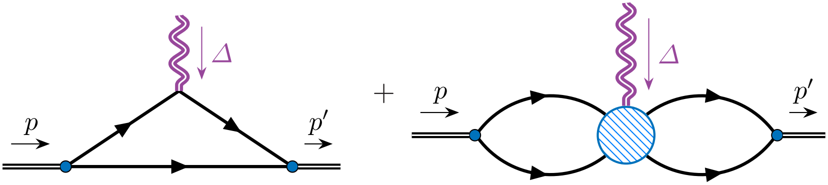
$$\langle p'|\hat{T}^{\mu\nu}(0)|p\rangle = 2P^\mu P^\nu A(t) + \frac{1}{2}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})D(t)$$

  - $A(t)$ encodes momentum density
  - $D(t)$ encodes stress distributions (anisotropic pressures)
  - Mix of both encodes energy density



- Sum of nucleon impulse and two-body currents:

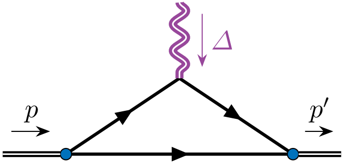
$T_D^{\mu\nu}(p,p') =$



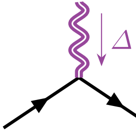
  - Two-body current uniquely determined via Noether's second theorem.
- Diagrams can be evaluated *exactly* in the separable model!
  - Symbolic algebra program needed though—results are *long* (hundreds of lines of generated Fortran code)
  - Results are covariant too.

Energy-momentum currents: triangle diagram

- Nucleon impulse given by **triangle diagram**



- Can use standard nucleon form factors in vertex;

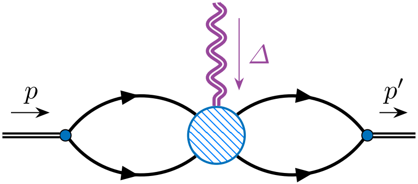


$$= \frac{\gamma^{\{\mu}P^{\nu\}}}{2}A_N(t) + \frac{\mathrm{i}P^{\{\mu}\sigma^{\nu\}\Delta}}{4m_N}B_N(t) + \frac{\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu}}{4m_N}D_N(t)$$

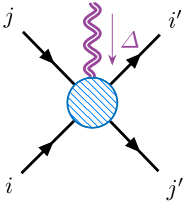
  - Using Mamo-Zahed model for form factors.

Energy-momentum currents: bicycle diagram

- Two-body currents given by **bicycle diagram**



- Derived from Noether's second theorem—*assuming pointlike nucleons*:



$$= -\frac{1}{2}\sum_X g_X \left\{ \widetilde{h}_X^{*\{\mu}(k,q)k^{\nu\}}\,\widetilde{f}_X(k') - \widetilde{f}_X(k)\widetilde{h}_X^{*\{\mu}(k',q)k'^{\nu\}} \right\}$$

$$\times \left(\gamma_X^{\nu} C\tau_2\right)_{i'j'} \left(C^{-1}\tau_2\overline{\gamma}_{X\nu}\right)_{ij} - g^{\mu\nu} \times \text{kernel}$$

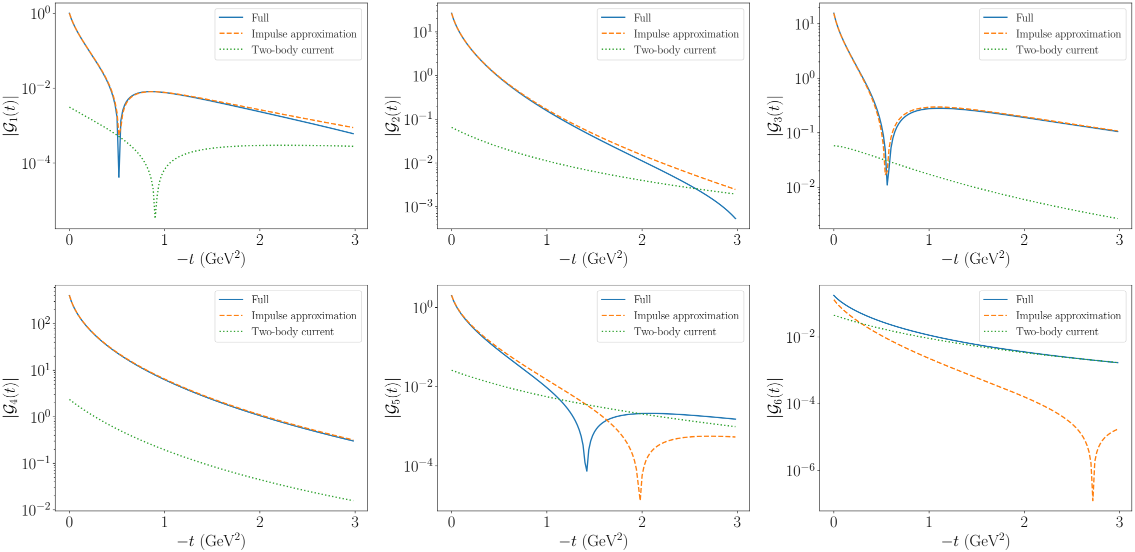
  - **How to fold in nucleon structure?**
  - Trying $A_N(t)$ at first but there are issues.

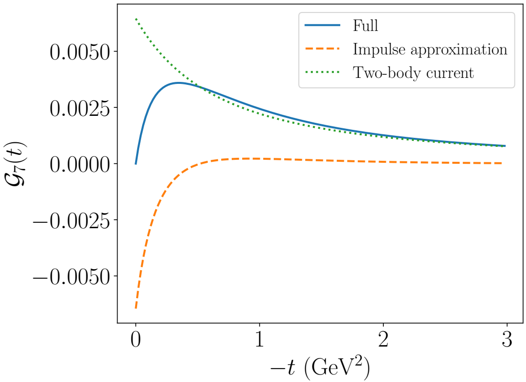
- Spin-one breakdown:

$$\begin{aligned}
\langle p', \lambda' | T_{\mu\nu}(0) | p, \lambda \rangle = & -2 P_\mu P_\nu \left[(\epsilon'^* \epsilon) \mathcal{G}_1(t) - \frac{(\Delta\epsilon'^*)(\Delta\epsilon)}{2M_\mathrm{D}^2} \mathcal{G}_2(t) \right] \\
& - \frac{1}{2} (\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu}) \left[(\epsilon'^* \epsilon) \mathcal{G}_3(t) - \frac{(\Delta\epsilon'^*)(\Delta\epsilon)}{2M_\mathrm{D}^2} \mathcal{G}_4(t) \right] \\
& + P_{\{\mu} \left(\epsilon'^*_{\nu\}} (\Delta\epsilon) - \epsilon_{\nu\}} (\Delta\epsilon'^*) \right) \mathcal{G}_5(t) \\
& + \frac{1}{2} \left[\Delta_{\{\mu} \left(\epsilon'^*_{\nu\}} (\Delta\epsilon) + \epsilon_{\nu\}} (\Delta\epsilon'^*) \right) - \epsilon'^*_{\{\mu} \epsilon_{\nu\}} \Delta^2 - g_{\mu\nu} (\Delta\epsilon'^*)(\Delta\epsilon) \right] \mathcal{G}_6(t) \\
& + \epsilon'^*_{\{\mu} \epsilon_{\nu\}} M_\mathrm{D}^2 \mathcal{G}_7(t) + g_{\mu\nu} M_\mathrm{D}^2 (\epsilon'^* \epsilon) \mathcal{G}_8(t) + \frac{1}{2} g_{\mu\nu} (\Delta\epsilon'^*)(\Delta\epsilon) \mathcal{G}_9(t)
\end{aligned}$$

  - Well, that's a lot. (Nine form factors!)
  - $\mathcal{G}_{7-9}(t) = 0$ required by energy/momentum conservation!

The conserved form factors





- $\mathcal{G}_7(t)$ should vanish at all t.
  - Follows from energy conservation.
- Only works out at $t = 0$.
- $t = 0$ values related to LCD (**check!**)

$$\int_0^1 \mathrm{d}y\, y\, f_T^{\mathrm{tri}}(y) = \mathcal{G}_7^{\mathrm{tri}}(t)$$

$$\int_0^1 \mathrm{d}y\, y\, f_T^{\mathrm{bi}}(y) = \mathcal{G}_7^{\mathrm{bi}}(t)$$

- Part of the t dependence from nucleon form factors.
  - Was I wrong to multiply bicycle diagram by $A_N(t)$?
  - If so, then what's right—and how to derive?
- Important to resolve before moving on to GPDs—$\mathcal{G}_7(t)$ appears in GPD sum rules!

Outlook

- Presented a covariant model of deuteron structure.
  - Separable kernel.
  - Bethe-Salpeter equation solvable in Minkowski spacetime.
  - Covariance means GPDs *will obey polynomiality*.

- Reproduced known deuteron properties in this framework.
  - Necessary sanity check.
  - **Two-body currents** (bicycle diagrams) must be accounted for!

- Much more to be done:
  - Energy momentum tensor and gravitational form factors.
  - **Generalized parton distributions** (the main purpose of this project!)

Thank you for your time!