Theoretical Simulation of the Virtual Meson Production in the Forward Direction

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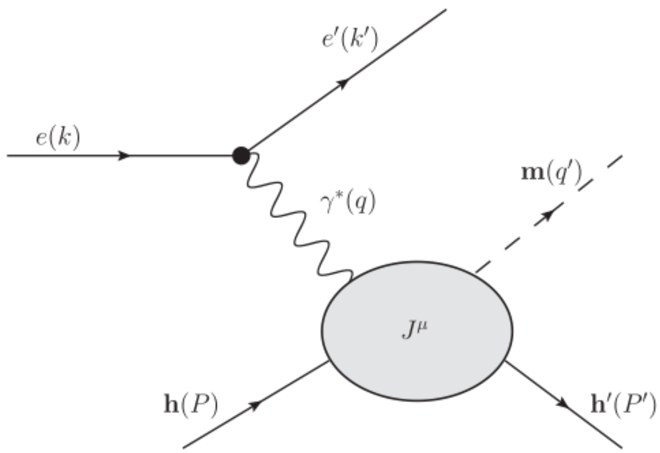
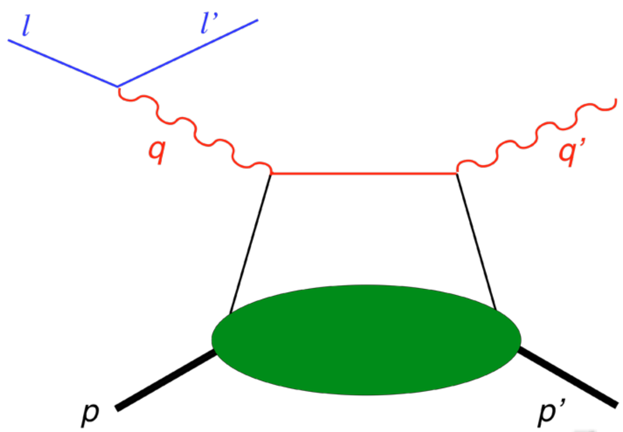
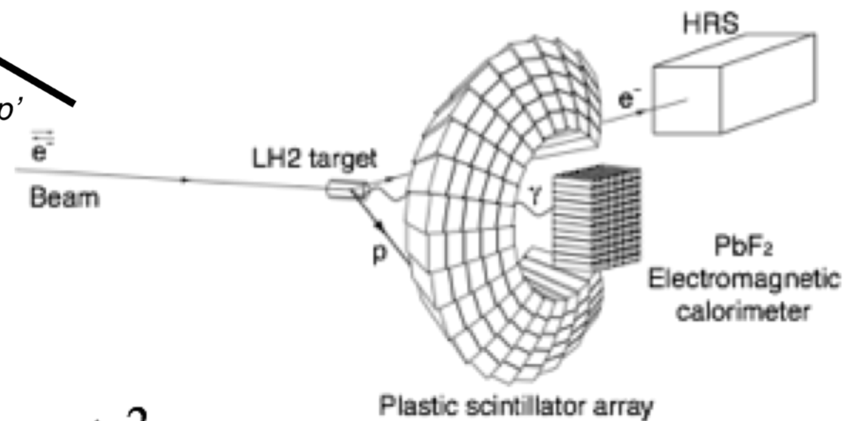


May, 23, 2022

Better Work in Forward Direction





GPD **LFD**

$$t = \Delta^2 = -\frac{\zeta^2 M^2 + \Delta_\perp^2}{1-\zeta}; \Delta^+(\equiv \Delta^0 + \Delta^3) = \zeta P^+; \Delta_\perp^2 > \Delta_{\perp\min}^2 \neq 0$$

PHYSICAL REVIEW D **105,** 096014 (2022)

Analysis of virtual meson production in a (1 + 1)-dimensional scalar field model

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Light-front dynamic analysis of the longitudinal charge density using the solvable scalar field model in (1 + 1) dimensions

Yongwoo Choi,[1] Ho-Meoyng Choi,[2,*] Chueng-Ryong Ji,[3,†] and Yongseok Oh[1,4,‡]

Outline

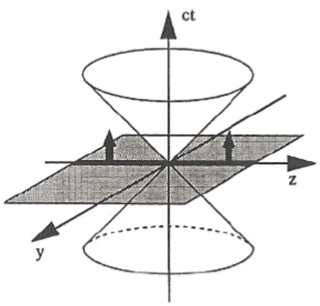
- Dirac's Proposition for Relativistic Dynamics

Instant Form Dynamics(IFD) vs. Light-Front Dynamics(LFD)

- Link between IFD and LFD : **IMF $\neq$ LFD**
- Virtual Meson Production off a Scalar Target
- Benchmarking GPD Applicability in Forward Direction
- GPD Sum Rule and Valence/Nonvalence Decomposition
- Conclusion and Outlook

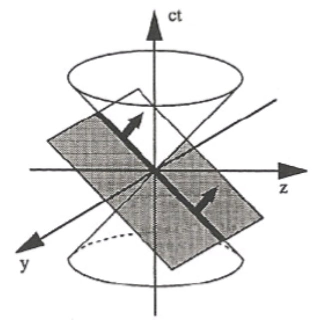
Dirac's Proposition for Relativistic Dynamics

$k_1^+ = k_2^+ = 0$


The instant form

1949

$$\begin{array}{ccc} Equal\ t & & Equal\ \tau \\ p^0 & \leftrightarrow & p^- = p^0 - p^3 \\ (p^1, p^2) & \leftrightarrow & \vec{p}_\perp \\ p^3 & \leftrightarrow & p^+ = p^0 + p^3 \end{array}$$

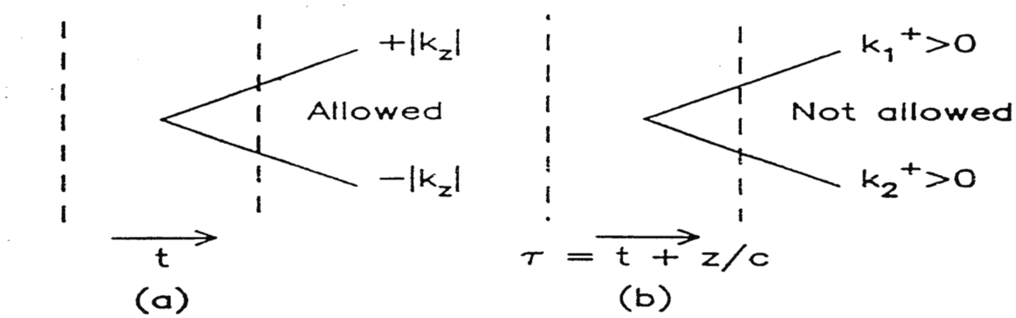

The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$

IFD

Instant Form Dynamics



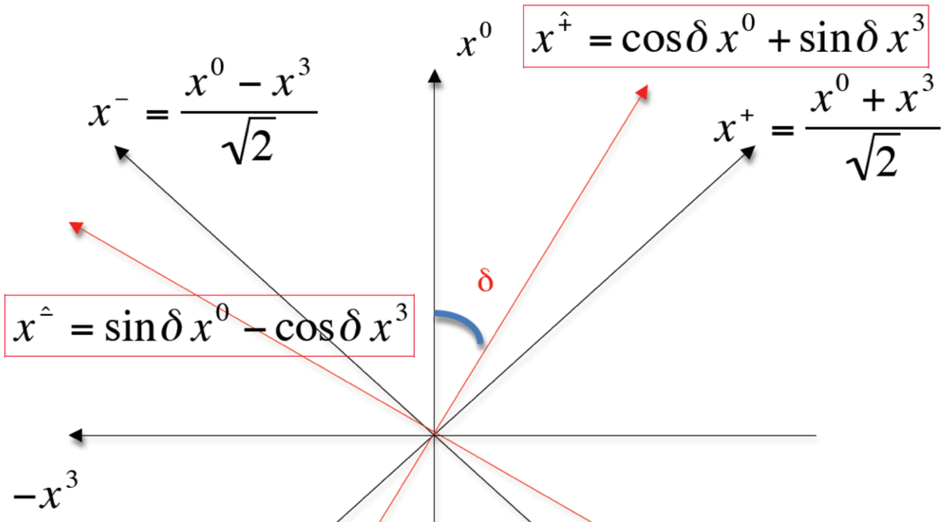
Except zero-modes

$k_1^+ = k_2^+ = 0$

LFD

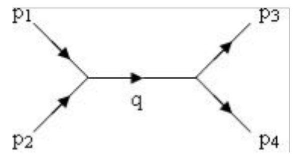
Light-Front Dynamics

Interpolation between IFD and LFD



$$(IFD) \quad 0 \le \delta \le \frac{\pi}{4} \quad (LFD)$$
$$1 \ge \quad C \equiv \cos(2\delta) \quad \ge 0$$

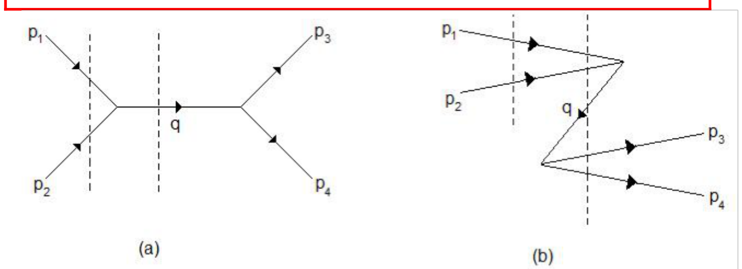
K. Hornbostel, PRD45, 3781 (1992) – RQFT
C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly
C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps
C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges
Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors
C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED
B.Ma and C.Ji, PRD104, 036004(2021) – QCD_{1+1}



$$\delta = 0$$
$$p_0 = p^0$$
$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$
$$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$$
$$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$$

$$\delta = \pi/4$$
$$p_+ = p^-$$
$$p_- = p^+$$



$$\frac{1}{2q^0}\left(\frac{1}{p_1^0+p_2^0-q^0} - \frac{1}{p_1^0+p_2^0+q^0}\right)$$

$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}}+\frac{\mathbb{S}q_{\hat{-}}-\omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}}+\frac{\mathbb{S}q_{\hat{-}}+\omega_q}{\mathbb{C}}}\right)$$

$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2\right)}$$

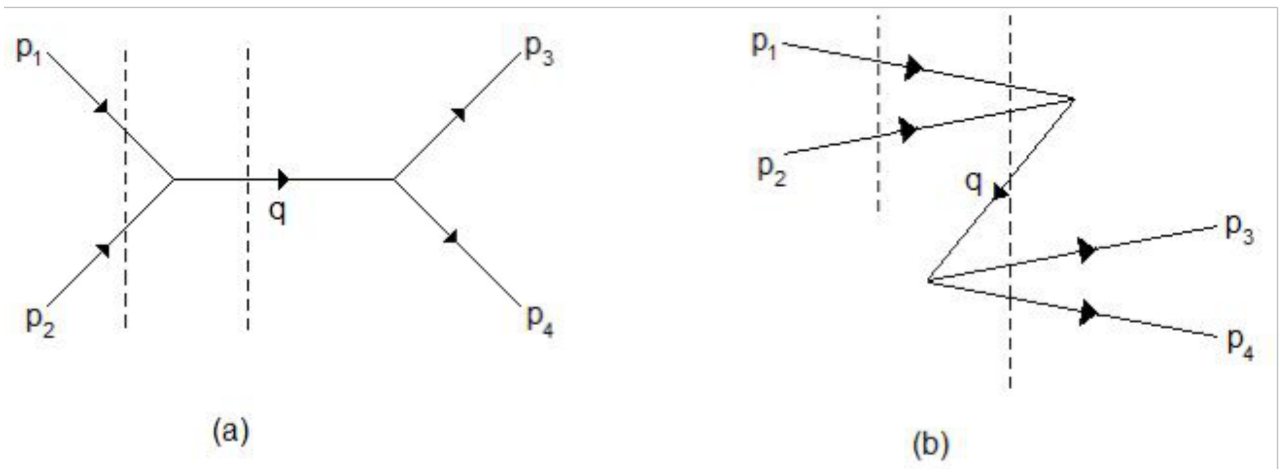
$$\mathbb{C} = \cos 2\delta$$

$$\mathbb{S} = \sin 2\delta$$

$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{\mathbf{P}}_\perp^2+m^2)}{2P^+}\right\}}$$

$$\frac{\mathbb{S}q_{\hat{-}}+\omega_q}{\mathbb{C}} \to \frac{2}{\mathbb{C}} - \frac{\vec{\mathbf{q}}_{\hat{\perp}}^2+m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$
$$\to \infty \quad \text{as } \mathbb{C} \to 0$$

Infinite Momentum Frame (IMF) Approach



$$\frac{1}{E_1 + E_2 - Eq}$$

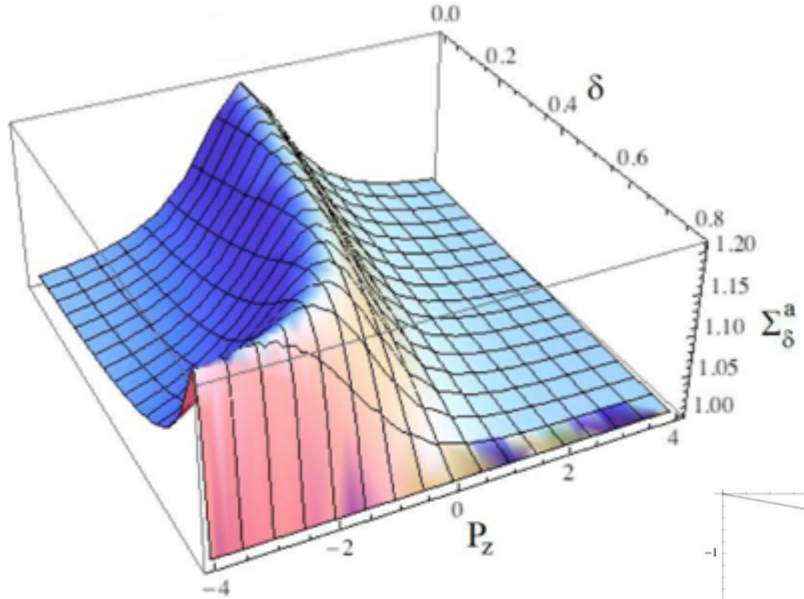
$$-\frac{1}{Eq + E_3 + E_4}$$

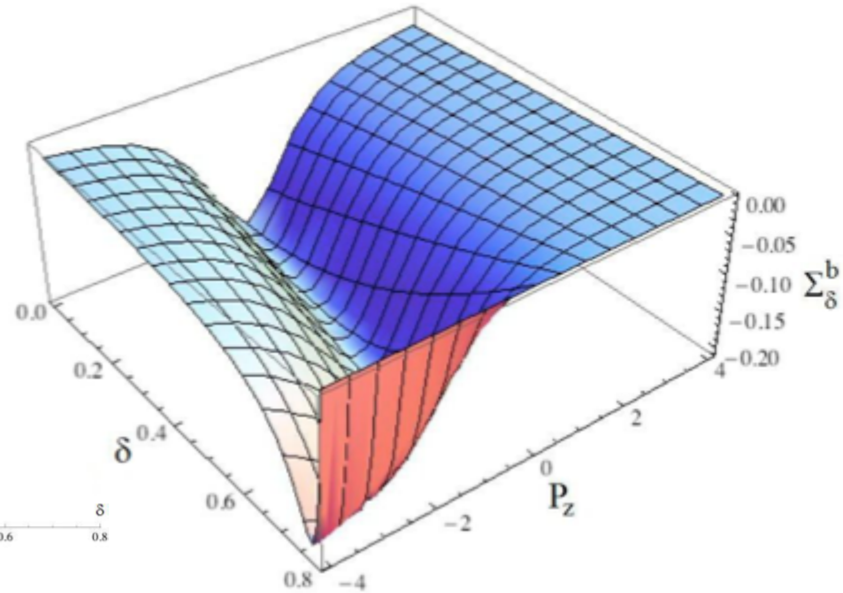
$$= -\frac{1}{Eq + E_1 + E_2}$$

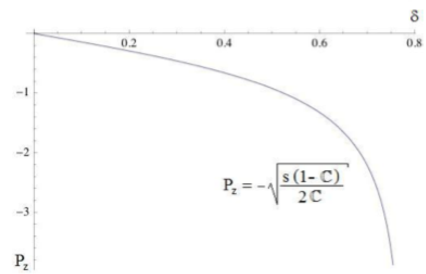
$$\rightarrow \quad 0$$

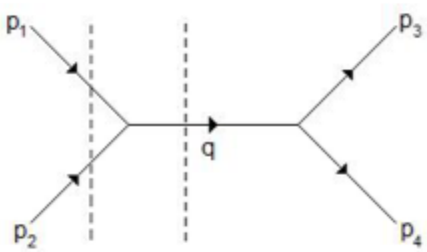
S.Weinberg, PR158,1638(1967)
"Dynamics at Infinite Momentum"

Note that this is still in the instant form (IFD).





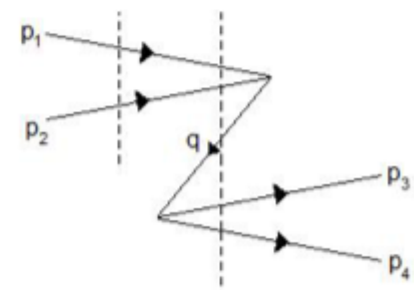




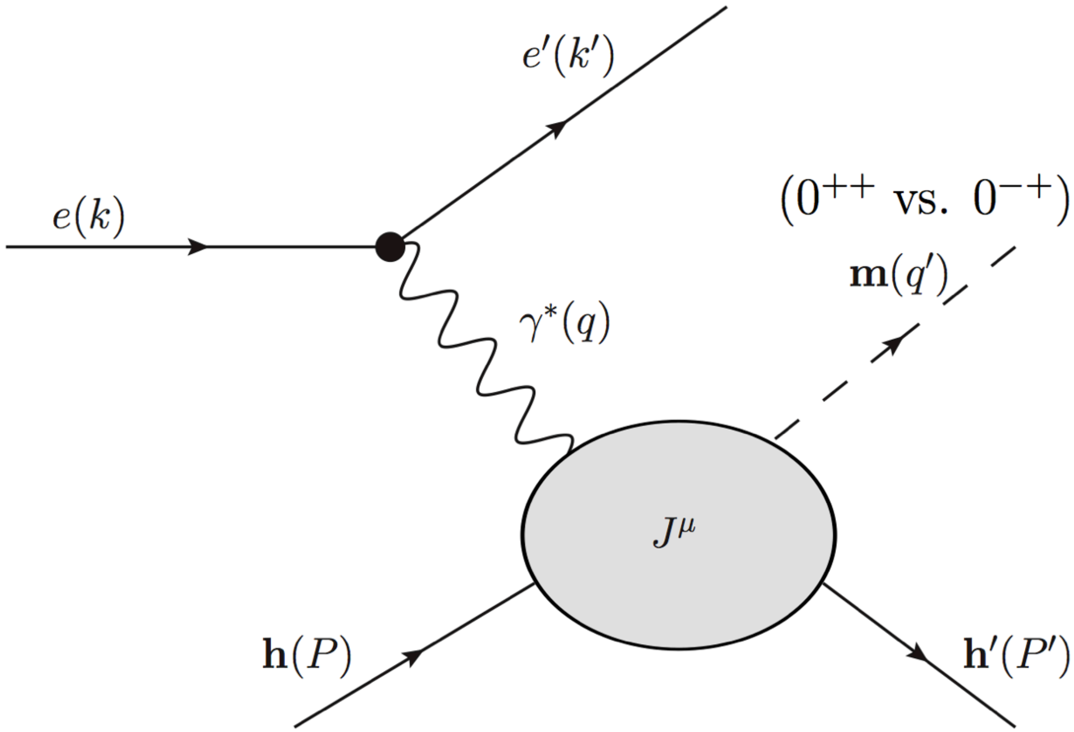
$\Sigma(a)+\Sigma(b)=1/(s-m^2)$; $s=2\ GeV^2$, $m=1GeV$

J-shape peak & valley : $P_z = -\sqrt{\frac{s(1-C)}{2C}}$; $C = \cos(2\delta)$

As $C \rightarrow 0$, $P^+ = P^0+P_z \rightarrow 0$ leads to LF Zero-modes.



Virtual Meson Production off a Scalar Target



C.Ji, H.-M.Choi, A.Lundeen, B.Bakker, PRD99,116008(2019)

Salient Features

- No interference with the Bethe-Heitler process
- Consistency between our benchmark BSA prediction for 0^{-+} meson production off the scalar target with the data of the exclusive coherent electroproduction of the π^0 off ^4He measured at JLab Hall B
- General formulation of hadronic amplitudes

  in Meson Production off the Scalar Target (0^{++} vs. 0^{-+})
- Comparison/Contrast with the leading twist GPD formulation.

Beam-Spin Asymmetry of Exclusive Coherent Electroproduction of the π^0 Off ^{4}He

Frank Thanh Cao, Ph.D.

University of Connecticut, 2019

To understand the partonic structure of nucleons in nuclei, extracting the beam spin asymmetry (BSA) from exclusive processes is an important measurement to get at the so-called Generalized Parton Distributions (GPDs) that describe the partons behavior inside the nucleon. In particular, BSA in Deeply Virtual Meson Production (DVMP) can offer valuable constraints on the transverse GPDs which are not accessible through Deeply Virtual Compton Scattering (DVCS).

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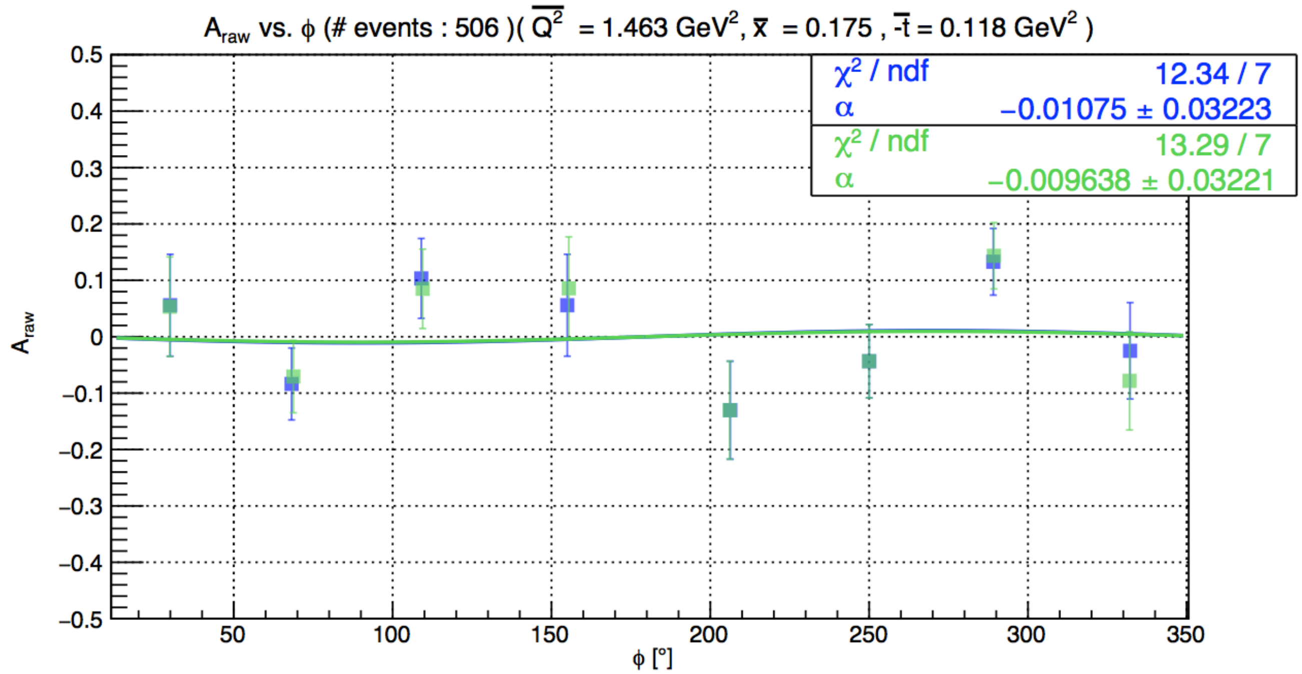
This benchmark measurement is in agreement with symmetry arguments presented in a recent theoretical formulation [2] that offers a framework complementary to that of the GPDs and gives confidence in the assumptions made for future studies of exclusive nuclear processes.



$$A_{LU}(\phi) = A_{LU}^{90^\circ} \sin\phi$$

$$A_{LU}^{90^\circ} = -1.08 \pm 3.22\,(\text{stat.}) \pm 2.83\,(\text{sys.})\,\%$$

4-26-2019

Beam-Spin Asymmetry of Exclusive Coherent Electroproduction of the π0 Off 4He

Frank Thanh Cao

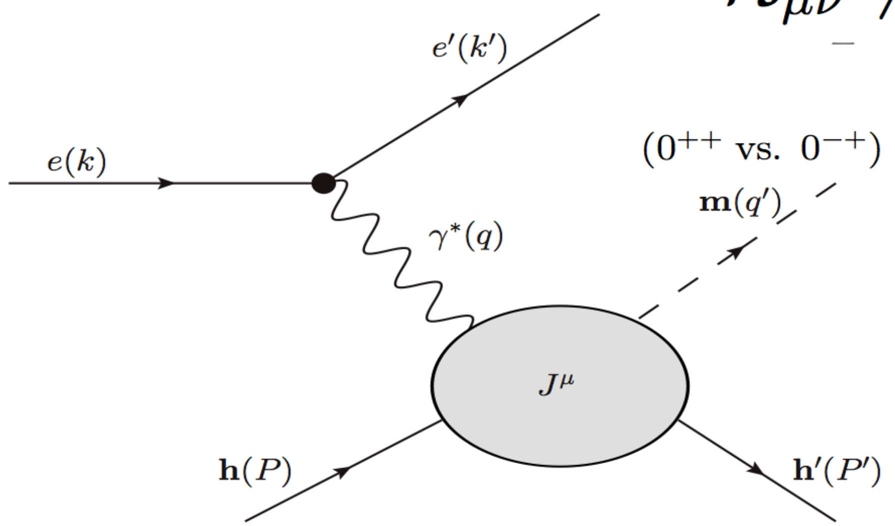
University of Connecticut - Storrs, franktcao@gmail.com

$$\langle|\mathcal{M}|^2\rangle \;=\; \left(\frac{e^2}{q^2}\right)^2 \mathcal{L}^{\mu\nu}\mathcal{H}_{\mu\nu}$$

$$\mathcal{L}^{\mu\nu} = q^2\left[g^{\mu\nu} + \frac{2}{q^2}(k^\mu k'^\nu + k'^\mu k^\nu)\right] + 2ih\epsilon^{\mu\nu\alpha\beta}k_\alpha k'_\beta$$

$$\mathcal{H}_{\mu\nu} = J^\dagger_\mu J_\nu$$

$$\mathcal{H}_{\mu\nu} \neq \mathcal{H}_{\nu\mu}$$



$$\boxed{J^{\mu}_{PS} = F_{PS}\epsilon^{\mu\nu\alpha\beta} q_{\nu}\bar{P}_{\alpha}\Delta_{\beta}}$$

$$\begin{aligned} \mathcal{H}_{\mu\nu} &= J^{\dagger}_{\mu} J_{\nu} \\ &= |F_{PS}|^2 \epsilon_{\mu\alpha\beta\gamma}\epsilon_{\nu\alpha'\beta'\gamma'} q^{\alpha}\bar{P}^{\beta}\Delta^{\gamma} q^{\alpha'}\bar{P}^{\beta'}\Delta^{\gamma'} \\ &= \mathcal{H}_{\nu\mu} \end{aligned}$$

$$\boxed{\epsilon^{\mu\nu\alpha\beta} k_{\alpha} k'_{\beta} \mathcal{H}_{\mu\nu} = 0}$$

$$\frac{d\sigma^{PS}_{h=+1} - d\sigma^{PS}_{h=-1}}{d\sigma^{PS}_{h=+1} + d\sigma^{PS}_{h=-1}} = 0$$

Pseudoscalar(0^{-+}) Meson vs. Scalar(0^{++}) Meson

$$\epsilon^{\mu\nu\alpha\beta} \quad \text{vs.} \quad d^{\mu\nu\alpha\beta} = g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta}$$

C.Ji & B.Bakker, PoS QCDEV2017,038(2017);
B.Bakker & C.Ji, Few Body Syst. 58,no.1,8(2017)

$$F_{PS}(Q^2, t, x)$$

$$\boxed{J^\mu_S = (S_q q_\alpha + S_{\bar{P}} \bar{P}_\alpha) d^{\mu\nu\alpha\beta} q_\beta \Delta_\nu}$$

$$\boxed{J^\mu_{PS} = F_{PS}\epsilon^{\mu\nu\alpha\beta} q_\nu \bar{P}_\alpha \Delta_\beta}$$

$$F_1 = S_q - S_{\bar{P}}$$
$$F_2 = S_{\bar{P}}$$

$$F_1(Q^2, t, x) \qquad F_2(Q^2, t, x)$$

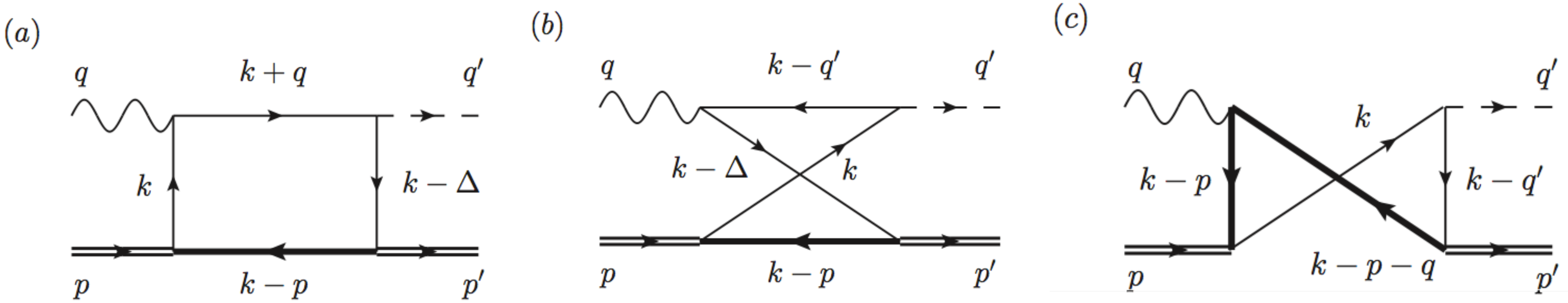
$$J^\mu_S = F_1(q^2\Delta^\mu - q^\mu q\cdot\Delta) + F_2[(\bar{P}\cdot q + q^2)\Delta^\mu - (\bar{P}^\mu + q^\mu) q\cdot\Delta]$$

$$q \;\; ; \;\; \bar{P} = P + P' \;\; ; \;\; \Delta = P - P' = q' - q$$

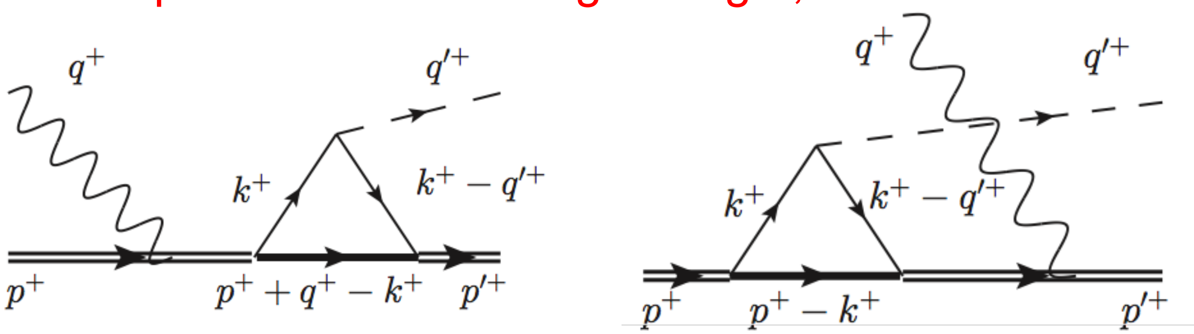
Scalar Field Model Simulation of VMP in Forward Direction

$$\text{“}\gamma^* + {}^4\text{He} \rightarrow f_0(980) + {}^4\text{He}\text{”}$$

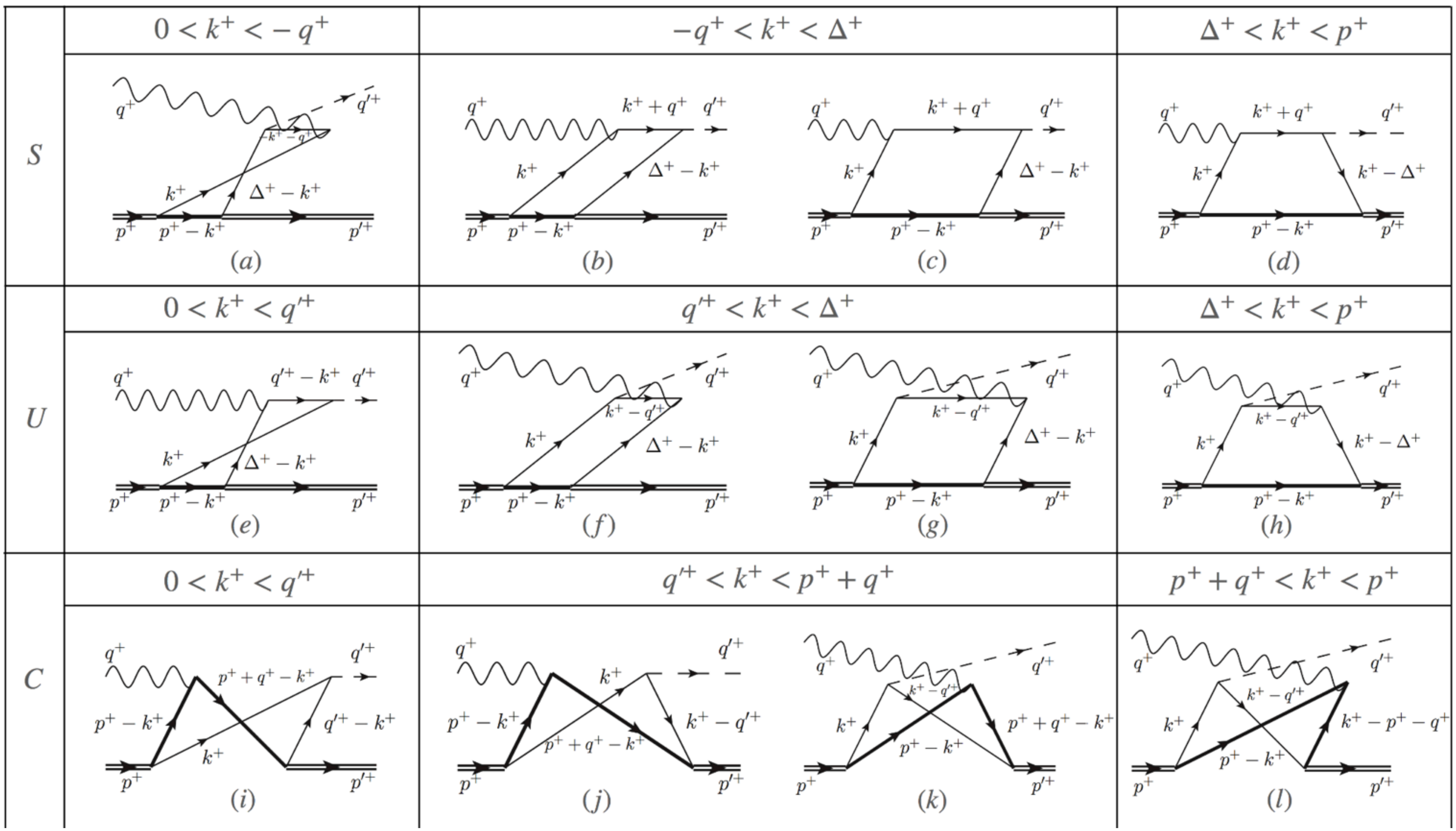
$$\mathcal{M}^{\mu(1+1)}_{\text{tot}} = [(\Delta \cdot q)q^\mu - q^2\Delta^\mu]\mathcal{F}$$

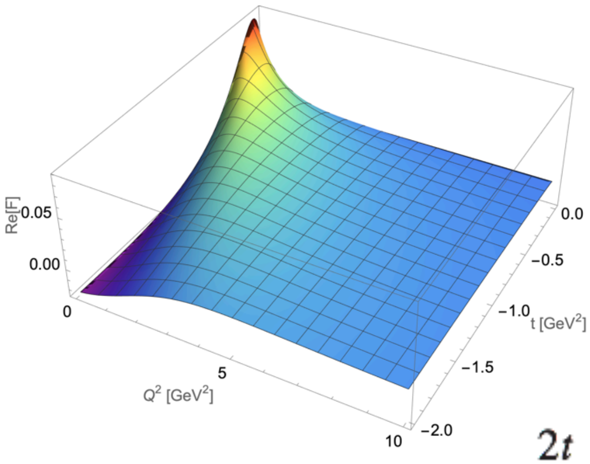


Two more amplitudes for the charged target, but not for the neutral target



Light-Front Time-Ordered Amplitudes

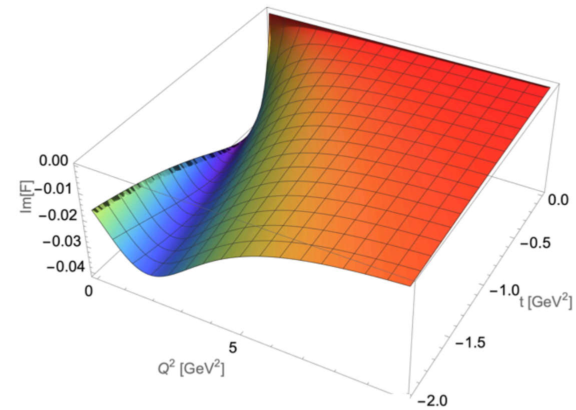




Compton Form Factor

$$\mathcal{M}^{\mu(1+1)}_{\text{tot}} = [(\Delta \cdot q) q^{\mu} - q^2 \Delta^{\mu}] \mathcal{F}$$

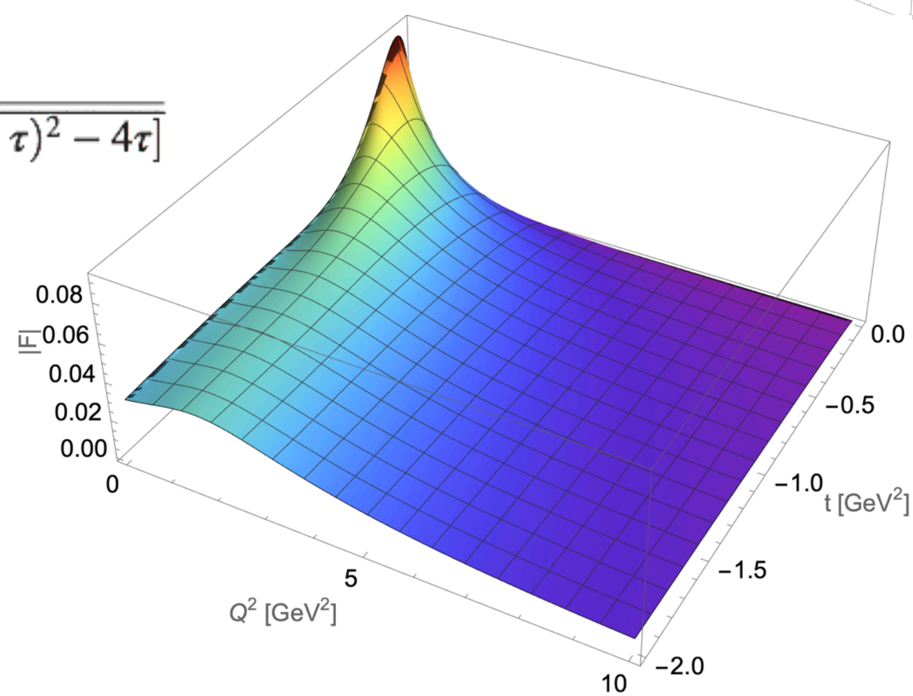
$$\mathcal{F}^{\text{VMP}}_{c}(Q^2, t) = \frac{\mathcal{M}^{\mu}_{\text{charged}}}{(\Delta \cdot q) q^{\mu} - q^2 \Delta^{\mu}}$$

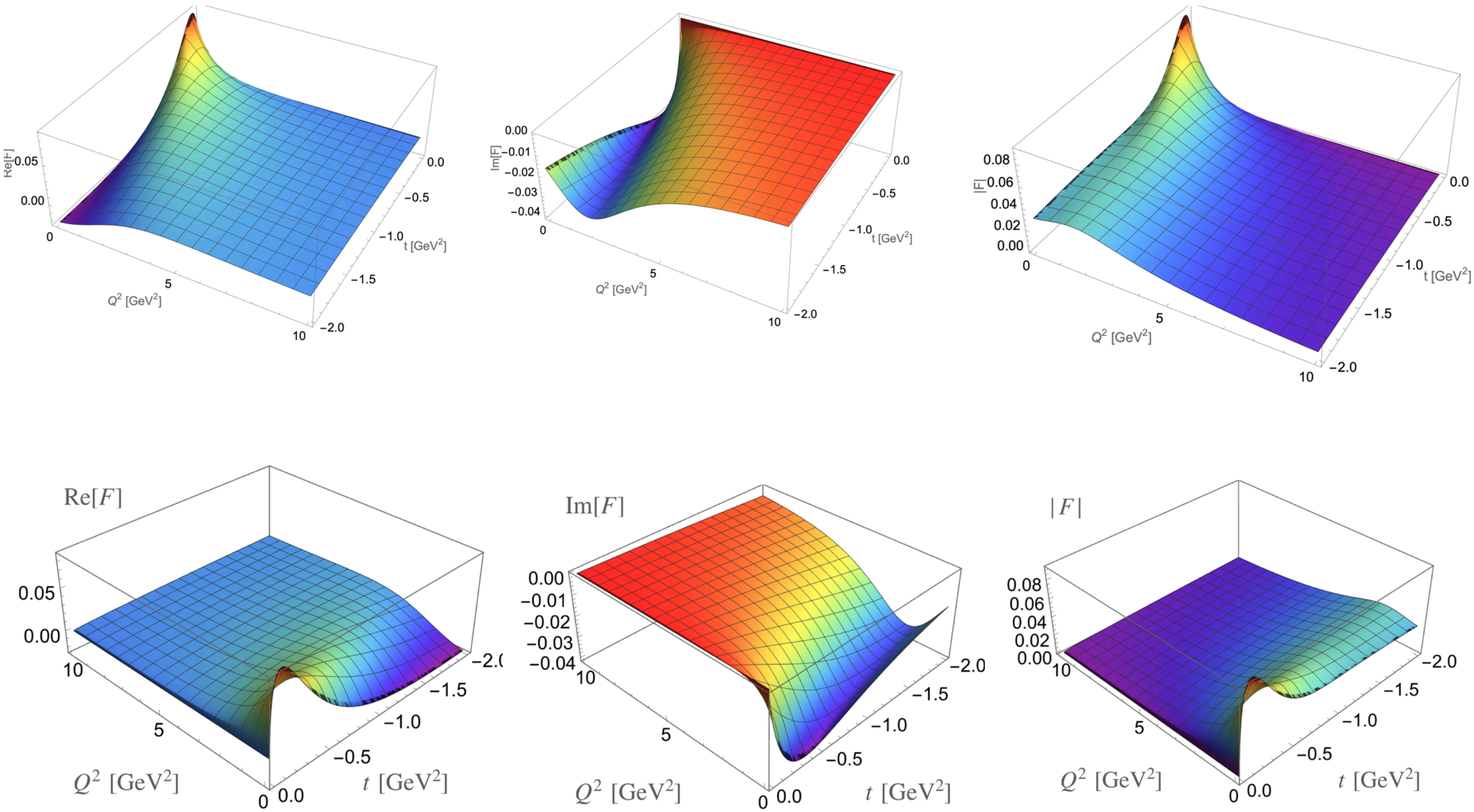


$$x_{\text{Bj}} = \frac{2t}{t(1+\mu_s+\tau) - \sqrt{t(t-4M_T^2)[(1+\mu_s+\tau)^2 - 4\tau]}}$$

$$\zeta = \frac{1}{2M_T^2}\left(t + \sqrt{t^2 - 4tM_T^2}\right)$$

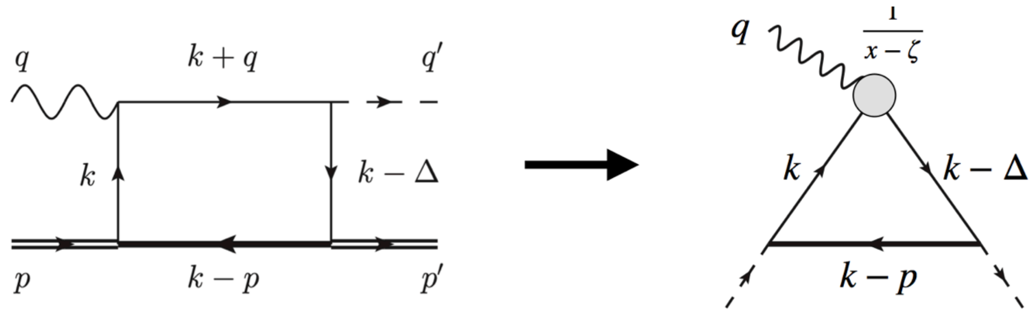
$$\mu_s = M_S^2/Q^2 \quad \text{and} \quad \tau = -t/Q^2$$





Re[F]
0.05
0.00
0
5
10
Q² [GeV²]
0.0
−0.5
−1.0
−1.5
−2.0
t [GeV²]
Im[F]
0.00
−0.01
−0.02
−0.03
−0.04
|F|
0.08
0.06
0.04
0.02

DVMP Reduction to GPD in S-channel with + Current



$$\mathcal{M}_s^\mu \sim \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - m^2} \boxed{\frac{2k^\mu + q^\mu}{(k+q)^2 - m^2}} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}$$

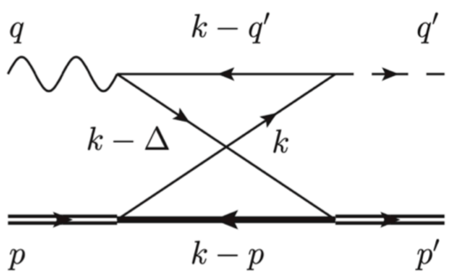
For a plus current of a virtual photon,

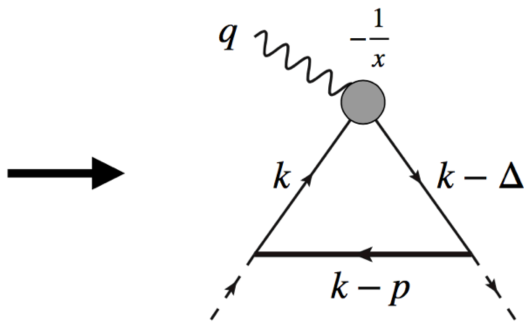
$$\boxed{\frac{2k^+ + q^+}{(k^+ + q^+)(k^- - k_t^-)}} \frac{1}{k^2 - m^2} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2} \quad \textbf{where} \quad k_t^- = -q^- + \frac{m_{Q_1}^2}{k^+ + q^+} - i\frac{\epsilon}{k^+ + q^+}$$

For large Q^2 : $\boxed{\quad\quad} \simeq \frac{1}{(x-\zeta)} \frac{\zeta'}{Q^2} (2x - \zeta) + \mathcal{O}\left(\frac{1}{Q^4}\right)$

$$= \boxed{\frac{1}{x-\zeta}} \frac{\zeta'}{Q^2} \boxed{\frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}}$$

DVMP Reduction to GPD in U-channel with + Current





$$\mathcal{M}_u^\mu \sim \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - m^2} \boxed{\frac{2k^\mu - \Delta^\mu - q'^\mu}{(k-q')^2 - m^2}} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}$$

For a plus current of a virtual photon,

$$\boxed{\frac{2k^+ - \Delta^+ - q'^+}{(k^+ - q'^+)(k^- - k_u^-)}} \frac{1}{k^2 - m^2} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}$$ **where** $$k_u^- = q'^- + \frac{m_{Q_1}^2}{k^+ - q'^+} - i\frac{\epsilon}{k^+ - q'^+}$$

For large Q^2 : $$\boxed{} \simeq -\frac{1}{x}\frac{\zeta'}{Q^2}(2x - \zeta) + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

$$= \boxed{-\frac{1}{x}} \frac{\zeta'}{Q^2} \boxed{\frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}}$$

$$\mathcal{M}^{+}_{Leading} = \frac{1}{4\pi}\,\frac{\zeta'}{Q^2}\left(\frac{1}{x-\zeta} - \frac{1}{x}\right)\frac{1}{k^2-m^2}\,\frac{2k^+ - \Delta^+}{p^+}\,\frac{1}{(k-\Delta)^2 - m^2}\,\frac{1}{(k-p)^2 - M^2}$$

$$A^+ = (\Delta\cdot q)q^+ - q^2\Delta^+ = \frac{1}{2}Q^2\zeta p^+\left[1 + \frac{t}{Q^2} + \cdots\right]$$

$$\frac{\mathcal{M}^{+}_{Leading}}{A^{+}_{Leading}} = \frac{1}{2\pi}\,\frac{1}{Q^4}\left(\frac{1}{x-\zeta} - \frac{1}{x}\right)\frac{1}{k^2-m^2}\,\frac{2k^+ - \Delta^+}{p^+}\,\frac{1}{(k-\Delta)^2 - m^2}\,\frac{1}{(k-p)^2 - M^2}$$

DVMP Reduction to GPD with - Current works as well.

$$\frac{\mathcal{M}^{-}_{Leading}}{A^{-}_{Leading}} = \frac{1}{2\pi}\,\frac{1}{Q^4}\left(\frac{1}{x-\zeta} - \frac{1}{x}\right)(2x-\zeta)\,\frac{1}{k^2-m^2}\,\frac{1}{(k-\Delta)^2 - m^2}\,\frac{1}{(k-p)^2 - M^2}$$

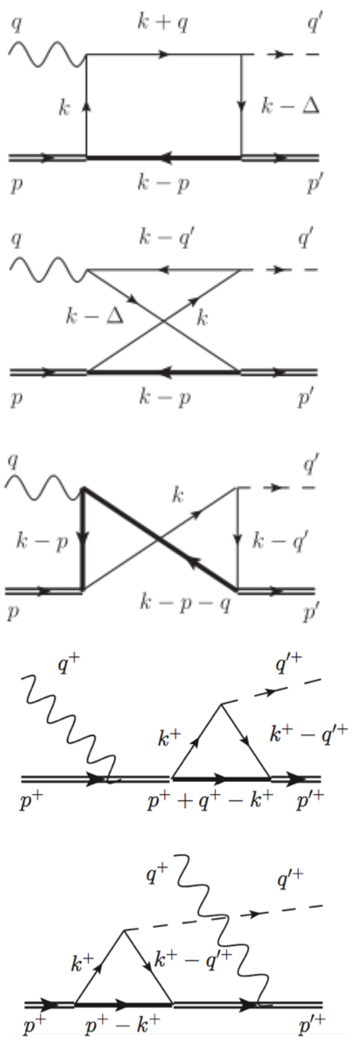
$$M^\mu = A^\mu F$$

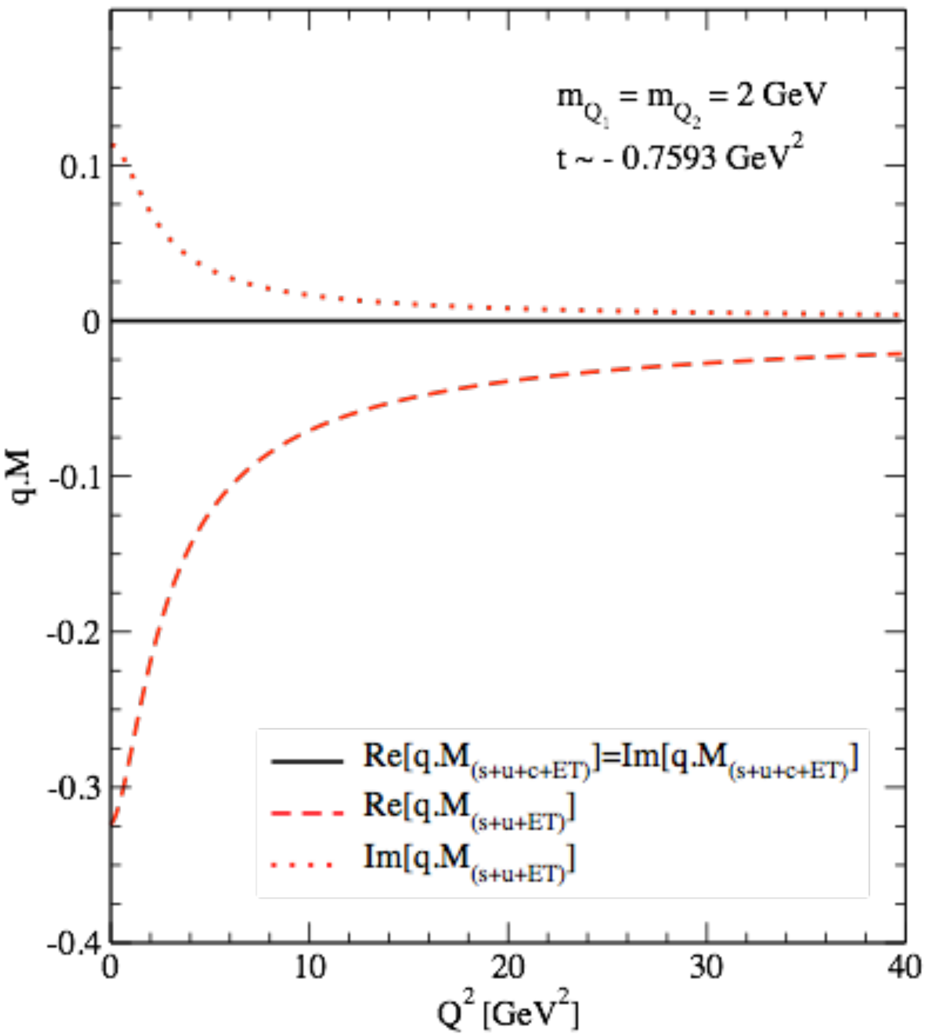
$$\mathscr{M}^\mu_{Leading} \sim \frac{q^\mu - 2q'^\mu}{q^2} \boxed{\left(\frac{1}{x-\zeta} - \frac{1}{x}\right)} \boxed{\frac{1}{k^2 - m^2} (2x - \zeta) \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}}$$

$$(\Delta \cdot q)\, q^\mu - q^2\, \Delta^\mu \longrightarrow \frac{Q^2}{2} (2\, q'^\mu - q^\mu)$$

Gauge Invariance
works asymptotically:

$$q' \cdot q \rightarrow q^2 / 2$$



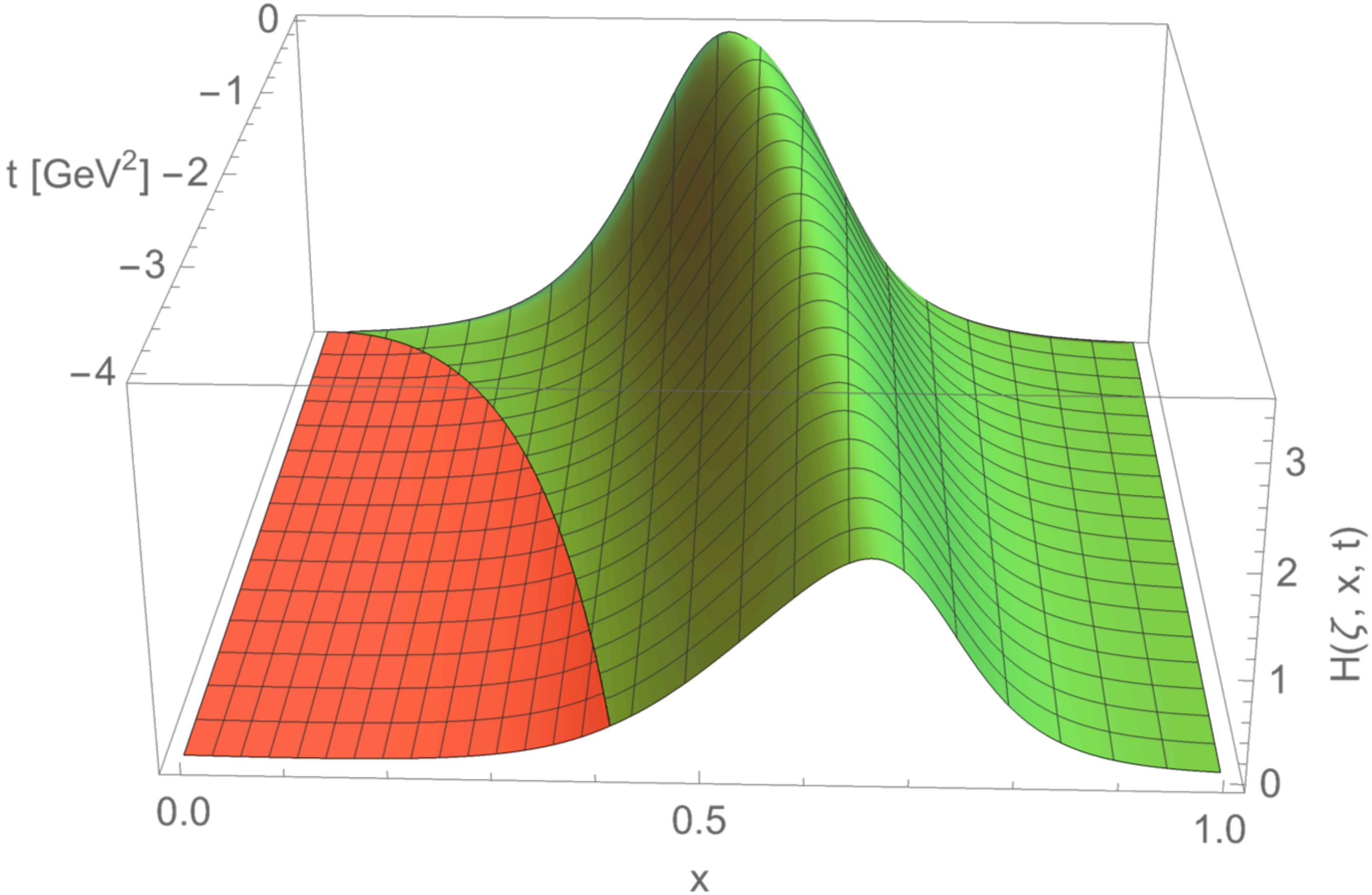


$$(\Delta \cdot q)\, q^{\mu} - q^{2}\, \Delta^{\mu}$$

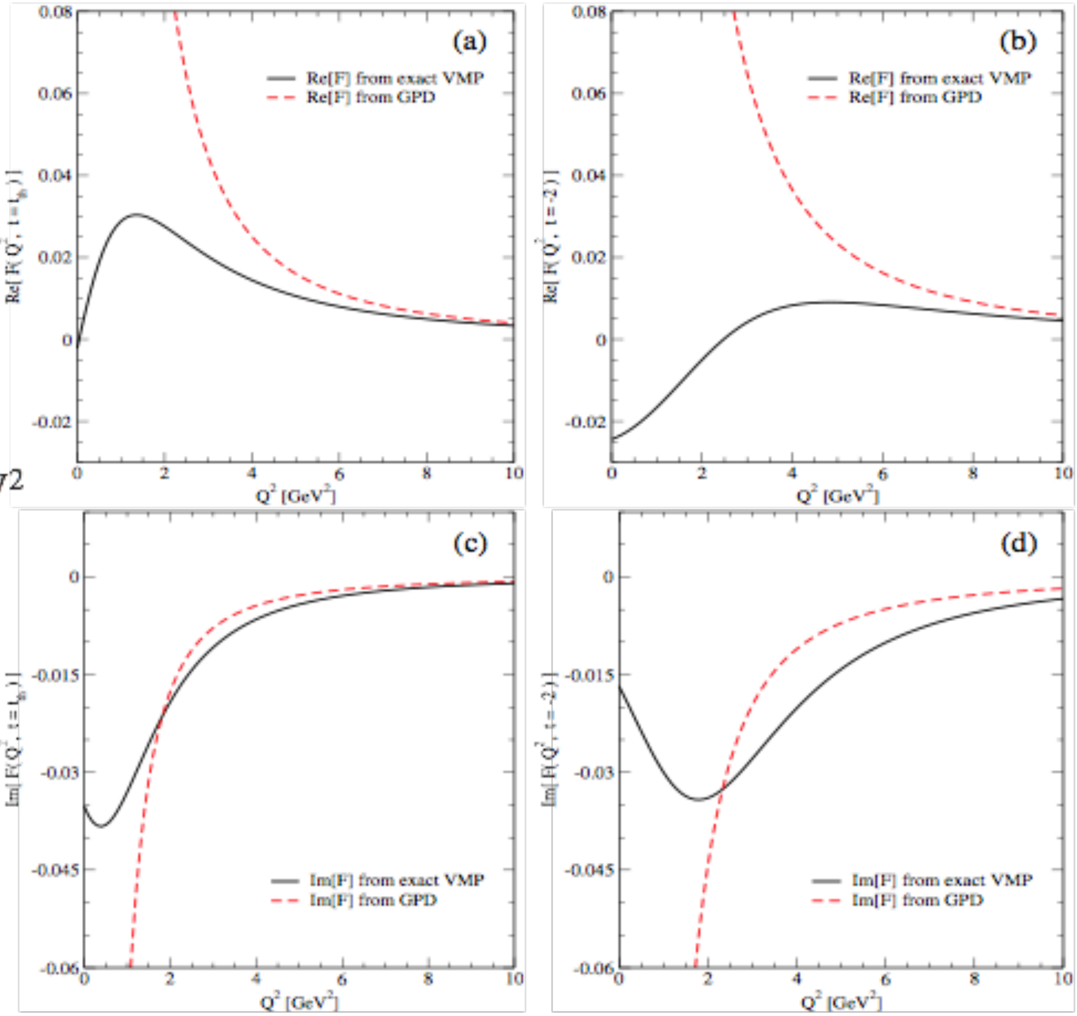
$$\downarrow$$

$$\frac{Q^{2}}{2}\,(2\, q'^{\mu} - q^{\mu})$$

$$\mathcal{M}^{\pm \mathrm{DVMP}}_{s+u} = \frac{e_{Q_1}\zeta}{4\pi Q^2}\int_0^1 dx\left(\frac{1}{x-\zeta} - \frac{1}{x}\right)H(\zeta, x, t)$$



$t = t_{\text{th}} \simeq -0.7593\,\text{GeV}^2$

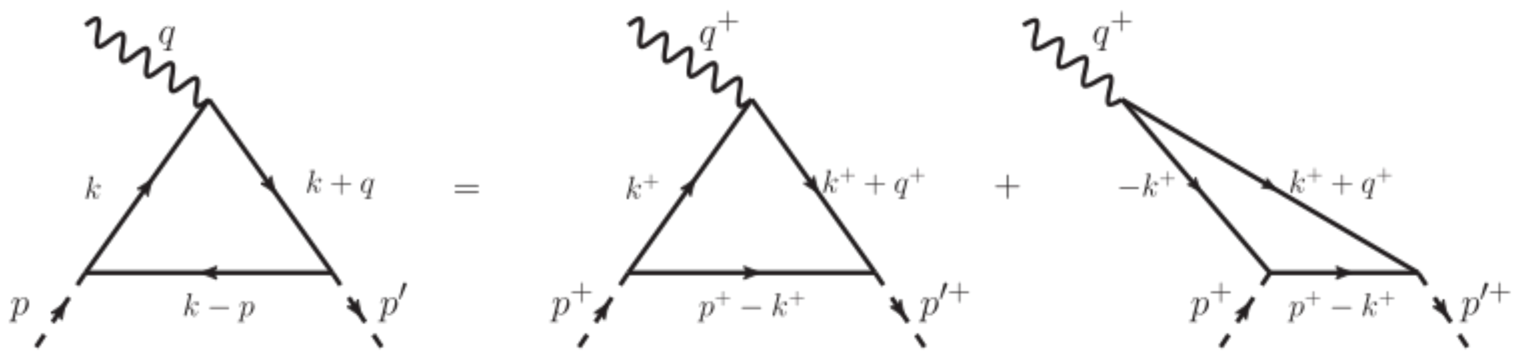


$t = -2\,\text{GeV}^2$

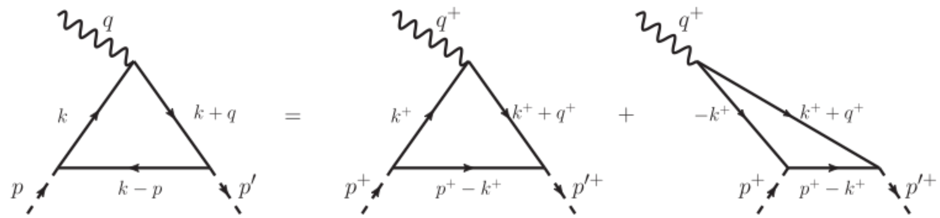
$-t/Q^2 \lesssim 0.1$

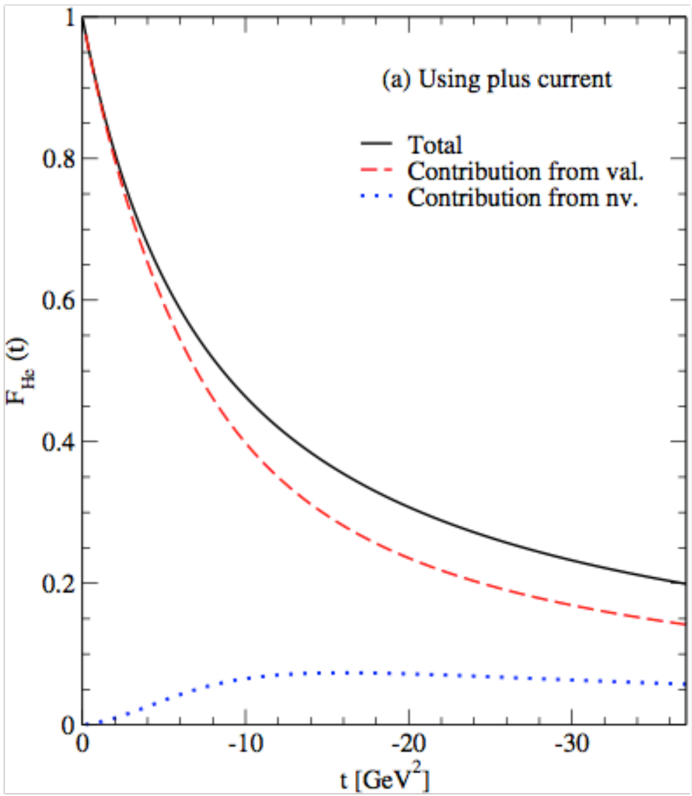
$$F_{\mathcal{M}}(t) = \int_0^1 \frac{dx}{1-\zeta/2} H(\zeta, x, t).$$

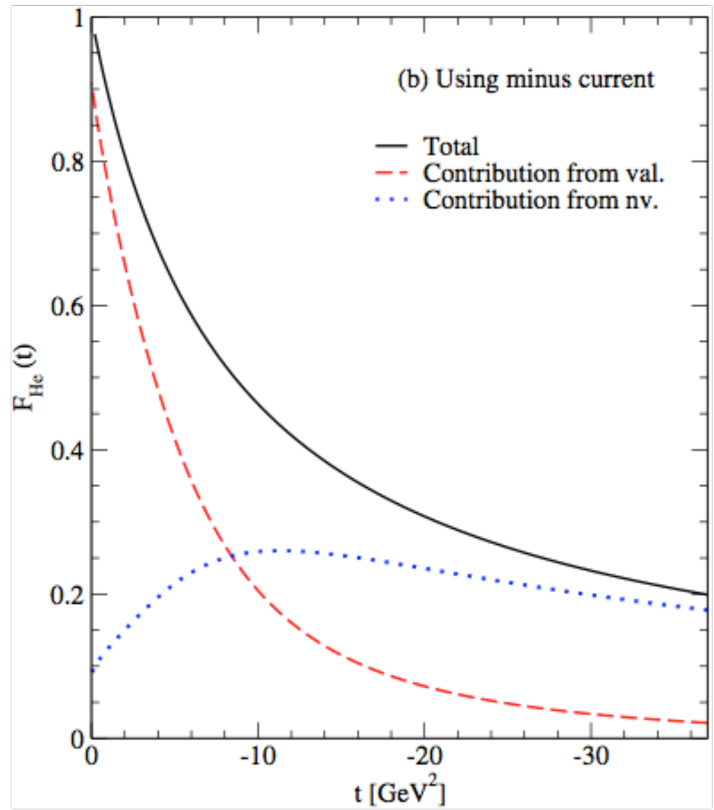
$$H(\zeta,x,t) = \begin{cases} H_{\mathrm{ERBL}}(\zeta,x,t), & \text{for } 0 \le x \le \zeta, \\ H_{\mathrm{DGLAP}}(\zeta,x,t), & \text{for } \zeta \le x \le 1 \end{cases}$$



$$J_S^{\mu}(0) = (p + p')^{\mu} F_{\mathcal{M}}^{S}(q^2)$$


q
k
k + q
=
p
k − p
p′
q+
k+
k+ + q+
+
p+
p+ − k+
p′+
−k+


(a) Using plus current
Total
Contribution from val.
Contribution from nv.
F_He (t)
t [GeV^2]
1
0.8
0.6
0.4
0.2
0
-10
-20
-30


(b) Using minus current
Total
Contribution from val.
Contribution from nv.
F_He (t)
t [GeV^2]
1
0.8
0.6
0.4
0.2
0
-10
-20
-30

Decomposition of the Form Factor

$$\sum_{i=V,NV} J_i^\mu(t) = (P+P')^\mu \sum_{i=V,NV} F_i(t) = (2P^\mu - \Delta^\mu) \sum_{i=V,NV} F_i(t)$$

$$J_V^\mu(t) = \int_{\Delta^+}^{P^+} dk^+ \int dk^- \frac{2k^\mu - \Delta^\mu}{D_V}$$

$$J_{NV}^\mu(t) = \int_0^{\Delta^+} dk^+ \int dk^- \frac{2k^\mu - \Delta^\mu}{D_{NV}}$$

$$\frac{2k^\mu - \Delta^\mu}{2P^\mu - \Delta^\mu} = \frac{2x-\zeta}{2-\zeta} \quad only \quad if \quad \Delta^\mu = \zeta P^\mu \quad or \quad q^\mu = q'^\mu + \zeta P^\mu$$

Note here that $(q-q')^2 = \Delta^2 = t = \zeta^2 M^2 > 0$

while $t < 0$ in DVMP.

Conclusion and Outlook

- Unless small $|t|/Q^2$, "Cat's ears" contribution should not be neglected.
- Sum rule correspondence between DGLAP/ERBL GPDs and Valence/Nonvalence contributions to the form factor works only for a certain current component.
- Form factor decomposition depends on the current component although the form factor itself is independent of the choice of the current component.(Democracy in current components)
- 3+1 D extension with BSA investigation is underway.
- Application to the energy-momentum tensor decomposition appears feasible.