A lattice QCD calculation of the off-forward Compton amplitude and generalised parton distributions

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For the CSSM/QCDSF/UKQCD collaboration

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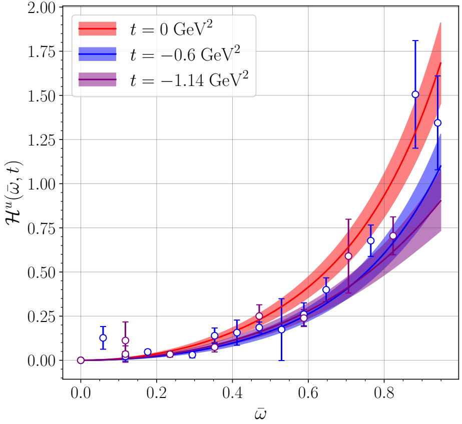
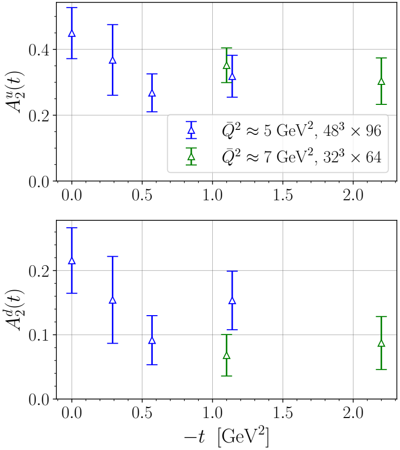
THE UNIVERSITY of ADELAIDE

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER



The $\mathcal{H}$ Compton form factor.



GPD moments.

Outline

1. **Background:**
   What are GPDs? Why are we interested in lattice calculations?
2. **Outline of method:**
   Novel application of Feynman-Hellmann methods
3. **Established results:**
   Presented in AHG et al., PRD 105, 2022
4. **New results**
   Preliminary!

1. GPDs—what are they and why are they interesting?
2. The Feynman-Hellmann method
3. Established results
   presented in AHG et al., PRD 105, 2022
4. New Results
   Preliminary!
5. Conclusions and Outlook

Outline of the problem

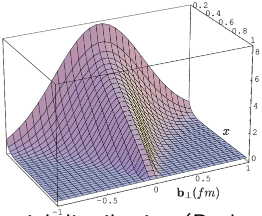
- Extensions of parton distribution functions (PDFs), related to elastic form factors
- Contain a staggering amount of physical information: a solution to proton spin puzzle, the spatial distributions of hadron constituents, and more

However...

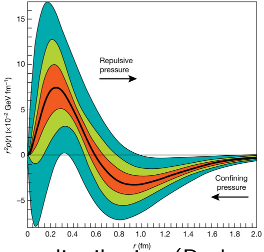
- Difficult to measure experimentally
- and difficult to calculate on the lattice

In this talk:

- a new lattice method to calculate GPDs (Feynman-Hellmann)
- with strong parallels to experimental measurements

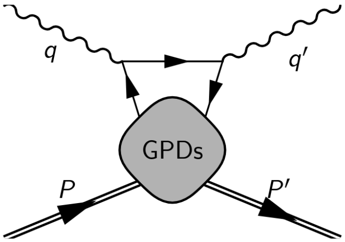


Quark spatial distribution (Burkardt, 2002)



Proton pressure distribution (Burkert et al., 2018)

What are generalised parton distributions?



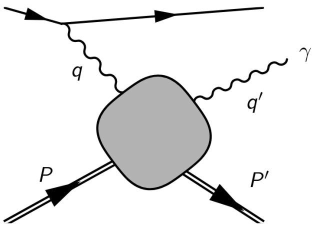
Emission and absorption of quark in high-energy nucleon, with $P \neq P'$

Light-cone matrix element:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \not{n} \psi_q(\lambda n/2) | P \rangle$$
$$= H^q(x,\xi,t) \bar{u}(P') \not{n} u(P)$$
$$+ E^q(x,\xi,t) \bar{u}(P') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} u(P).$$

- H^q and E^q are helicity-conserving and -flipping GPDs (analogous F_1 and F_2).
- $t = (P' - P)^2$ momentum transfer.
- x, ξ are momentum fractions.

Measurement of GPDs



- Deeply virtual Compton scattering: $e^- + p \to e^- + p + \gamma$.
- Measure the off-forward Compton amplitude
- Compton form factors at large $-q^2$

$$\text{CFF} = \int_{-1}^{1} dx \left(\frac{1}{x - \xi + i\varepsilon} \pm \frac{1}{x + \xi + i\varepsilon} \right) \text{GPD}$$

Difficulties:

- deconvolution problem,
- spanning kinematics,
- lack of theoretical constraints.

Lattice calculations:

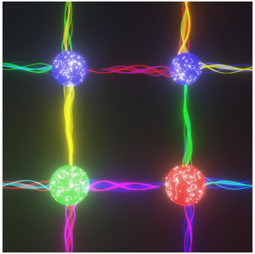
- provide theoretical constraints,
- access unphysical kinematics ($\xi = 0$),
- exclude models,

Our aim: calculate this OFCA with lattice QCD for $\xi = 0$. **Previous calculations:** focus on leading-order.

Lattice QCD

QCD path integral

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}A_\mu \mathcal{D}\bar{\psi}\mathcal{D}\psi \mathcal{O} e^{iS_{\text{QCD}}}.$$



To evaluate this numerically:

1. discrete spacetime,
2. Wick rotation $t \to -i\tau$, $e^{iS_{\text{QCD}}} \to e^{-S^E_{\text{QCD}}}$,
3. generate gauge configurations according to $e^{-S^E_{\text{QCD}}}$.

Then, the path integral can be evaluated as a weighted sum over gauge configurations:

$$\langle \mathcal{O} \rangle \approx \frac{1}{N} \sum_{i=0}^{N} \mathcal{O}_i.$$

Why can't we calculate parton distributions directly?

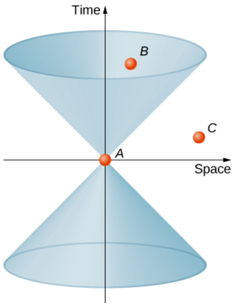
Wick rotated separation:

$$x^2 = (-i\tau)^2 - |\vec{x}|^2 = -\tau^2 - |\vec{x}|^2 < 0.$$

All spacelike.
But parton distributions are lightlike correlation functions:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P^{(\prime)} | \bar{\psi}_q(-\lambda n/2) \not{n} \psi_q(\lambda n/2) | P \rangle,$$

since $n^2 = 0$. $\therefore$ can only calculate related quantities.



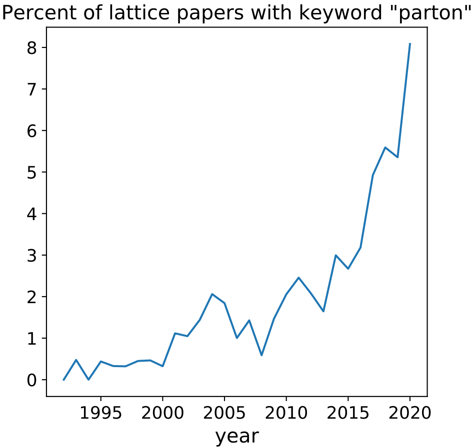
Hunt for lattice parton distributions

A lot of related quantities:

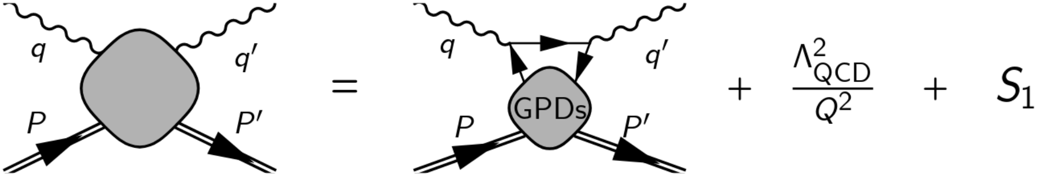
- Moments from 3-pt functions
- Quasi- and pseudo-distributions
- Lattice cross sections
- Heavy-quark OPE
- and others...

GPD studies:

- Quasi: NPB 952 (2020), PRL 125 (2020), PRL 127 (2021)
- Many 3-pt calculations of leading $n = 1, 2, 3$ moments (insufficient for full reconstruction)



Why the Compton amplitude?

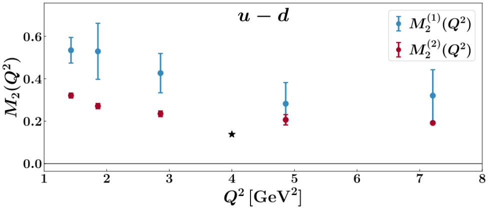


- 3-pt moments and quasi leading-order
- But in DVCS expt., hard scale relatively small: $Q^2 < 10$ GeV^2
- Unknown subtraction function, S_1

From lattice OFCA, can we get:

- Q^2 dependence [Latt. 2021 PoS 324]
- higher-order terms [Latt. 2019 PoS 278]
- subtraction function [Latt. 2021 PoS 028]

For the *forward* ($P = P'$) Compton amplitude, we have calculated these properties with Feynman-Hellmann.



Summary

- GPDs contain LOTS of physical information.
- They are hard to measure from experiment.
- They are hard to calculate on the Lattice.
- We want lattice calculation of Compton amplitude (more overlap with experiment) $\rightarrow$ GPDs

1. GPDs—what are they and why are they interesting?
2. The Feynman-Hellmann method
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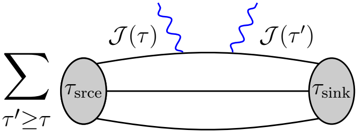
Why Feynman-Hellmann?

Lattice OFCA:

$$T_{\mu\nu} = \sum_{z_\mu} e^{\frac{i}{2}(q+q')\cdot z} \langle P' | T\{j_\mu(z) j_\nu(0)\} | P \rangle.$$

Requires 4-pt function:

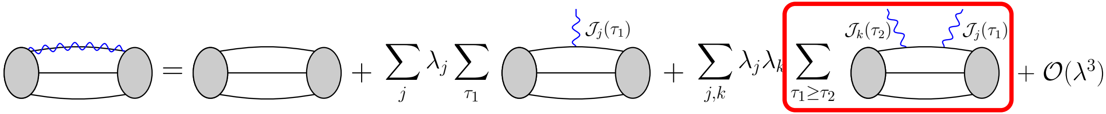
$$\langle \chi(z_4) j_\mu(z_3) j_\nu(z_2) \chi^\dagger(z_1) \rangle$$



- New inversion for each (τ, τ')
- Large T: $\tau_{\text{sink}} \gg \tau'$, $\tau_{\text{srce}} \ll \tau$
- Expensive!

Feynman-Hellmann perturbed Dirac operator: $i\not{D} - m \longrightarrow i\not{D} - m - \underbrace{\lambda_1 \mathcal{J}(q_1) - \lambda_2 \mathcal{J}(q_2)}_{\text{background fields}}$

Expand around $\lambda_1, \lambda_2 = 0$:



Then $\lambda_1 \lambda_2$ term will be OFCA.

In more detail

Calculate perturbed quark propagator:

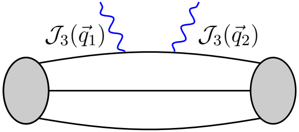
$$S_{(\lambda_1,\lambda_2)}(x_n - x_m) = \left[M - \lambda_1 \cos(\vec{q}_1 \cdot \vec{x})\gamma_3 - \lambda_2 \cos(\vec{q}_2 \cdot \vec{x})\gamma_3\right]^{-1}_{n,m}.$$

Two couplings, λ_1, λ_2; two momenta, $\vec{q}_1$ and $\vec{q}_2$; choose γ_3, which gives T_{33} component.

$$S_{\vec{\lambda}} = \underbrace{S}_{\text{unperturbed}} + \sum_i \lambda_i \underbrace{S\mathcal{J}_3(\vec{q}_i)S}_{\text{three-point}} + \sum_{i,j} \lambda_i\lambda_j \underbrace{S\mathcal{J}_3(\vec{q}_i)S\mathcal{J}_3(\vec{q}_j)S}_{\text{four-point}} + \mathcal{O}(\lambda^3)$$

Put into nucleon propagator: $\mathcal{G}^{dd}_{\vec{\lambda}} \simeq \langle S^u S^u S^d_{\vec{\lambda}} \rangle$, or $\mathcal{G}^{uu}_{\vec{\lambda}} \simeq \langle S^u_{\vec{\lambda}} S^u_{\vec{\lambda}} S^d \rangle$.

$$\mathcal{G}_{(\lambda,\lambda)} + \mathcal{G}_{(-\lambda,-\lambda)} - \mathcal{G}_{(-\lambda,\lambda)} - \mathcal{G}_{(\lambda,-\lambda)} \simeq \lambda^2$$

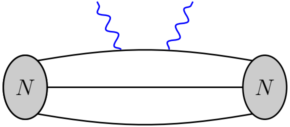


$$+ \; \mathcal{O}(\lambda^4)$$

The $(\lambda_1)^2$ and $(\lambda_2)^2$ terms give forward Compton amplitudes. The $\lambda_1\lambda_2$ term gives OFCA.

Off-forward Compton amplitude from Feynman-Hellmann

Ground state saturation

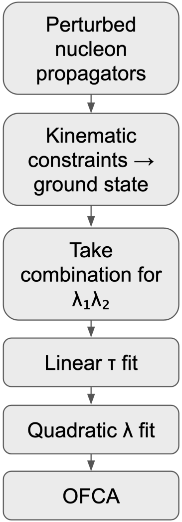


$|\vec{p}| = |\vec{p} + \vec{q}_1 - \vec{q}_2|$ (equal energy but momentum transfer)

Feynman-Hellmann relation

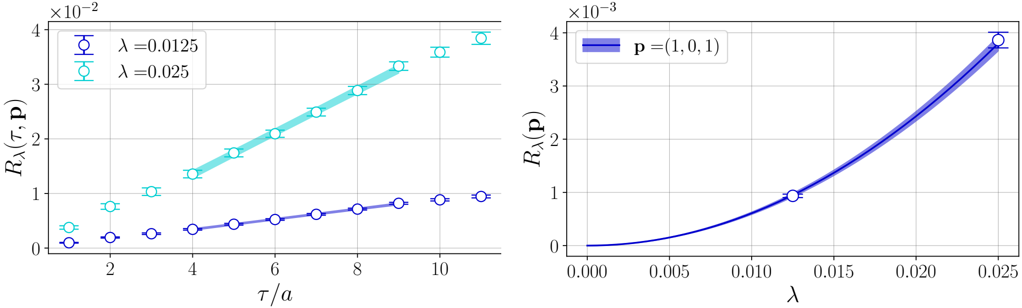
$$R_\lambda \equiv \frac{\mathcal{G}_{(\lambda,\lambda)} + \mathcal{G}_{(-\lambda,-\lambda)} - \mathcal{G}_{(-\lambda,\lambda)} - \mathcal{G}_{(\lambda,-\lambda)}}{\mathcal{G}_0} \simeq 2\lambda^2 \tau \frac{T_{33}}{E_N}.$$

- T_{33} is OFCA for $\mu = \nu = 3$.
- Linear in τ, Euclidean time; quadratic in λ.



Fits and signal quality

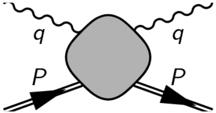
$$R_\lambda(\tau) \simeq 2\lambda^2 \tau \frac{T_{33}}{E_N}.$$



Extract T_{33} for a given sink momentum, **p**. Now, what do we do with it?

Forward Compton amplitude from Feynman-Hellmann

Forward Compton amplitude:



- Previously calculated with Feynman-Hellmann [CSSM/QCDSF PRL 118 (2017), PRD 102 (2020)]
- Same calculation but with one λ and $\vec{q}$.
- Extract $T_{33}(\vec{p}, \vec{q})$, forward Compton amplitude.

Spin-independent:

$$T_{\mu\nu}(p,q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathcal{F}_1(\omega, Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathcal{F}_2(\omega, Q^2)}{p\cdot q},$$

$$\mathcal{F}_1(\omega, Q^2) - \mathcal{F}_1(0, Q^2) = 2\omega^2 \int_0^1 dx \frac{2x F_1(x, Q^2)}{1 - x^2\omega^2 - i\epsilon},$$

$$\mathcal{F}_2(\omega, Q^2) = 4\omega \int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}$$

- From FH, $\mathcal{F}_{1,2}$ and subtraction function $S_1(Q^2) = \mathcal{F}_1(0, Q^2)$
- Inverse Bjorken variable, $\omega = 2\vec{p}\cdot\vec{q}/\vec{q}^2$
- DIS structure functions $F_{1,2}$ become PDFs at $Q^2 \to \infty$.

Parton distribution moments

On lattice, Euclidean CA. To relate to Minkowski,

$$E_X(\vec{p}\pm\vec{q}) > E_N(\vec{p}), \quad \Rightarrow \quad \omega = \left|\frac{2\vec{p}\cdot\vec{q}}{\vec{q}^2}\right| < 1$$

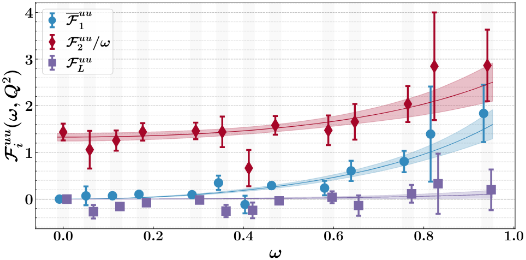
For $|\omega| < 1$,

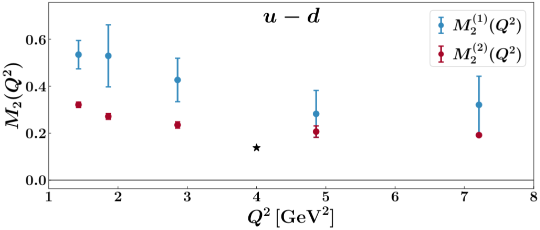
$$\overline{\mathcal{F}}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx \frac{2xF_1(x,Q^2)}{1 - x^2\omega^2 - i\epsilon},$$

$$\overline{\mathcal{F}}_1(\omega, Q^2) = 4 \sum_{n=2,4,6}^{\infty} \omega^n M_n(Q^2)$$

$$M_n(Q^2) \xrightarrow{Q^2\to\infty} \int_0^1 dx x^{n-1} q(x).$$

Euclidean Compton amplitude → moments of physical Compton amplitude.





Perturbative expansion of OFCA

Off-forward is very similar, but more complicated...

$$T_{\mu\nu} = \frac{1}{2\bar{P}\cdot\bar{q}}\left[-\left(h\cdot\bar{q}\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\right)g_{\mu\nu} + \frac{1}{\bar{P}\cdot\bar{q}}\left(h\cdot\bar{q}\mathcal{H}_2 + e\cdot\bar{q}\mathcal{E}_2\right)\bar{P}_\mu\bar{P}_\nu + \mathcal{H}_3 h_{\{\mu}\bar{P}_{\nu\}}\right] + \dots$$

OFCA has 18 Compton form factors, but we want to relate these to GPDs. Lots of perturbative expansions of OFCA. A classic from X. Ji, PRL 78 (1997):

$$T^{\mu\nu}(P,q;P',q') = -\frac{1}{2}(n^\mu\tilde{n}^\nu + n^\nu\tilde{n}^\mu - g^{\mu\nu})\int_{-1}^{1} dx\left(\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi+i\epsilon}\right)$$
$$\times\left[H(x,\xi,t)\bar{u}(P')\tilde{\not{n}}u(P) + E(x,\xi,t)\bar{u}(P')\frac{i\sigma^{\alpha\beta}\tilde{n}_\alpha\Delta_\beta}{2M}u(P)\right].$$

- But n^μ and $\tilde{n}^\mu$ are lightlike vectors.
- Almost all published expansions of OFCA use light-cone kinematics (especially for nucleon).
- But it can't be compared to a Euclidean lattice calculation.

GPD moments from the Compton amplitude

$$T_{\mu\nu} = \frac{1}{2\bar{P}\cdot\bar{q}}\left[-\left(h\cdot\bar{q}\mathcal{H}_1 + e\cdot\bar{q}\mathcal{E}_1\right)g_{\mu\nu} + \frac{1}{\bar{P}\cdot\bar{q}}\left(h\cdot\bar{q}\mathcal{H}_2 + e\cdot\bar{q}\mathcal{E}_2\right)\bar{P}_\mu\bar{P}_\nu + \mathcal{H}_3 h_{\{\mu}\bar{P}_{\nu\}}\right] + \dots$$

My master's:

- Matching non-perturbative tensor decomposition (e.g. Tarrach) to leading-order GPD moments
- Even though we're interested in non-perturbative structure, want high-energy limit ($\bar{Q}^2 \to \infty$) to guide us.

$$\mathcal{H}_1(\bar{\omega},t) - S_1 = \int_{-1}^{1} dx \frac{2x}{(x-1/\bar{\omega})^2 + i\varepsilon} H(x,t) = 2\sum_{n=2,4,6}^{\infty} \bar{\omega}^n A_{n,0}(t), \quad |\bar{\omega}|<1$$

$$\mathcal{E}_1(\bar{\omega},t) + S_1 = \int_{-1}^{1} dx \frac{2x}{(x-1/\bar{\omega})^2 + i\varepsilon} E(x,t) = 2\sum_{n=2,4,6}^{\infty} \bar{\omega}^n B_{n,0}(t), \quad |\bar{\omega}|<1$$

To fit GPD moments

$$\overbrace{\sum_{\text{spins}} \Gamma u' T^{33} \bar{u}}^{\text{Feynman-Hellmann}} \propto \sum_{n=2,4,6}^{\infty} \bar{\omega}^n \Big[\underbrace{N_\Gamma^A A_{n,0}(t) + N_\Gamma^B B_{n,0}(t)}_{\text{GPD moments}}\Big] + \mathcal{O}\left(\frac{1}{Q^2}\right).$$

Summary

- Feynman-Hellmann efficient method to calculate four-point functions in lattice QCD.
- Can be extended to off-forward kinematics.
- We can extract the off-forward Compton amplitude, and relate it to GPDs.

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Calculation Details

N_f	$\kappa_l \ \kappa_s$	$L^3 \times T$	a [fm]	M_π [MeV]
$2+1$	0.1209	$32^3 \times 64$	0.07	470

- SU(3) flavour symmetric point—u, d and s quarks have same mass.
- Heavy pion mass: ~ 470 MeV, compared to the physical point ~ 140 MeV (π^+).

Feynman-Hellmann details:

- To isolate $\lambda_1\lambda_2$, we calculate perturbed propagators with: $(\lambda, \lambda), \quad (-\lambda, -\lambda), \quad (\lambda, -\lambda), \quad (-\lambda, \lambda)$.
- Two magnitudes: $\lambda = 0.0125, 0.025$.
- Each set of perturbed propagators, insert two momenta: $\vec{q}_1$ and $\vec{q}_2$.
- These momenta define the kinematics accessible for the calculation.

Kinematics

Our momentum scalars are:

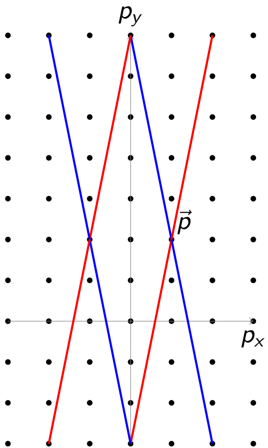
- $$\bar{\omega} = \frac{4\vec{p}\cdot(\vec{q}_1 + \vec{q}_2)}{(\vec{q}_1 + \vec{q}_2)^2}, \quad \xi \propto \vec{q}_1^{\,2} - \vec{q}_2^{\,2}.$$
- We always choose $|\vec{q}_1| = |\vec{q}_2|$, so $\xi = 0$.
- Momentum transfers:

$$t = -(\vec{q}_1 - \vec{q}_2)^2, \quad \bar{Q}^2 = \frac{1}{4}(\vec{q}_1 + \vec{q}_2)^2.$$

t determines how off-forward, while $\bar{Q}^2 \to \infty$ isolates leading-order GPDs

Note: we need to keep source/sink momenta equal magnitude: $|\vec{p}| = |\vec{p} + \vec{q}_2 - \vec{q}_1|$

Right: $\bar{\omega} = 0.8$, $\bar{Q}^2 = 25$, $t = -4$ (lattice units).



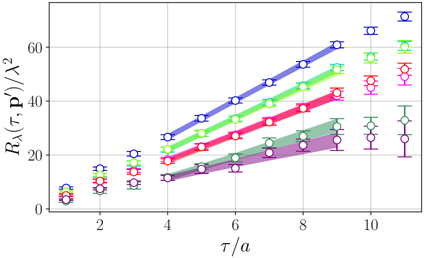
Data Sets

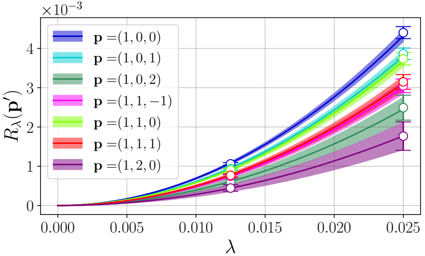
Set	$\mathbf{q}_1, \mathbf{q}_2$	t [GeV^2]	$\bar{Q}^2$ [GeV^2]	N_{meas}
#1	$(1,5,1)$, $(-1,5,1)$	-1.10	7.13	996
#2	$(4,2,2)$, $(2,4,2)$	-2.20	6.03	996

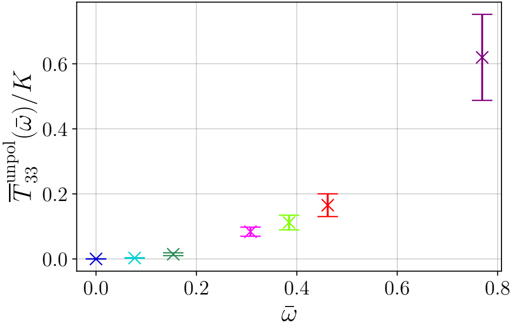
Note that the two data sets have different $\bar{Q}^2$.

- Can compare this the forward Compton amplitude, from K. U. Can et al PRD 102 (2020).
- In those results, $Q^2 = 7.13$ GeV^2, $t = 0$.

Applying Feynman-Hellmann



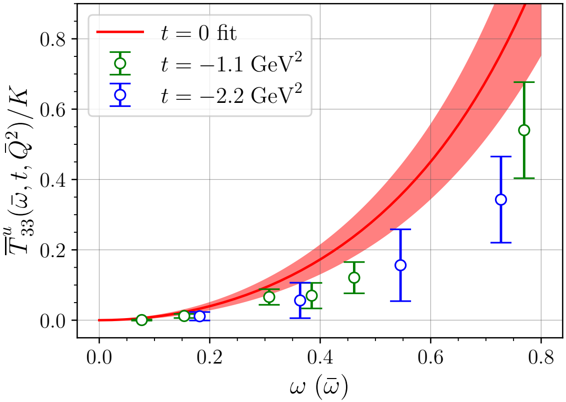




By varying the sink momentum, $\vec{p}$, we vary scaling variable:

$$\bar{\omega} = \frac{4\vec{p} \cdot (\vec{q}_1 + \vec{q}_2)}{(\vec{q}_1 + \vec{q}_2)^2}$$

Compton Amplitude



- Red curve is forward Compton amplitude: $t = 0$, $Q^2 = 7.13$ GeV^2 (PRD 102, 2020)
- up quarks, with unpolarised spin-parity projector: $T_\uparrow + T_\downarrow$

$$\overline{T}^{\text{unpol}}(\bar{\omega}, t) = 2 \sum_{n=2,4,6}^{\infty} \bar{\omega}^n \Big[A^u_{n,0}(t) + \frac{t}{2m(E+m)} B^u_{n,0}(t)\Big]$$

- Dominated by $\mathcal{H}$ CFF, equivalently $A_{n,0}$ moments
- Decrease with $-t$.

Fitting Moments

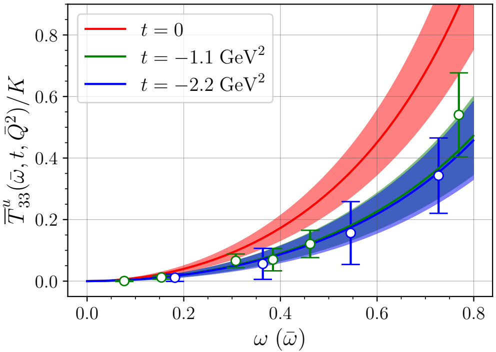
Fit our data to

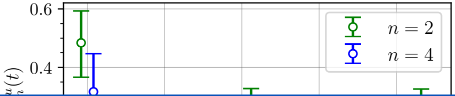
$$f_{N_{\text{max}}}(\bar{\omega}) = M_2\bar{\omega}^2 + M_4\bar{\omega}^4 + \ldots + M_{2N_{\text{max}}}\bar{\omega}^{2N_{\text{max}}}$$

To prevent over-fitting, use MCMC with monotonic decreasing priors (not rigorous): $M_2 \geq M_4 \geq \ldots \geq M_{2N_{\text{max}}-2} \geq M_{2N_{\text{max}}} \geq 0$.
Extract the linear combination of moments

$$M_n(t) \approx A_{n,0}(t) + \frac{t}{8m^2}B_{n,0}(t).$$

- Fit $N_{\text{max}} = 3$; limited by number of $\bar{\omega}$ points
- $n = 4$ moments never calculated before!
- $n = 2$ consistent with 3-pt moments at similar pion mass





Summary

- Preliminary calculation was successful
- Extracted linear combination of GPD moments ($A_{n,0}$ and $B_{n,0}$)

However,

- We would like A and B moments (equivalently $\mathcal{H}$ and $\mathcal{E}$ CFFs) separately
- More $\bar{\omega}$ values
- Better motivated fitting priors (not monotonic)

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   Preliminary!
5. Conclusions and Outlook

New data

Set	t [GeV^2]	$\bar{Q}^2$ [GeV^2]	N_{meas}
	0	4.86	10000
#1	-0.29	4.79	1000
#2	-0.57	4.86	1000
#3	-1.14	4.86	1000

For $L^3 \times T = 48^3 \times 96$, $m_\pi = 410$ MeV.

New kinematics:

- Choose momentum transfer $\vec{q}_1 - \vec{q}_2$ in same direction as EM current
- Allows use to access a linear combination of $\mathcal{H}_1$ and $\mathcal{E}_1$ (only 2 CFFs)
- Use two different spin-parity projectors—isolate each CFF.

Benefits of new calculation:

- Many more $\bar{\omega}$ values (larger lattice)
- More $-t$ values and smaller
- Can isolate $\mathcal{H}_1$ and $\mathcal{E}_1$ and therefore A and B moments

Separating $\mathcal{H}$ and $\mathcal{E}$

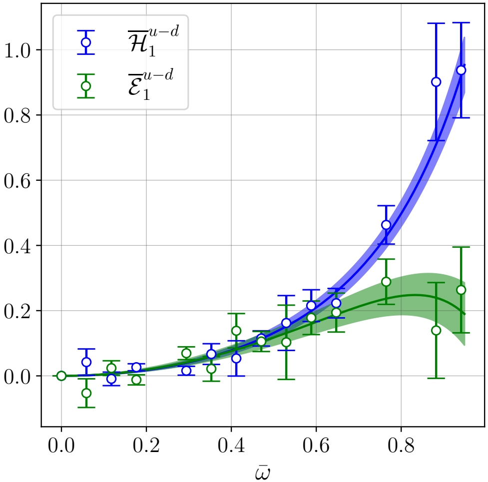
New kinematics:

$$\mathrm{tr}\left\{\Gamma \bar{u}' T_{33} u\right\} = N_\Gamma^h \mathcal{H}_1 + N_\Gamma^e \mathcal{E}_1$$

Γ is spin-parity projector.
Then, similar to elastic FFs,

$$\begin{pmatrix} \mathfrak{Re}\, T_{33}^{\mathrm{unpol}} \\ \mathfrak{Im}\, T_{33}^{\mathrm{pol}} \end{pmatrix} = \begin{pmatrix} N^h_{\mathrm{unpol}} & N^e_{\mathrm{unpol}} \\ N^h_{\mathrm{pol}} & N^e_{\mathrm{pol}} \end{pmatrix} \begin{pmatrix} \mathcal{H}_1 \\ \mathcal{E}_1 \end{pmatrix}$$

Linear combination of $\mathcal{H}$ and $\mathcal{E}$ not as orthogonal as we'd like, but still workable



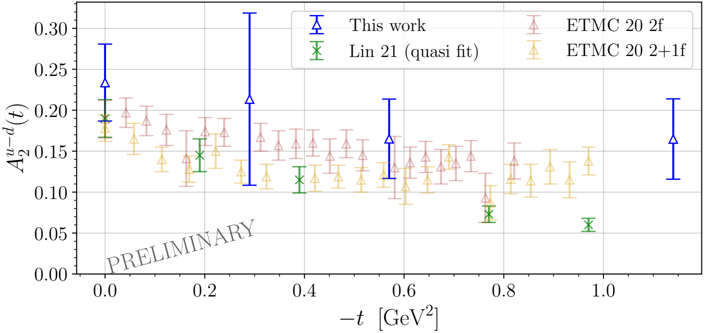
Isovector results for $t = -0.57$ GeV^2.

New moments

Model-independent GPD positivity constraints [Pobylitsa, PRD 65, 2002]:

$$\left|A_{n,0}(t)\right| \leq a_n, \quad \left|B_{n,0}(t)\right| \leq \frac{2m}{\sqrt{-t}} a_n$$

- Note: different m_π and $\bar{Q}^2 \approx 5\ \text{GeV}^2$
- Different systematics between all three
- But still consistent—less so with quasi at larger $-t$



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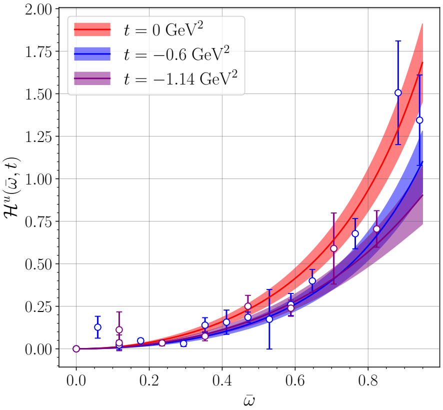
Conclusion:

- New method to calculate OFCA → GPDs
- Allows strong parallels with experiment: scaling, subtraction function, higher-twist
- Calculation of leading moments of $\mathcal{H}_1$ and $\mathcal{E}_1$

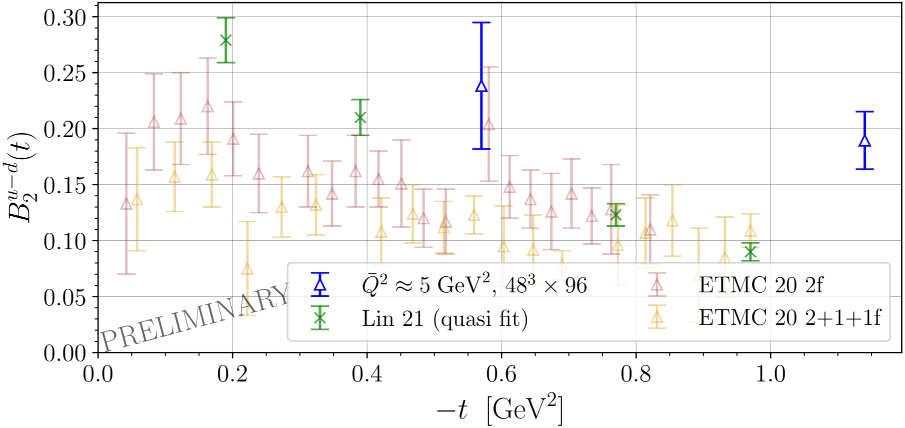
Outlook:

- Beyond leading moments: GPD model fits, inversion methods
- Off-forward analogue of $\mathcal{F}_2$: test off-forward Callan-Gross—test higher-order
- More $\bar{Q}^2$ values: $2 - 10$ GeV^2
- More t values (drop equal energy condition, extend FH)



Thank you for listening—questions?

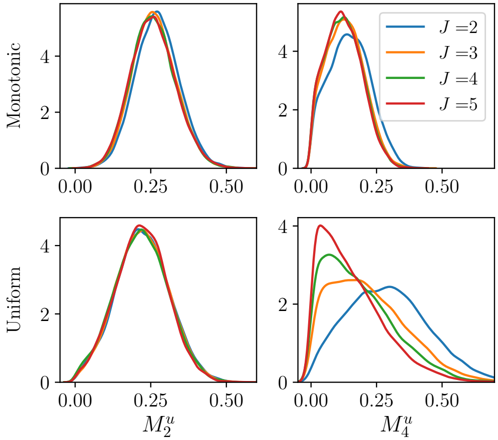
$\mathcal{E}_1$ moments



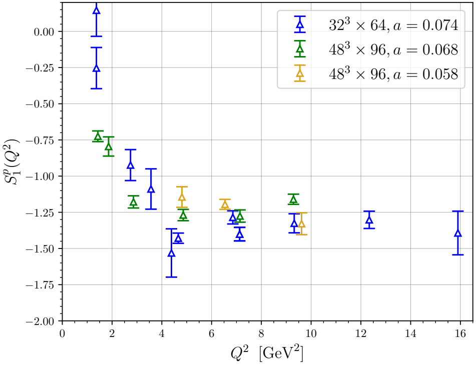
Markov chain Monte Carlo fits

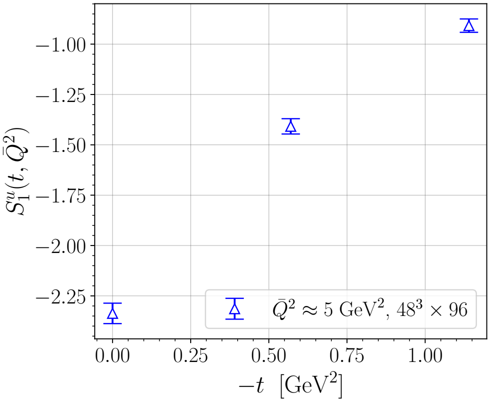
Compare fits:

- J is number of moments.
- Uniform priors are $[0, 100]$
- For $32^3 \times 64$ dataset—few $\bar{\omega}$ values.
- Yet to repeat with new data set or new priors.



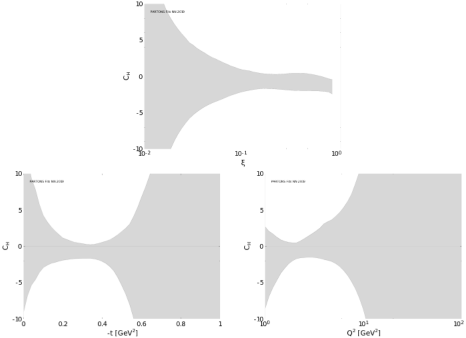
Subtraction Term



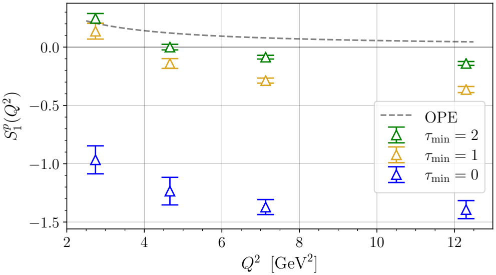


OPE predicts $S_1(Q^2) \to 0$ with $Q^2 \to \infty$.

Subtraction Term



Subtraction function from DVCS; input for calculation of proton pressure distribution.



Varies with discretisation: see forthcoming proceedings.