GPDs and nucleon spin structures

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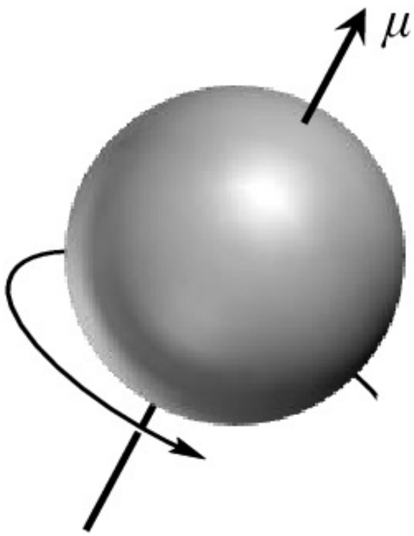
YG, X. Ji and K. Shiells [2109.10373]
YG, X. Ji, B. Kriesten and K. Shiells [2202.11114]

Outline

- Spin structure of nucleons and GPDs
- Angular momentum from GPDs
- Deeply virtual Compton scattering
- Some numeric studies with JLab kinematics
- Conclusion

Proton spin puzzle

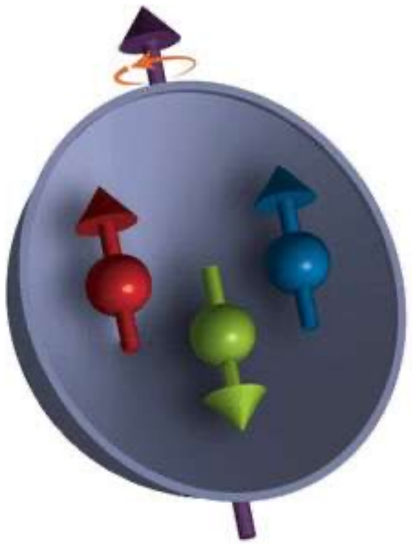
Our understanding of the nucleon spin has developed a lot.



Rutherford atomic model

Spin ½ point-like nucleus ?

$$(i\partial\!\!\!/ - m)\psi = 0$$



Naive quark model

Adding three spin ½ quarks?

European Muon Collaboration (EMC)

Quarks and gluons in QCD

Complicated composite particles!

Mechanical properties of nucleons

The mechanical properties of nucleon are encoded in the QCD energy momentum tensor.

$$\langle P' \left| T^{\mu\nu}_{q,g} \right| P \rangle = \bar{u}\left(P'\right) \Bigg[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\frac{\bar{P}^{(\mu} i\sigma^{\nu)}\,\alpha_\alpha}{2M_N}$$

$$+ C_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M_N} + \bar{C}_{q,g}(t) M_N g^{\mu\nu} \Bigg] u(P)$$

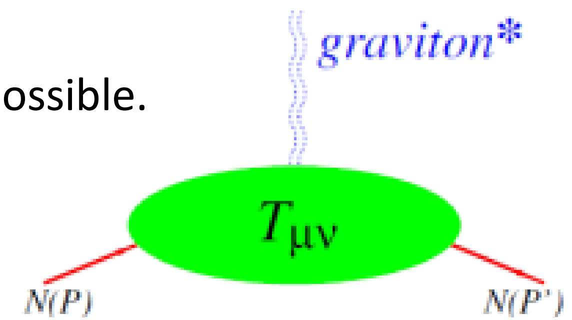
X. Ji 1996

Momentum $P_{q/g}(t) = A_{q/g}(t)$

Angular momentum (AM) $J_{q/g}(t) = \frac{1}{2}\left(A_{q/g}(t) + B_{q/g}(t)\right)$

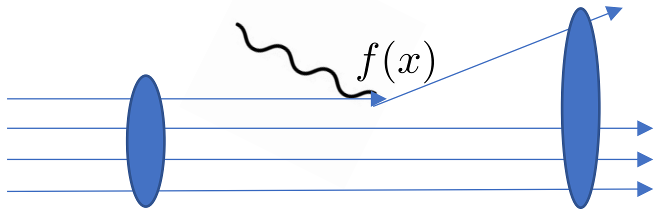
Directly probing the energy momentum tensor seems to be impossible.

Asymptotic freedom!



Parton and twist

$\lim_{P \to \infty}$



Feynman's parton model

in infinite momentum frame (IMF).

Nucleon structures are described by partons which can be probed in experiment.

QCD Sum rule – summing over all contributions of parton.

Yet we don't have particles with infinite momentum practically.

$$f(x, Q) = f^{(2)}(x) + \frac{\Lambda}{Q} f^{(3)}(x) + \cdots$$

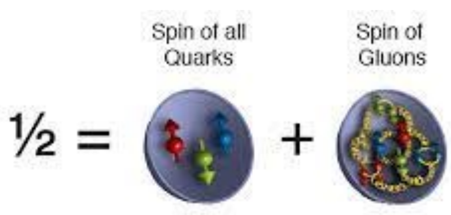
Leading twist (twist-two) – enhanced by boost (move along with nucleon)

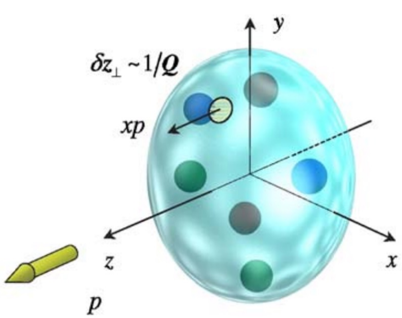
Sub-leading twist (twist-three) – boost invariant

Spin and nucleon 3-D structure

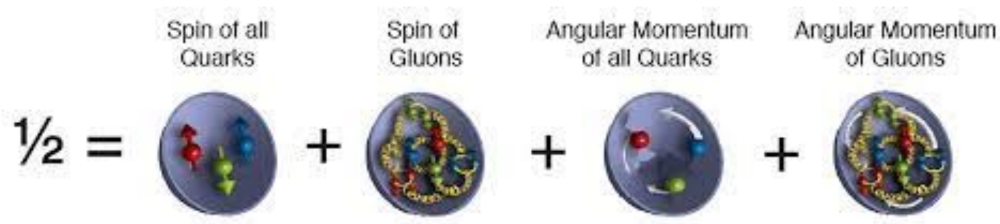
Simple parton picture is not enough for the nucleon spin!

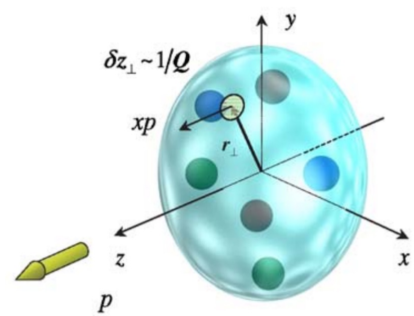
Without transverse dimension





With transverse dimension

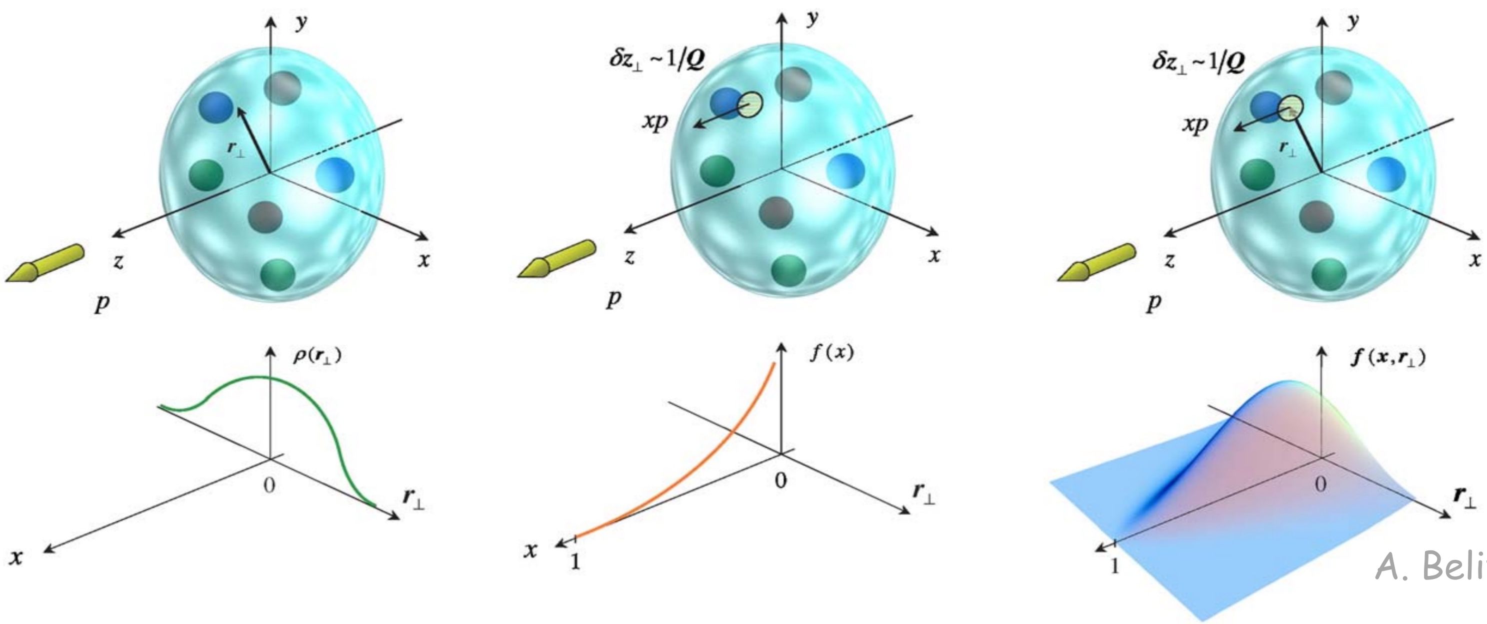




AM sum rules are related to parton with trans. displacement!

Generalized parton distributions

GPDs are essentially the combination of elastic form factors and PDFs.



A. Belitsky et. al. 2005

$$F(\Delta) \qquad\qquad f(x) \qquad\qquad f(x,\Delta) = f(x,\xi,t=\Delta^2)$$

$$\rho(r) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3} F(\Delta) e^{i\Delta\cdot r}$$

$$\rho(x, r_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} f(x,\Delta_\perp) e^{i\Delta_\perp\cdot r_\perp}$$

Nucleon tomography in the impact parameter space

M. Burkardt 2003

Angular momentum from GPDs

AM sum rule and GPDs

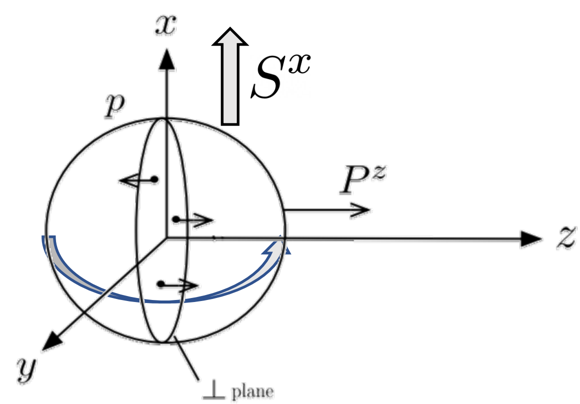
Therefore, the mechanical properties can be expressed as the sum of all partons (GPDs)

$$A_{q/g}(t) = \int \mathrm{d}x \; x H_{q/g}(x, \xi = 0, t) \qquad B_{q/g}(t) = \int \mathrm{d}x \; x E_{q/g}(x, \xi = 0, t)$$

Ji sum rule $$J_{q/g}(t) = \int \mathrm{d}x \frac{1}{2} x \left(H_{q/g}(x, \xi, t) + E_{q/g}(x, \xi, t) \right)$$

$H_{q/g}(x, \xi, t), E_{q/g}(x, \xi, t)$: Measurable twist-two GPDs.

- Independence of frame and gauge choices
- Apply to both trans. and long. polarization (Covariance)
- Parton AM densities in trans. polarization



Decomposition of spin

The total angular momentum can be decomposed into the spin and the orbital AM part.

$$L_q = J_q - \frac{1}{2}\Delta\Sigma$$

(Gluons?)

Ji 1996

Ji sum rule

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

- Covariant and frame-independent
- Apply to both trans. and long. polarization
- Parton AM densities in trans. polarization

Jaffe & Manohar 1990

Jaffe-Manohar Sum rule

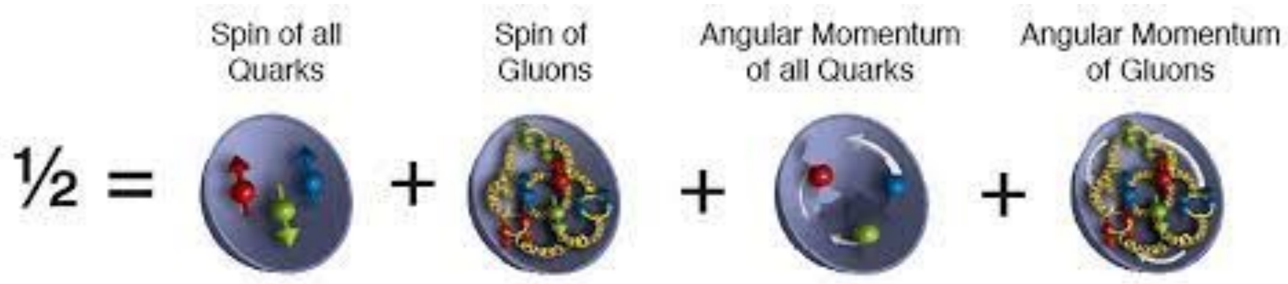
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \ell_q^z + \ell_g^z$$

- Works in the infinite momentum frame
- Apply to long. polarization only
- Parton AM densities in long. polarization

Splitting the gluon spin and OAM is subtle which is easier in the IMF!

ℓ_q^z, ℓ_g^z canonical OAMs that describe the transverse motion of parton in the IMF.

Sum rules with canonical OAM



Y. Guo et. al. 2021

Jaffe-Manohar Sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \ell_q^z + \ell_g^z$$

Longitudinal AM densities of parton

$$J^z_{q/g} = \sum_{i\in q/g} r_i^y p_i^x - r_i^x p_i^y$$

They are twist-three (hard)!

GJS sum rule (2021)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^T + \Delta G^T + \ell_q^T + \ell_g^T$$

Transverse AM densities of parton

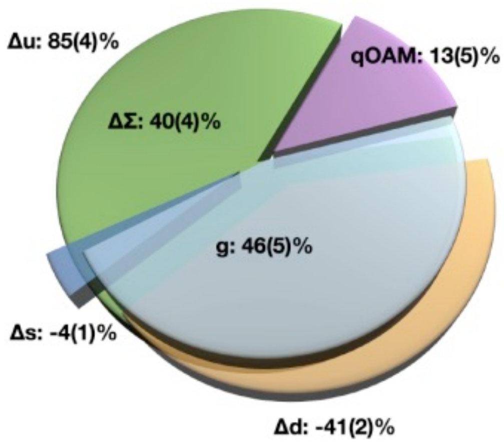
$$J^x_{q/g} = \sum_{i\in q/g} r_i^y p_i^z - r_i^z p_i^y$$

Two sum rules can be derived due to the

breaking of rotational symmetry in z and x/y.

How to get the nucleon AM?

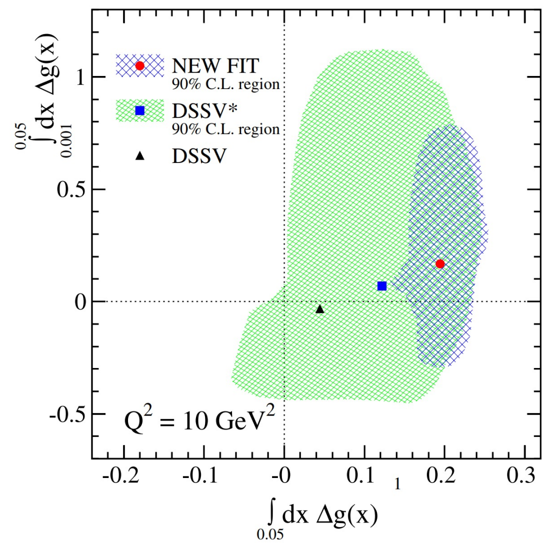
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q^z + J_g$$



Lattice calculation of proton spin

Chi-QCD Collaborations 2021

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \ell_q^z + \ell_g^z$$



Experiment constraints on ΔG

D. de Florian et. al. 2014

Experiments seem to be less constraining?

Not a fair comparison – Experiment always have the partonic structures.

How to get the partonic AM?

Hard since everyone only knows parts of them.

$$J_{q/g}(t) = \int \mathrm{d}x \frac{1}{2} x \left(H_{q/g}(x, \xi, t) + E_{q/g}(x, \xi, t) \right)$$

- ❑ Forward scattering or global fitted PDF

  know nothing about E(x)

- ❑ Lattice calculation of form factors

  know nothing about x-dependence

- ❑ Lattice calculation of GPDs (LaMET) Ji 2013

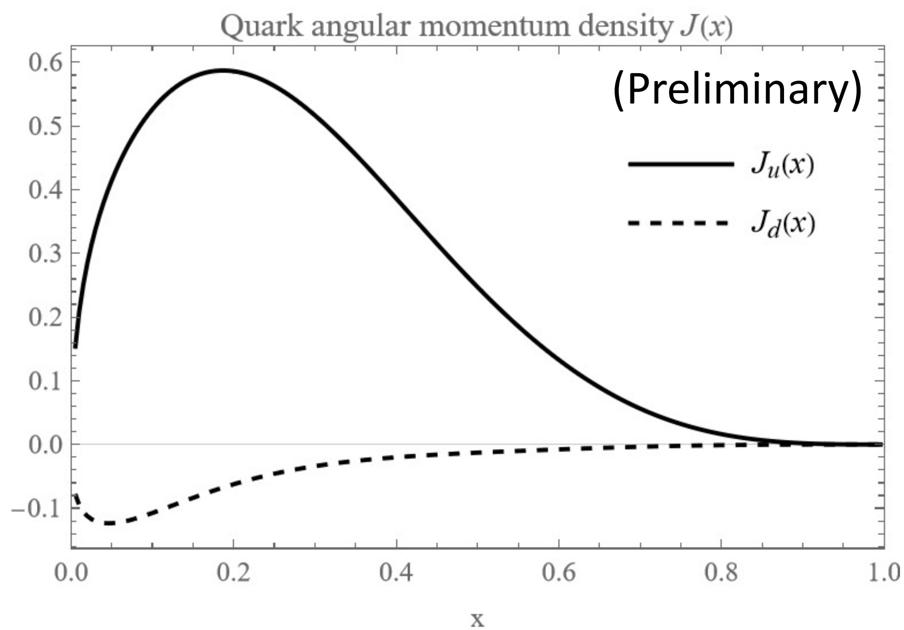
  Hard to get the end-point region with precision

- ❑ Experiment measurements on GPDs (DVCS, DVMP)

  Relations to GPDs are indirect.

How to get the partonic AM?

Global fitting program of GPDs to combine them all!



Y. Guo et. al. in progress

GPDs through Universal Moment Parameterization (GUMP)

Extension of the KM model to include the other constraints.

(Kumericki & Muller 2007, 2009)

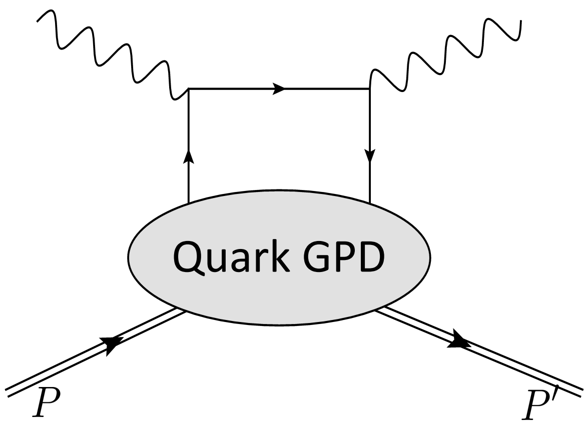
What about the twist-three OAMs?

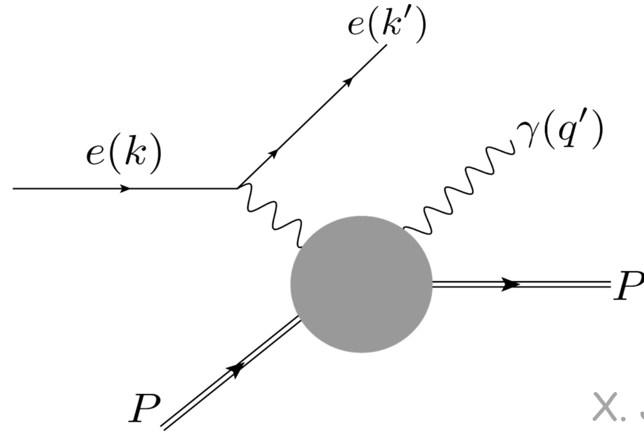
We don’t have the complete arsenal for them yet.

Deeply virtual Compton scattering

Quark distribution and DVCS

The quark distributions can be probed with Deeply virtual Compton scattering.



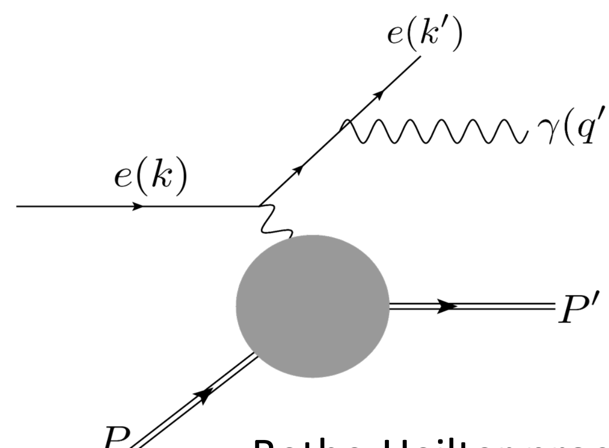


X. Ji 1996

Deeply virtual Compton scattering

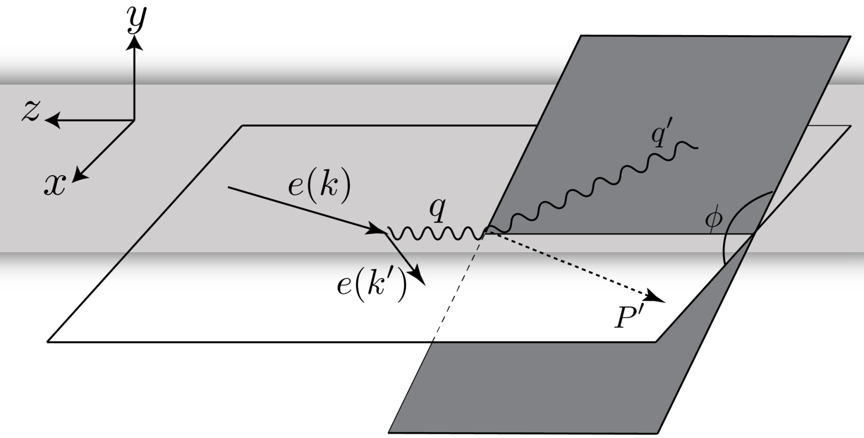
Unfortunately, it's always mixed with the

QED-driven Bethe-Heilter process



Bethe-Heilter process

DVCS cross-sections



The cross-section can be expressed in terms of the squared scattering amplitude

$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}|t| \mathrm{d}\phi \mathrm{d}\phi_S} = \frac{\alpha_{\mathrm{EM}}^3 x_B y^2}{16\pi^2 Q^4 \sqrt{1+\gamma^2}} |\mathcal{T}|^2 \ ,$$

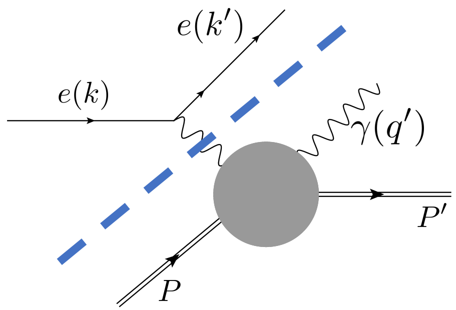
$$|\mathcal{T}|^2 = |\mathcal{T}_{\mathrm{BH}}|^2 + |\mathcal{T}_{\mathrm{DVCS}}|^2 + T_{\mathrm{BH}}^* \mathcal{T}_{\mathrm{DVCS}} + \mathcal{T}_{\mathrm{DVCS}}^* \mathcal{T}_{\mathrm{BH}}$$

$$\mathrm{d}\sigma_{\mathrm{Total}} = \mathrm{d}\sigma_{\mathrm{BH}} + \mathrm{d}\sigma_{\mathrm{DVCS}} + \mathrm{d}\sigma_{\mathrm{INT}} \ .$$

The BH part concerns about the elastic form factors only and won’t be discussed in detail.

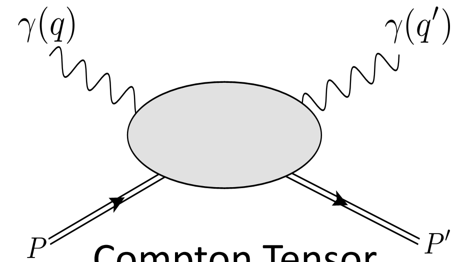
DVCS amplitude and Compton tensor

The DVCS amplitude can be split into the leptonic (QED) part and hadronic (QCD) part



hadronic

part



Compton Tensor

$$T^{\mu\nu} = i \int \mathrm{d}^4 z e^{i(q+q')z/2} \langle P' | J^\mu(-z/2) J^\nu(z/2) | P \rangle$$

Without loss of generality, the Compton tensor can be decomposed into some quantities (Compton form factors) and their corresponding tensor structures.

$$T^{\mu\nu} = \mathscr{T}^{\mu\nu}_{(2)} \mathcal{F} + \widetilde{\mathscr{T}}^{\mu\nu}_{(2)} \widetilde{\mathcal{F}} + \mathscr{T}^{\mu\nu;\rho}_{(3)} \mathcal{F}^{\perp}_{\rho}(x) + \widetilde{\mathscr{T}}^{\mu\nu;\rho}_{(3)} \widetilde{\mathcal{F}}^{\perp}_{\rho}(x) + \cdots$$

$\mathscr{T}^{\mu\nu}_{(i)}$ Compton tensor Coefficients

$$\mathscr{T}^{\mu\nu}_{(2)} = -\frac{1}{2}\left[g^{\mu\nu} - \frac{q^\mu q'^\nu + q^\nu q'^\mu}{q \cdot q'} + \frac{q'^\mu q'^\nu q^2}{(q \cdot q')} \right]$$

$\mathcal{F}$ Related to Compton form factors

$$\mathcal{F} = \bar{u}(P', S') \left[\not{n} \mathcal{H}(\xi, t) + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} \mathcal{E}(\xi, t) \right] u(P, S)$$

$$\mathcal{H}(\xi, t) = \int_{-1}^{1} \mathrm{d}x C(x, \xi) H(x, \xi, t)$$

Cross-section formulas

Ji (Ji 1996)

BMK01 (Belitsky, Muller, & Kirchner 2001)

BMK10 (Belitsky & Muller 2010)

BMJ (Belitsky, Muller & Ji 2012)

BMMP (Braun, Manashov, Muller & Pirnay 2014)

VA (B. Kriesten et. al. 2019)

This work (Y. Guo et. al. 2021)

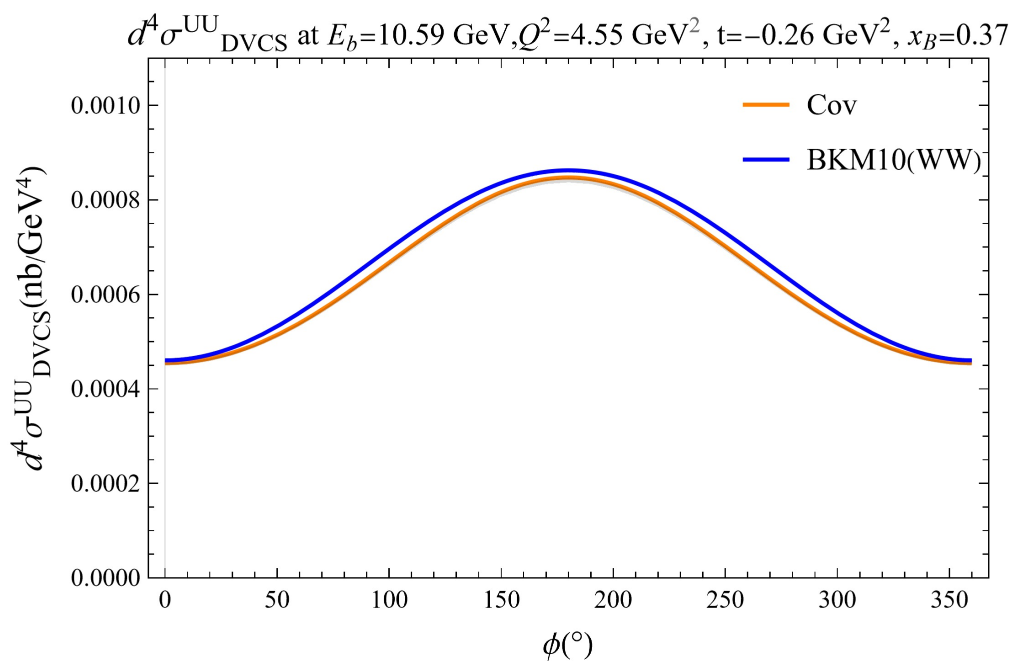
Kinematical corrections

Tensions here

Independent check

Differences of formulas

- Extra cos(phi) factor in the VA interference cross-sections than BMK’s (and ours).
  - Different choices of light-cone vectors.
  - Different choices of final photon gauge fixing condition.
    - Twist expansion generally breaks gauge invariance.



Our results agree with the BMK10 ones up to twist-four effects.

Some concerns in DVCS analysis

DVCS analysis are very undetermined!

$$\mathrm{d}^4\sigma = \mathrm{d}^4\sigma(x_B, t, Q, \phi, \mathcal{F}(x_B, t, Q))$$

Fixed at each kinematical point

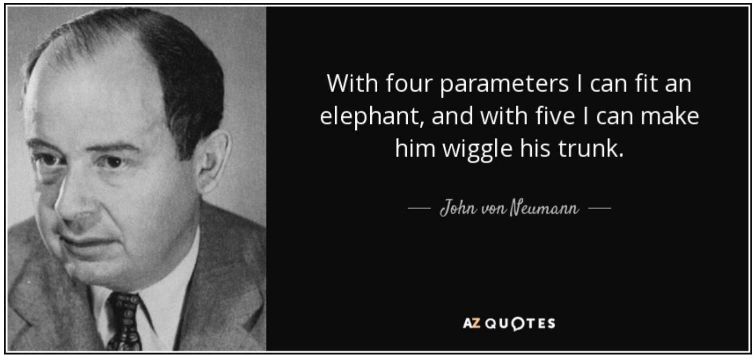
How many measurements we can have? – A lot (different ϕ)

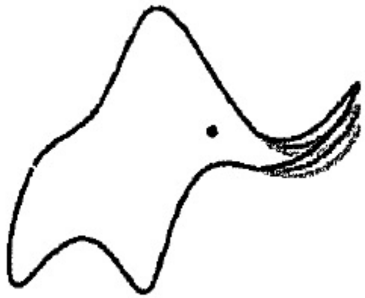
How many independent measurements we can have? – Very limited!

$$\mathrm{d}^4\sigma^{\mathrm{UU}}(x_B, t, Q, \phi, \mathcal{F}) \approx \mathrm{d}^4\sigma_0'^{\mathrm{UU}}(x_B, t, Q, \mathcal{F}) + \cos(\phi)\mathrm{d}\sigma_1'^{\mathrm{UU}}(x_B, t, Q, \mathcal{F})$$

$$\mathrm{d}^4\sigma^{\mathrm{LU}}(x_B, t, Q, \phi, \mathcal{F}) \approx \sin(\phi)\mathrm{d}^4\sigma_1'^{\mathrm{LU}}(x_B, t, Q, \mathcal{F})$$

8 twist-2 CFFs $\Longleftrightarrow$ effectively 3 measurements.





General strategy for DVCS analysis

(K. Shiells et. al. 2021)

It's crucial to have enough independent measurements!

- More polarization (12 independent meas. for all polarization)
- Beam charge asymmetry

…

The situation gets worse with twist-3 CFFs (another 8 of them)

General strategy for DVCS analysis

- ❖ At large enough Q, measure the twist-two CFFs.
- ❖ With the above input, fit the twist-three CFFs at lower Q .

Some numeric studies

A glance of the effects of twist-3 CFFs

Twist-three CFFs approximated by the Wandzura-Wilczek relation.

Prop. to twist-2 GPD

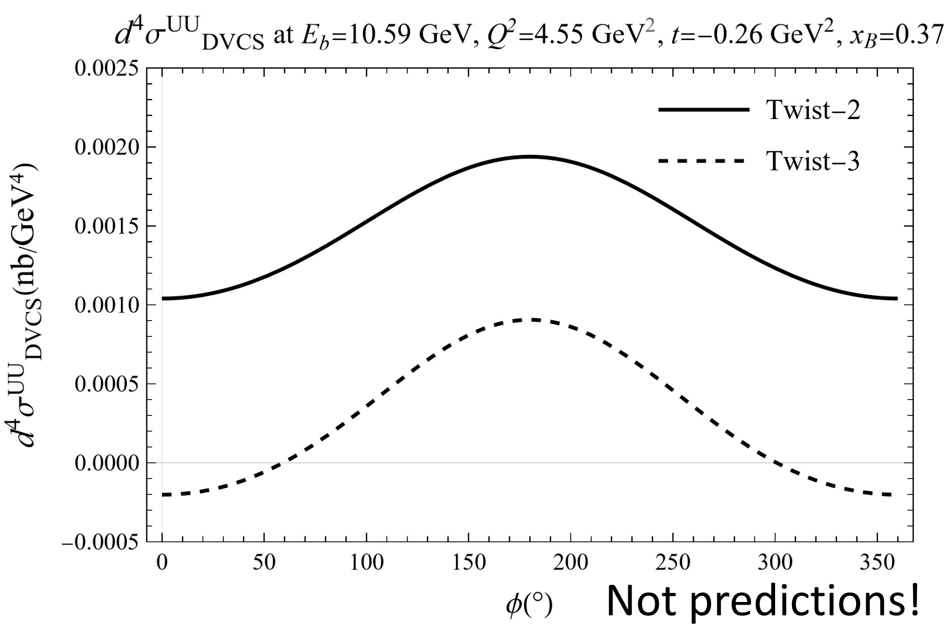
Genuine twist-3 GPD

$$F^{\perp}_{\rho}(x,\xi,t) \approx \frac{\Delta^{\mu}_{\perp}}{n\cdot\Delta}F(x,\xi,t) + \int_{-1}^{1}\mathrm{d}u W_{+}(x,u,\xi)G^{\mu}(u,\xi,t) + i\tilde{\epsilon}^{\mu\nu}\int_{-1}^{1}\mathrm{d}u W_{-}(x,u,\xi)\widetilde{G}_{\nu}(u,\xi,t) + \cdots ,$$

Twist-3 GPD

WW Kernel

twist-2 GPDs



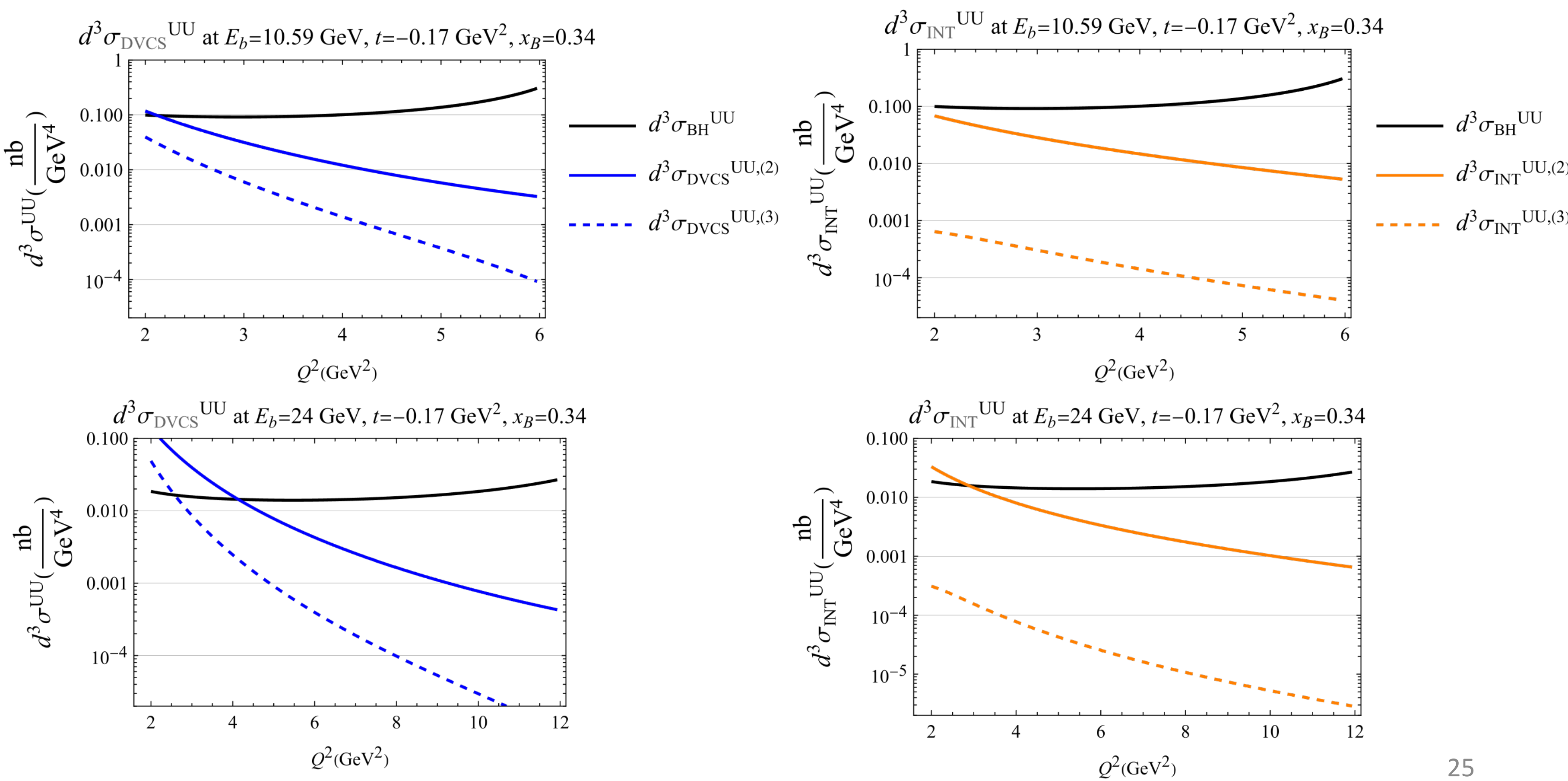
Not predictions!

Twist-3 effects can be important for small Q.

Q-dependence of twist-3 effects

Naturally, we can go to high Q to avoid the twist-three effects.

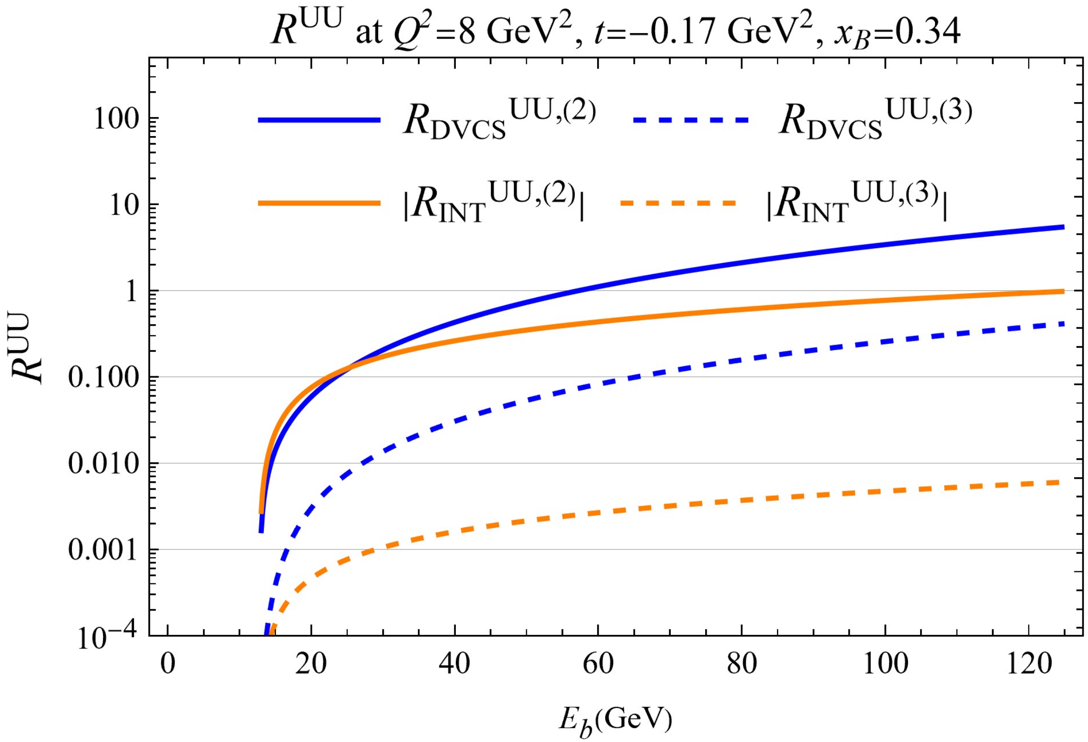
However, we always have the BH background which dominates at large Q.



E_b-dependence of twist-3 effects

It looks like we can't go too high for Q^2?

$$R_i^{\mathrm{UU}}(x_B, Q^2, t) = \frac{\mathrm{d}^3\sigma_i^{\mathrm{UU}}}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}|t|} \left(\frac{\mathrm{d}^3\sigma_{\mathrm{BH}}^{\mathrm{UU}}}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}|t|} \right)^{-1} ,$$



BH contributions are suppressed for large E_b ($s = M^2 + 2ME_b$)

Kinematical x_B- and t-dependence

The x_B and t dependence of CFFs are non-trivial and unpredictable.

The kinematical coefficients of cross-sections is defined without the CFFs

$$\mathrm{d}^3\sigma^{\mathrm{UU},(3)}_{\mathrm{DVCS}}(x_B,t,Q^2,\cdots)=\mathrm{d}^3\widetilde{\sigma}^{\mathrm{UU},(3)}_{\mathrm{DVCS}}(x_B,t,Q^2,\cdots)\times\mathcal{F}^{(3)}_{\mathrm{UU}}(x_B,t)\ ,$$

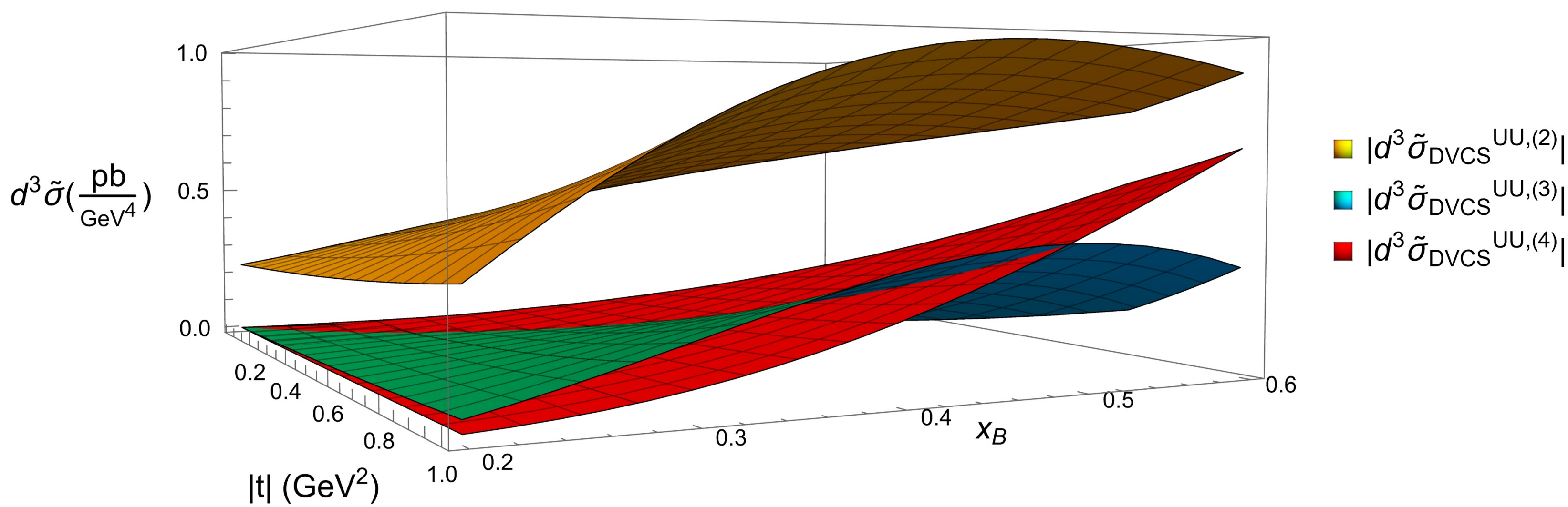
Those coefficients are essentially the scalar coefficients

$$\mathrm{d}^3\widetilde{\sigma}^{\mathrm{UU},(3)}_{\mathrm{DVCS}}(x_B,t,Q^2,\cdots)=\frac{\Gamma}{Q^4}\int\mathrm{d}\phi\ 4h^{\mathrm{U}}_{(3)}(\phi,x_B,t,Q^2,\cdots)\ ,$$

Different azimuthal dependence can be projected out.

x_B- and t-dependence

The higher twist effects at $E_b = 10.59\ \mathrm{GeV}$ and $Q^2 = 4\ \mathrm{GeV}^2$ are then calculated



The power counting is usually $\mathcal{O}\left(\frac{t}{Q^2}\right)$ or $\mathcal{O}\left(\frac{x_B^2 M^2}{Q^2}\right)$

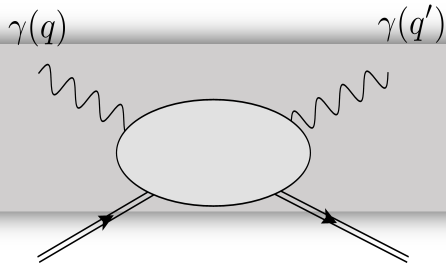
Twist-three effects are more suppressed with larger Q, though the BH contribution will dominate with larger Q (less sensitive to CFFs).

Summary

- AM and OAM are encoded in twist-two and twist-three GPDs.
- It's important to combine all the constraints on GPDs to complete the whole picture of angular momentum of parton.
- In DVCS, the number of independent measurements is crucial.
- It's more practical to extract the leading-twist CFFs first with large Q.
- Measurements at larger x_B and t require larger Q suppression.

Thanks!

Helicity amplitude



The Compton tensor can be expressed in other equivalent way

$$T^{\mu\nu} = \mathscr{T}^{\mu\nu}_{(2)}\mathcal{F} + \widetilde{\mathscr{T}}^{\mu\nu}_{(2)}\widetilde{\mathcal{F}} + \mathscr{T}^{\mu\nu;\rho}_{(3)}\mathcal{F}^{\perp}_{\rho}(x) + \widetilde{\mathscr{T}}^{\mu\nu;\rho}_{(3)}\widetilde{\mathcal{F}}^{\perp}_{\rho}(x) + \cdots$$

$$T_{\lambda\lambda'} \equiv \epsilon_{\nu}(q,\lambda)T^{\mu\nu}\epsilon^{*}_{\mu}(q',\lambda')$$

$$T^{\mu\nu} = \sum_{\lambda,\lambda'} \epsilon^{*\nu}(q,\lambda)\epsilon^{\mu}(q',\lambda')T_{\lambda\lambda'}$$

- ✓ Intuitive physical picture.
- ✓ Power suppression of helicity amplitude.

$$T^{++} \sim \mathcal{O}(Q^0), T^{+0} \sim \mathcal{O}(Q^{-1}), T^{+-} \sim \mathcal{O}(Q^{-2})$$

- ✓ Helicity amplitudes are invariant.

- ❑ Photon helicities will be summed over.
- ❑ Not exactly twist-separated.
- ❑ The helicity vectors could be different.

Twist-three effects in DVCS

Sources of twist-three effects

All the approximations are made to the Compton tensor.

$$T^{\mu\nu} = \mathscr{T}^{\mu\nu}_{(2)}\mathcal{F} + \widetilde{\mathscr{T}}^{\mu\nu}_{(2)}\widetilde{\mathcal{F}} + \mathscr{T}^{\mu\nu;\rho}_{(3)}\mathcal{F}^{\perp}_{\rho}(x) + \widetilde{\mathscr{T}}^{\mu\nu;\rho}_{(3)}\widetilde{\mathcal{F}}^{\perp}_{\rho}(x) + \cdots$$

Naively, the twist-three effects are only associated with those twist-three terms.

However, another twist-three effects are implicitly there.

To help understand this, consider the Taylor expansion of functions:

$$f(x) = \frac{1}{1-x} \approx 1 + x + \mathcal{O}(x^2) \quad (x \ll 1)$$

If one expands the same function at a different point

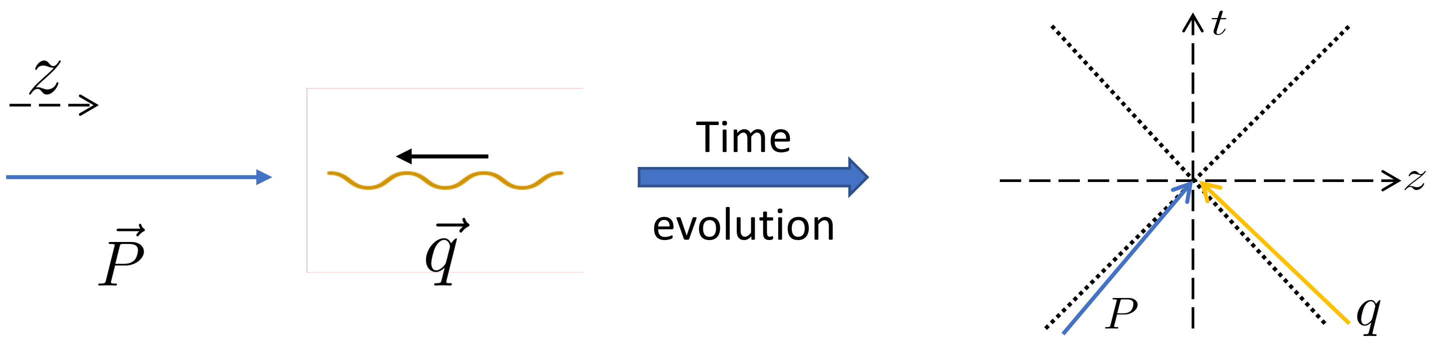
$$f(x) = \frac{1}{1-x} \approx \frac{10}{9} + \frac{100}{81}\left(x - \frac{1}{10}\right) + \mathcal{O}\left(\left(x - \frac{1}{10}\right)^2\right) \quad \left(\left|x - \frac{1}{10}\right| \ll 1\right)$$

Each term is different even though they add up to the same function.

Terms of different orders are ambiguous unless the point of expansion is specified!

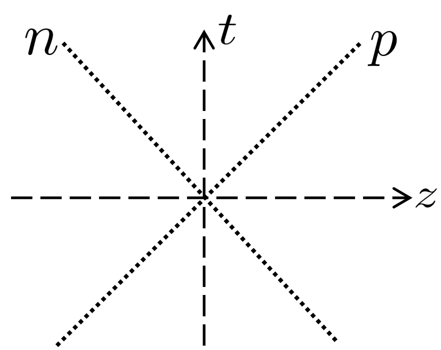
Light-cone vectors

In the center of mass frame, the photon and proton collide with each other



Two auxiliary vectors n and p are defined such that in the Bjorken limit

$$q \to n \qquad P \to p$$



Light-cone vectors are commonly used for twist separation.

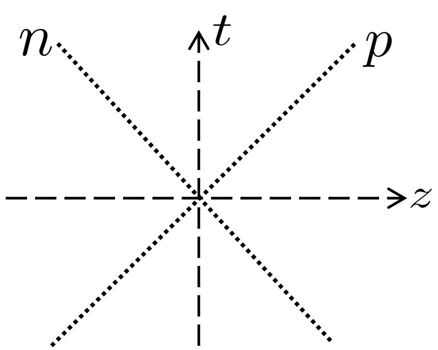
Choices of light-cone vectors

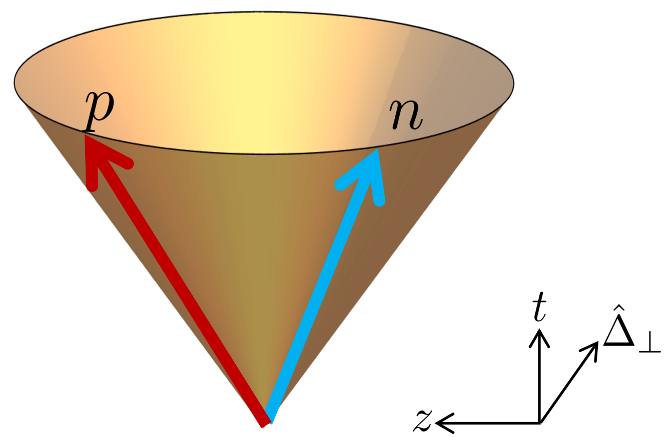
In an off-forward scattering process with non-zero momentum transfer,

the initial and final proton momenta are not the same $\Delta \equiv P' - P \neq 0$

You might align the light cone vectors with P or P' or any combination of them

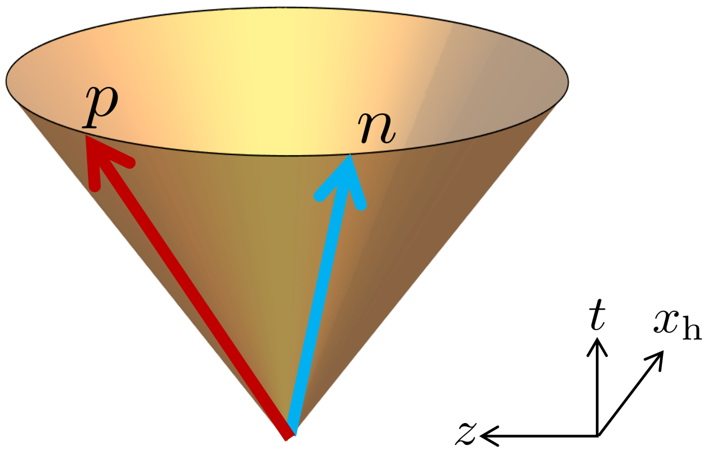
Rotate the light cone vectors in the direction of $\Delta \equiv P' - P$

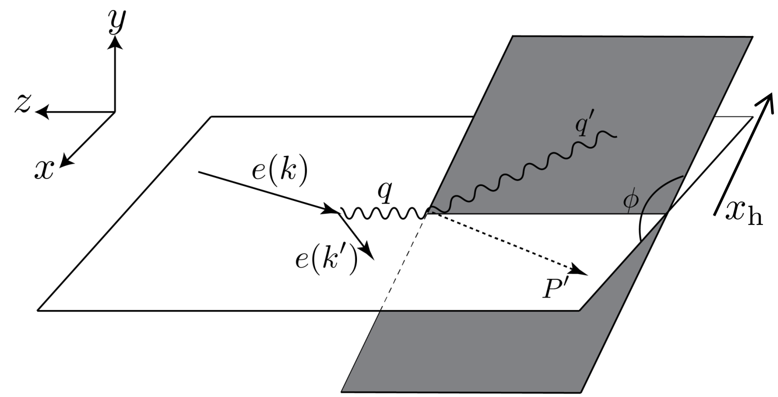




Two rotational degrees of freedom α, β

Degrees of freedom of LC vectors





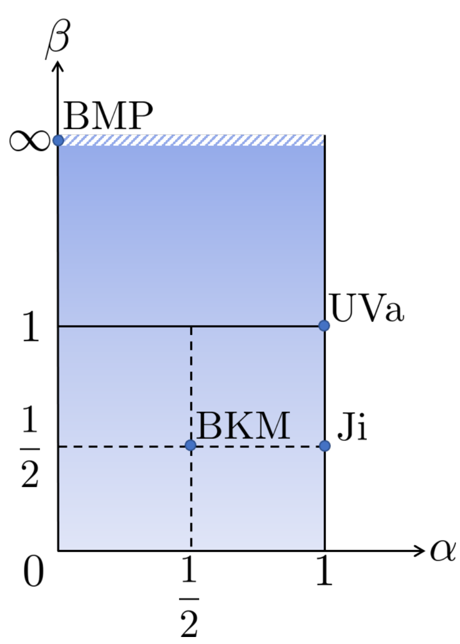
- Two LC vectors are light-like $n^2 = p^2 = 0$
- Their scales are fixed by the condition $n \cdot p = n \cdot \bar{P} = 1$
- Their directions can be chosen freely

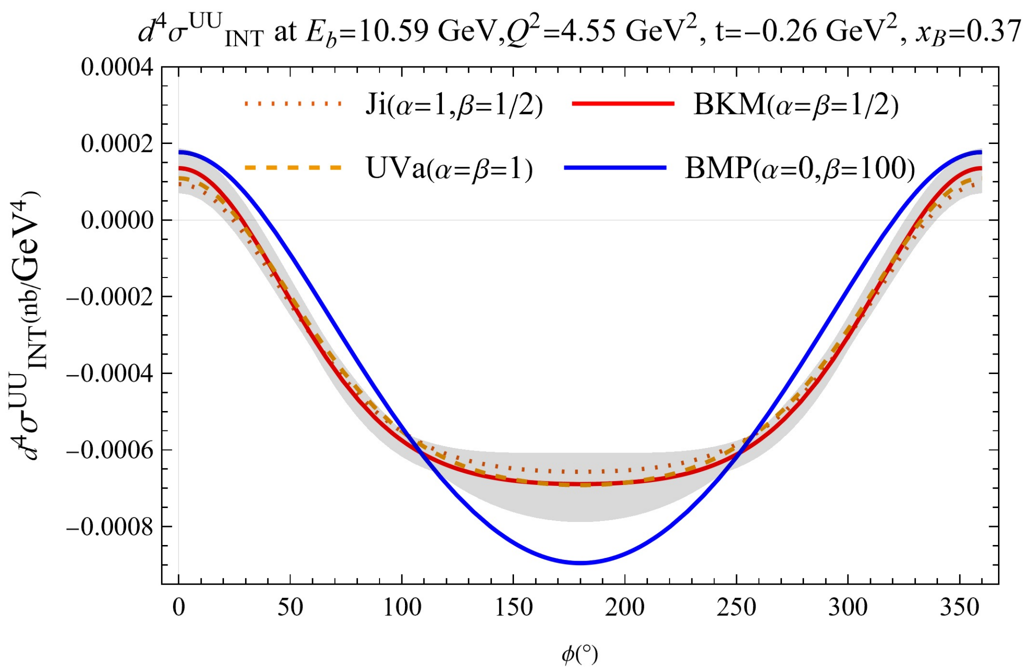
  - Two rotational degrees of freedom of LC vectors α, β

(How) Does it matter?

"Each term is different even though they add up to the same function."

- The cross-sections from twist-two CFFs are different if one chose different LC vectors.
- Adding the effects of twist-three CFFs, the differences cancel.





Wandzura-Wilczek relations

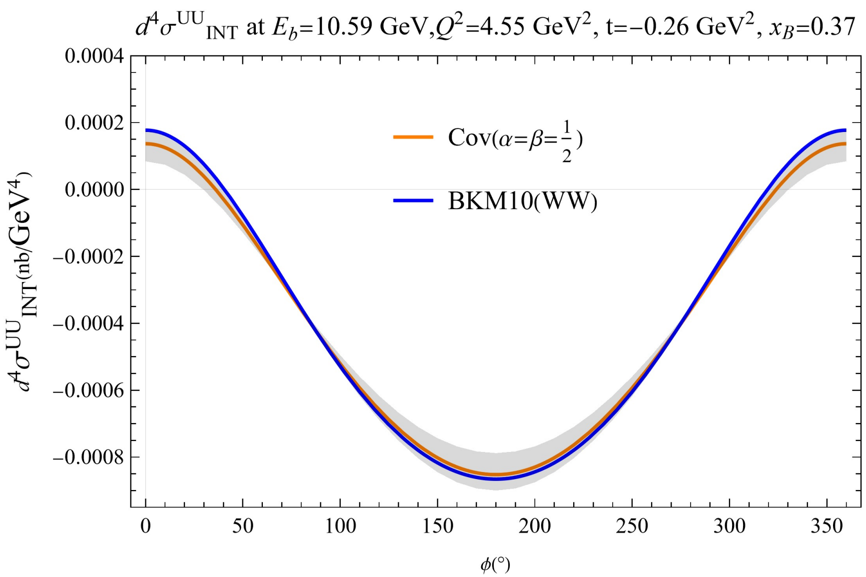
Nontrivial relations between twist-2 and twist-3 CFFs are required for the cancelation of LC dependence to happen.

$$T^{\mu\nu} = \mathscr{T}^{\mu\nu}_{(2)}\mathcal{F} + \widetilde{\mathscr{T}}^{\mu\nu}_{(2)}\widetilde{\mathcal{F}} + \mathscr{T}^{\mu\nu;\rho}_{(3)}\mathcal{F}^{\perp}_{\rho}(x) + \widetilde{\mathscr{T}}^{\mu\nu;\rho}_{(3)}\widetilde{\mathcal{F}}^{\perp}_{\rho}(x) + \cdots$$

The relations between them are results of Lorentz symmetry and equation of motion.

$$\mathcal{F}^{\perp}_{\rho} \approx \frac{\Delta^{\mu}_{\perp}}{n\cdot\Delta}\mathcal{F} + \cdots\ , \qquad \widetilde{\mathcal{F}}^{\perp}_{\rho} \approx \frac{\Delta^{\mu}_{\perp}}{n\cdot\Delta}\widetilde{\mathcal{F}} + \cdots\ ,$$

Terms involving convolutions of GPD, and genuine higher-twist terms are not shown.

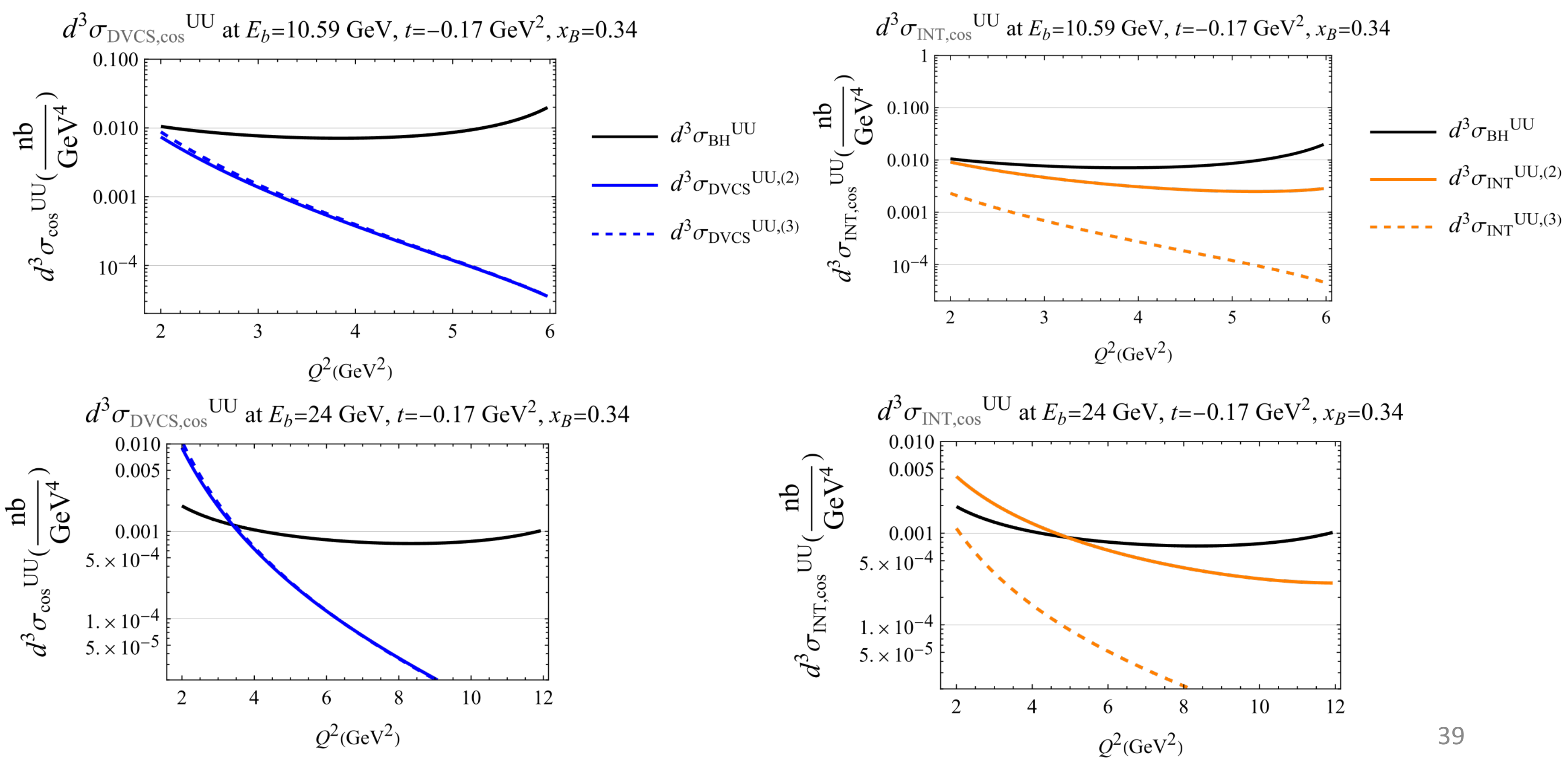


The band are now twist-four effects!

Q-dependence of twist-3 effects (cos phi)

Naturally, we can go to high Q to avoid the twist-three effects.

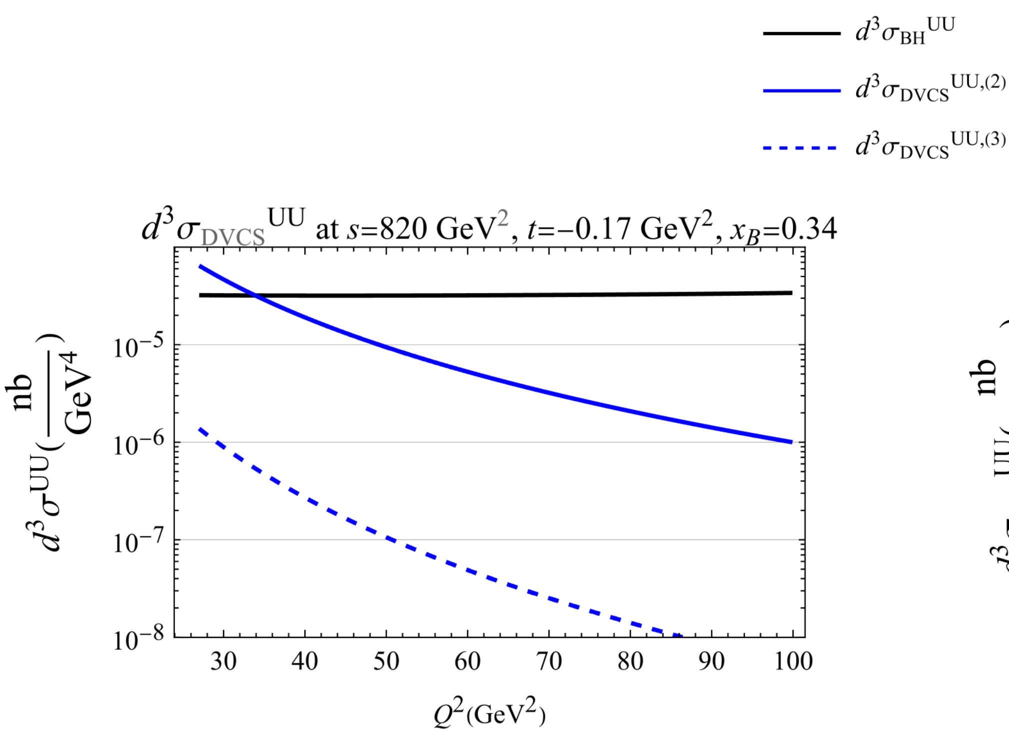
However, we always have the BH background which dominates at large Q.

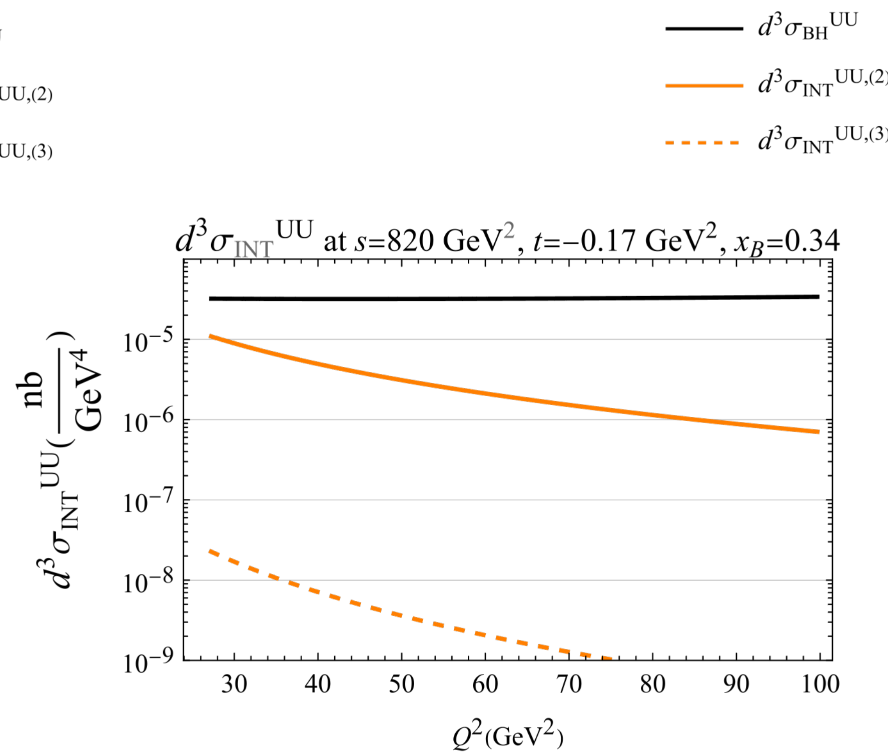


Q-dependence of twist-3 effects (EIC)

Naturally, we can go to high Q to avoid the twist-three effects.

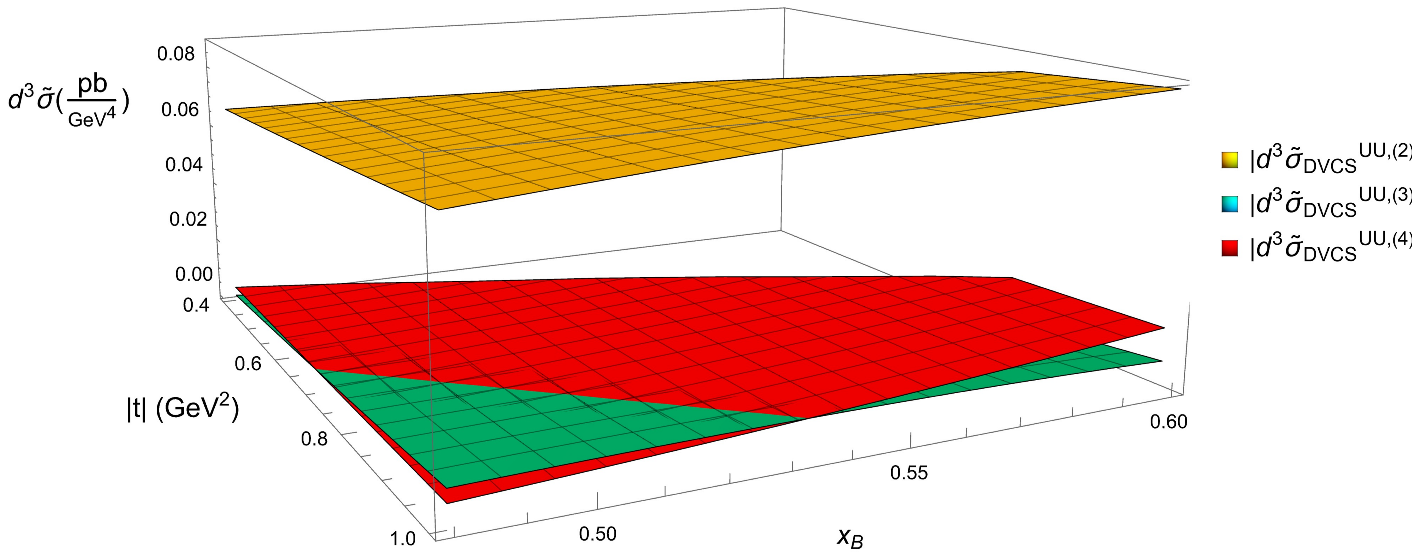
However, we always have the BH background which dominates at large Q.





x_B- and t-dependence at large Q

Higher twist effects are more suppressed at higher Q $E_b = 10.59\ \mathrm{GeV}$ and $Q^2 = 8\ \mathrm{GeV}^2$



Though with such large Q^2, the BH contribution will be dominated.