UNIVERSITÄT BONN

bctp Bethe Center for Theoretical Physics

Next-to SV resumed Drell-Yan cross section beyond leading-logarithmic accuracy

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based on work in collaboration with
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and
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Introduction

- **What is Next-to-SV (NSV) corrections ?**

  - In QCD improved parton model, **Hadron Collider :** transformed into "**Parton collider**" via Parton distribution functions (pdfs) :

$$\sigma(\tau, q^2) = \sigma_0(\mu_R^2) \sum_{ab=q,\bar{q},g} \int dx_1 dx_2 dz f_a(x_1, \mu_F^2)\ \Delta_{ab}(z, q^2, \mu_R^2, \mu_F^2)\ f_b(x_2, \mu_F^2) \delta(\tau - z x_1 x_2)$$

  - **Definitions** :
    - Δ_{ab} : Finite Partonic Coefficient Function (CF), q: scale of the process,
    - $\sqrt{\hat{s}}$: partonic centre of mass energy , $z = \frac{q^2}{\hat{s}}$: partonic scaling variable.

  - The Partonic Coefficient Function **near threshold**, $z \to 1$:

$$\Delta_{ab} \overset{z\to 1}{\approx} a_i \left[\frac{\ln^i(1-z)}{1-z}\right]_+ + b\ \delta(1-z) + c_i \ln^i(1-z) + d$$

- **Soft-Virtual (SV)**
- **Resummation to N³LL accuracy**

- **Next-to-soft virtual (NSV)**
- **Resummation to LL accuracy**

Introduction

- **Why Next-to-SV corrections ?**
  - **Significant contributions to the hadronic cross-section : Because of large coefficients**

    [Anastasiou, Duhr, Dulat et al.(`14,`19, `20)]

a_s^3	$\ln^5(1-z)$	$\left[\frac{\ln^5(1-z)}{1-z}\right]_+$	$\ln^4(1-z)$	$\left[\frac{\ln^4(1-z)}{1-z}\right]_+$	TOTAL NLP	TOTAL LP
$gg \to H$	117.95%	96.72%	103.36%	20.648%	25.83%	-2.28%
Drell-Yan	8.59%	5.44%	9.82%	2.62%	1.49%	0.02%

% to the total cross-section relative to the leading order contribution

  - **But these logarithms give large contributions in certain kinematic region : Spoils perturbativity of the series**
  - **Resolution : Find a way to resum NSV logarithms beyond Leading logarithms (LL).**

Previous Works

- Early attempts :
  - Kraemer, Laenen, Spira (98),
  - Akhoury, Sotiropoulos & Sterman (98)

- Important Results & Predictions using Physical Kernel Approach & explicit computation:
  - Moch , Vogt et al. (09-20),
  - Anastasiou, Duhr, Dulat et al.(14).

- Universality of NSV effects and LL Resummation:
  - Laenen, Magnea, et al. (08-19),
  - Grunberg & Ravindran (09),
  - Ball, Bonvini, Forte, Marzani, Ridolfi (13),
  - Del Duca et al. (17).

- Subleading Factorisation and LL Resummation at NSV using SCET:
  - Larkoski, Nelli , Stewart et al. (14) ,
  - Kolodrubetz, Moult, Neill ,Stewart et al. (17),
  - Beneke et al. (19-21).

Formalism

- **SV was well understood** through the seminal work of Sterman, Catani et.al.

- SV formalism was earlier applied for DY, Higgs production and DIS based on the following: **[Ravindran (`05, `06)]**
  - Collinear factorisation.
  - Renormalization Group Invariance
  - Logarithmic structure of perturbative quantities in dimensional regularisation.

- Extend the very formalism for **NSV logarithms of the Diagonal Channels for** color singlet processes.

- We start with the mass factorisation formula: ϵ : **Dimensional Regularization** parameter

$$\frac{1}{z}\hat{\sigma}_{ab}(z,\epsilon) = \sigma_0 \sum_{a'b'} \Gamma^T_{aa'}(\mu_F^2, z, \epsilon) \otimes \left(\Delta_{a'b'}(\mu_F^2, z, \epsilon)\right) \otimes \Gamma_{b'b}(\mu_F^2, z, \epsilon)$$

Partonic cross section containing only Initial state collinear singularities

Collinear Finite

Altarelli-Parisi Splitting Kernel (Collinear Singular)

Formalism

- **Since we want to obtain SV and NSV terms it is sufficient to keep terms which gives SV and/or NSV upon convolutions.**

- **Hence we can safely drop terms like :**

$$\Gamma^{(0)}_{qq} \otimes \Delta^{(1)}_{qg} \otimes \Gamma^{(1)}_{g\bar{q}} \longrightarrow (\mathbf{1-z})^{\alpha}, \forall \alpha > 0 \text{ NNSV terms}$$

- **We find that only Diagonal channel and Diagonal AP kernels contribute and so :**

$$\frac{1}{z}\hat{\sigma}_{c\bar{c}}(z,\epsilon) = \sigma_0 \Gamma_{cc}(\mu_F^2, z, \epsilon) \otimes \left(\Delta_{c\bar{c}}(\mu_F^2, z, \epsilon) \right) \otimes \Gamma_{\bar{c}\bar{c}}(\mu_F^2, z, \epsilon)$$

- **This gives rise to a decomposition Formula :**

$$\Delta_{c\bar{c}}(z, \epsilon, q^2 \mu_R^2, \mu_F^2) = \left(\Gamma^T\right)^{-1}_{cc} \otimes \left\{ \left(Z_{c,UV} \right)^2 |\hat{F}_c(Q^2, \epsilon)|^2 S_c(q^2, z, \epsilon) \right\} \otimes \left(\Gamma\right)^{-1}_{\bar{c}\bar{c}}$$

- **Each building block obeys first order differential equations and additional evolution equations w.r.t factorisation scale (μ_F, μ_R)**

Building blocks

- Form Factor, $\hat{F}_c$:
  - Captures virtual corrections.

[Sterman,Sen, Magnea]

  - Functional form :

$$\ln \hat{F}_c(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \sum_{j=-\infty}^{i+1} \mathcal{L}_c^{(i,j)} \frac{1}{\epsilon^j}$$

  - Expressed in terms of : $\mathcal{L}_c^{(i,j)} = \{A^c, B^c, f^c, \gamma^c, g^c\}$
  - Process-Independent : $\{A^c, B^c, f^c, \gamma^c\}$
  - Process-Dependent : $\{g^c\}$

- Overall Renormalization constant, $Z_{c,UV}$:
  - Functional form :

$$\ln Z_{c,UV}(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \sum_{j=1}^{i} \mathcal{Z}_c^{(i,j)} \frac{1}{\epsilon^j}$$

  - Expressed in terms of : $\mathcal{Z}_c^{(i,j)} = \{\gamma^c\}$
  - Process-Independent : $\{\gamma^c\}$: UV anomalous dimension
  - Renormalization scale : μ_R

Building blocks

- **Soft-Collinear Function, S_c:**
  - **Born normalized Soft and collinear contributions.**
  - **Functional form :**

[Ajjath, Ravindran, PM(`20)]

$$\ln S_c(\hat{a}_s, q^2, \mu^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2} \right)^{i\frac{\epsilon}{2}} S_\epsilon^i \, \varphi_c^{(i)}(z, \epsilon)$$

  - **$\{\epsilon\}$ dependency :**

$$\varphi_c^{(1)}(z,\varepsilon) = \frac{1}{\varepsilon}\mathcal{G}^c_{L,1}(z,\varepsilon)\,,$$
$$\varphi_c^{(2)}(z,\varepsilon) = \frac{1}{\varepsilon^2}\left(-\beta_0 \mathcal{G}^c_{L,1}(z,\varepsilon)\right) + \frac{1}{2\varepsilon}\mathcal{G}^c_{L,2}(z,\varepsilon)$$
$$\varphi_c^{(3)}(z,\varepsilon) = \frac{1}{\varepsilon^3}\left(\frac{4}{3}\beta_0^2 \mathcal{G}^c_{L,1}(z,\varepsilon)\right) + \frac{1}{\varepsilon^2}\left(-\frac{1}{3}\beta_1 \mathcal{G}^c_{L,1}(z,\varepsilon) - \frac{4}{3}\beta_0 \mathcal{G}^c_{L,2}(z,\varepsilon)\right) + \frac{1}{3\varepsilon}\mathcal{G}^c_{L,3}(z,\varepsilon)$$

  - **Hence using the RG evolution of strong coupling constant and the energy evolution equation of S_c we derive the functional form till 4-loop.**

Building blocks

- $\{z\}$ dependency :

SV	NSV
$\mathcal{G}^{c}_{L,1}(z,\varepsilon) = \frac{2A_1}{1-z} + \epsilon \frac{\mathcal{G}^{c,(1)}_{sv,1}}{1-z} + \mathcal{O}(\epsilon^2)$	$\mathcal{G}^{c}_{L,1}(z,\varepsilon) = 2D_1 + 2C_1 \ln(1-z) + \epsilon\ \mathcal{G}^{c,(1)}_{nsv,1}(z) + \mathcal{O}(\epsilon^2)$
$\mathcal{G}^{c}_{L,2}(z,\varepsilon) = \frac{2A_2}{1-z} - 2\beta_0 \frac{\mathcal{G}^{c,(1)}_{sv,1}}{1-z} + + \mathcal{O}(\epsilon)$	$\mathcal{G}^{c}_{L,2}(z,\varepsilon) = 2D_2 + 2C_2 \ln(1-z) - 2\beta_0 \mathcal{G}^{c,(1)}_{nsv,1}(z) + \mathcal{O}(\epsilon)$

- **Here the NSV coefficient is parametrised as :**

$$\mathcal{G}^{c,(j)}_{nsv,i}(z) = \sum_{k=0}^{i+j-1} \mathcal{G}^{c,(j,k)}_{nsv,i} \ln^k(1-z)$$

- **The Fixed Order result known till N^3LO demonstrate the above logarithmic structure and hence we propose an ansatz to all orders.**

- **The SV and NSV coefficients are determined from the explicit computations.**

PREDICTIONS

- **With these building blocks we have a structure for $\Delta_{c\bar{c}}$:**

$$\ln \Delta_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z, \varepsilon) = \left(\ln\left(Z_{UV,c}(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon) \right)^2 + \ln \left| \hat{F}_c(\hat{a}_s, \mu^2, Q^2, \varepsilon) \right|^2 \right) \delta(1-z)$$
$$+ \ln S_c(\hat{a}_s, \mu^2, q^2, z, \varepsilon) - 2C \ln \Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon) \,.$$

- **What do we achieve as a consequence to this decomposition:** $L_z^i = \ln^i(1-z)$

GIVEN	PREDICTIONS					
FO Coefficient	$\Delta_{c\bar{c}}^{(2)}$	$\Delta_{c\bar{c}}^{(3)}$	$\Delta_{c\bar{c}}^{(4)}$	$\Delta_{c\bar{c}}^{(5)}$	$\Delta_{c\bar{c}}^{(6)}$	$\Delta_{c\bar{c}}^{(i)}$
χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

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$$+ \ln S_c(\hat{a}_s, \mu^2, q^2, z, \varepsilon) - 2C \ln \Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon) \,.$$

- What do we achieve as a consequence to this decomposition: $L_z^i = \ln^i(1-z)$

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FO Coefficient	$\Delta_{c\bar{c}}^{(2)}$	$\Delta_{c\bar{c}}^{(3)}$	$\Delta_{c\bar{c}}^{(4)}$	$\Delta_{c\bar{c}}^{(5)}$	$\Delta_{c\bar{c}}^{(6)}$	$\Delta_{c\bar{c}}^{(i)}$
1-loop χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

PREDICTIONS

- With these building blocks we have a structure for $\Delta_{c\bar{c}}$:

$$\ln \Delta_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z, \varepsilon) = \left(\ln\left(Z_{UV,c}(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon) \right)^2 + \ln \left| \hat{F}_c(\hat{a}_s, \mu^2, Q^2, \varepsilon) \right|^2 \right) \delta(1-z)$$
$$+ \ln S_c(\hat{a}_s, \mu^2, q^2, z, \varepsilon) - 2C \ln \Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon) \, .$$

- What do we achieve as a consequence to this decomposition:

$$L_z^i = \ln^i(1-z)$$

	GIVEN	PREDICTIONS					
	FO Coefficient	$\Delta_{c\bar{c}}^{(2)}$	$\Delta_{c\bar{c}}^{(3)}$	$\Delta_{c\bar{c}}^{(4)}$	$\Delta_{c\bar{c}}^{(5)}$	$\Delta_{c\bar{c}}^{(6)}$	$\Delta_{c\bar{c}}^{(i)}$
1-loop	χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
2-loop	χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
	χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

PREDICTIONS

- **With these building blocks we have a structure for $\Delta_{c\bar{c}}$:**

$$\ln \Delta_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z, \varepsilon) = \left(\ln\left(Z_{UV,c}(\hat{a}_s, \mu^2, \mu_R^2, \varepsilon) \right)^2 + \ln\left| \hat{F}_c(\hat{a}_s, \mu^2, Q^2, \varepsilon) \right|^2 \right) \delta(1-z)$$
$$+ \ln S_c(\hat{a}_s, \mu^2, q^2, z, \varepsilon) - 2C \ln \Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \varepsilon) \, .$$

- **What do we achieve as a consequence to this decomposition:** $L_z^i = \ln^i(1-z)$

	GIVEN	PREDICTIONS					
	FO Coefficient	$\Delta_{c\bar{c}}^{(2)}$	$\Delta_{c\bar{c}}^{(3)}$	$\Delta_{c\bar{c}}^{(4)}$	$\Delta_{c\bar{c}}^{(5)}$	$\Delta_{c\bar{c}}^{(6)}$	$\Delta_{c\bar{c}}^{(i)}$
1-loop	χ_1	L_z^3	L_z^5	L_z^7	L_z^9	L_z^{11}	L_z^{2i-1}
2-loop	χ_2		L_z^4	L_z^6	L_z^8	L_z^{10}	L_z^{2i-2}
3-loop	χ_3			L_z^5	L_z^7	L_z^9	L_z^{2i-3}

CHECKS AND PREDICTIONS

- **But there are certain logarithms which we cannot predict completely from previous order informations.**
- **For instance : $\ln^3(1-z)$ coefficient at the 3rd order.**
- **Even though 2-loop cannot predict completely, but we find many color factors come from 2-loop.**
- **So let's see how far we can get it right !**

The left column stands for the exact result and the right for the predictions using two loop.

	$gg \to H$			Drell-Yan (DY)		$b\bar{b} \to H$	
C_A^3	$\frac{-111008}{27} + 3584\zeta_2$	$\frac{-110656}{27} + 3584\zeta_2 + \chi_1$	C_F^3	$2272 + 3072\zeta_2$	$2272 + 3072\zeta_2$	$736 + 3072\zeta_2$	$736 + 3072\zeta_2$
$C_A^2 n_f$	$\frac{6560}{9}$	$\frac{19616}{27} + \chi_2$	$C_F^2 n_f$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_3$	$\frac{19456}{27}$	$\frac{6464}{9} + \chi_3$
$C_A n_f^2$	$\frac{-256}{27}$	$\frac{-256}{27}$	$C_A C_F^2$	$\frac{-111904}{27} + 512\zeta_2$	$\frac{-37184}{9} + 512\zeta_2 + \chi_4$	$\frac{-111904}{27} + 512\zeta_2$	$\frac{-37184}{9} + 512\zeta_2 + \chi_4$
			$C_F n_f^2$	$\frac{-256}{27}$	$\frac{-256}{27}$	$\frac{-256}{27}$	$\frac{-256}{27}$
			$C_A C_F n_f$	$\frac{2816}{27}$	$\frac{2816}{27}$	$\frac{2816}{27}$	$\frac{2816}{27}$
			$C_A^2 C_F$	$\frac{-7744}{27}$	$\frac{-7744}{27}$	$\frac{-7744}{27}$	$\frac{-7744}{27}$

[Anastasiou et al.] [Duhr et al.]

CHECKS AND PREDICTIONS

- **Predictions till 7-loop for the first three logs for DY and $b\bar{b} \to H$, using 3-loop.**

$$\Delta_q^{NSV} = a_s \Delta_q^{NSV(1)} + a_s^2 \Delta_q^{NSV(2)} + a_s^3 \Delta_q^{NSV(3)} + a_s^4 \Bigg[\Bigg\{-\frac{4096}{3}C_F^4\Bigg\}L_z^7 + \Bigg\{\frac{39424}{9}C_F^3 C_A + \frac{19712}{3}C_F^4 - \frac{7168}{9}n_f C_F^3\Bigg\}L_z^6 + \Bigg\{-\frac{123904}{27}C_F^2 C_A^2 - \Bigg(\frac{805376}{27} - 3072\zeta_2\Bigg)C_F^3 C_A + \Bigg(9088 + 20480\zeta_2\Bigg)C_F^4 + \frac{45056}{27}n_f C_F^2 C_A + \frac{139520}{27}n_f C_F^3 - \frac{4096}{27}n_f^2 C_F^2\Bigg\}L_z^5 + \mathcal{O}(L_z^4)\Bigg]$$
$$+ a_s^5 \Bigg[\Bigg\{-\frac{8192}{3}C_F^5\Bigg\}L_z^9 + \Bigg\{\frac{51200}{3}C_F^5 - \frac{8192}{3}C_F^4 n_f + \frac{45056}{3}C_F^4 C_A\Bigg\}L_z^8 + \Bigg\{\Bigg(\frac{72704}{3} + \frac{229376}{3}\zeta_2\Bigg)C_F^5 - \Bigg(\frac{1120256}{9} - \frac{32768}{3}\zeta_2\Bigg)C_F^4 C_A - \frac{81920}{81}C_F^3 n_f^2 + \frac{194560}{9}C_F^4 n_f + \frac{901120}{81}C_F^3 C_A n_f - \frac{2478080}{81}C_F^3 C_A^2\Bigg\}L_z^7 + \mathcal{O}(L_z^6)\Bigg]$$
$$+ a_s^6 \Bigg[\Bigg\{-\frac{65536}{15}C_F^6\Bigg\}L_z^{11} + \Bigg\{\frac{167936}{5}C_F^6 - \frac{180224}{27}C_F^5 n_f + \frac{991232}{27}C_F^5 C_A\Bigg\}L_z^{10} + \Bigg\{\Bigg(\frac{145408}{3} + 196608\zeta_2\Bigg)C_F^6 + \frac{5054464}{81}C_F^5 n_f - \frac{327680}{81}C_F^4 n_f^2 - \Bigg(\frac{28997632}{81} - \frac{81920}{3}\zeta_2\Bigg)C_F^5 C_A + \frac{3604480}{81}C_F^4 C_A n_f - \frac{9912320}{81}C_F^4 C_A^2\Bigg\}L_z^9 + \mathcal{O}(L_z^8)\Bigg]$$
$$+ a_s^7 \Bigg[\Bigg\{-\frac{262144}{45}C_F^7\Bigg\}L_z^{13} + \Bigg\{\frac{2392064}{45}C_F^7 - \frac{1703936}{135}C_F^6 n_f + \frac{9371648}{135}C_F^6 C_A\Bigg\}L_z^{12} + \Bigg\{\Bigg(\frac{1163264}{15} + \frac{5767168}{15}\zeta_2\Bigg)C_F^7 + \frac{55115776}{405}C_F^6 n_f - \Bigg(\frac{315080704}{405} - \frac{262144}{5}\zeta_2\Bigg)C_F^6 C_A - \frac{917504}{81}C_F^5 n_f^2 + \frac{10092544}{81}C_F^5 C_A n_f - \frac{27754496}{81}C_F^5 C_A^2\Bigg\}L_z^{11} + \mathcal{O}(L_z^{10})\Bigg] + \mathcal{O}(a_s^8)\,,$$

$$\Delta_b^{NSV} = a_s \Delta_b^{NSV(1)} + a_s^2 \Delta_b^{NSV(2)} + a_s^3 \Delta_b^{NSV(3)} + a_s^4 \Big[\Delta_q^{NSV(4)} - 6144 C_F^4 L_z^5 + \mathcal{O}(L_z^4)\Big] + a_s^5 \Big[\Delta_q^{NSV(5)} - 16384 C_F^5 L_z^7 + \mathcal{O}(L_z^6)\Big] + a_s^6 \Big[\Delta_q^{NSV(6)} - 32768 C_F^6 L_z^9 + \mathcal{O}(L_z^8)\Big] + a_s^7 \Big[\Delta_q^{NSV(7)} - \frac{262144}{5}C_F^7 L_z^{11} + \mathcal{O}(L_z^{10})\Big] + \mathcal{O}(a_s^8)\ ,$$

Till 4-loop
[Vogt, Moch et al.],
[De Florian et al.],
[Das et all]

CHECKS AND PREDICTIONS

- **Predictions till 7-loop for the first three logs for gluon fusion, using 3-loop.**

$$
\begin{aligned}
\Delta_g^{NSV} = {} & a_s \Delta_g^{NSV(1)} + a_s^2 \Delta_g^{NSV(2)} + a_s^3 \Delta_g^{NSV(3)} \\
& + a_s^4 \Bigg[\Bigg\{ -\frac{4096}{3} C_A^4 \Bigg\} L_z^7 + \Bigg\{ \frac{98560}{9} C_A^4 - \frac{7168}{9} n_f C_A^3 \Bigg\} L_z^6 + \Bigg\{ \Bigg(-\frac{298240}{9} + 23552\zeta_2 \Bigg) C_A^4 \\
& + \frac{174208}{27} n_f C_A^3 - \frac{4096}{27} n_f^2 C_A^2 \Bigg\} L_z^5 + \mathcal{O}(L_z^4) \Bigg] + a_s^5 \Bigg[\Bigg\{ -\frac{8192}{3} C_A^5 \Bigg\} L_z^9 + \Bigg\{ \frac{96256}{3} C_A^5 \\
& - \frac{8192}{3} C_A^4 n_f \Bigg\} L_z^8 + \Bigg\{ \Bigg(-\frac{12283904}{81} + \frac{262144}{3} \zeta_2 \Bigg) C_A^5 + \frac{2569216}{81} C_A^4 n_f - \frac{81920}{81} n_f^2 C_A^3 \Bigg\} L_z^7 \\
& + \mathcal{O}(L_z^6) \Bigg] + a_s^6 \Bigg[\Bigg\{ -\frac{65536}{15} C_A^6 \Bigg\} L_z^{11} + \Bigg\{ \frac{9490432}{135} C_A^6 - \frac{180224}{27} C_A^5 n_f \Bigg\} L_z^{10} + \Bigg\{ \Bigg(\frac{671744}{3} \zeta_2 \\
& - \frac{4261888}{9} \Bigg) C_A^6 + \frac{8493056}{81} C_A^5 n_f - \frac{327680}{81} n_f^2 C_A^4 \Bigg\} L_z^9 + \mathcal{O}(L_z^8) \Bigg] \\
& + a_s^7 \Bigg[\Bigg\{ -\frac{262144}{45} C_A^7 \Bigg\} L_z^{13} + \Bigg\{ \frac{3309568}{27} C_A^7 - \frac{1703936}{135} C_A^6 n_f \Bigg\} L_z^{12} + \Bigg\{ \Bigg(-\frac{449429504}{405} \\
& + \frac{1310720}{3} \zeta_2 \Bigg) C_A^7 + \frac{11583488}{45} C_A^6 n_f - \frac{917504}{81} n_f^2 C_A^5 \Bigg\} L_z^{11} + \mathcal{O}(L_z^{10}) \Bigg] + \mathcal{O}(a_s^8) .
\end{aligned}
$$

Till 4-loop
[Vogt, Moch et al.],
[De Florian et al.],
[Das et all]

CHECKS AND PREDICTIONS

- **Predictions till 7-loop for the first three logs for gluon fusion, using 3-loop.**

In General :

$$\ln^k(1-z),\ n+1 \leq k \leq 2n-1$$

at order a_s^n

$$\Delta_g^{NSV} = \ldots$$

$$+\mathcal{O}(L_z^6)\Big] + a_s^6\Big[\Big\{-\frac{65536}{15}C_A^6\Big\}L_z^{11} + \Big\{\frac{9490432}{135}C_A^6 - \frac{180224}{27}C_A^5 n_f\Big\}L_z^{10} + \Big\{\Big(\frac{671744}{3}\zeta_2 - \frac{4261888}{9}\Big)C_A^6 + \frac{8493056}{81}C_A^5 n_f - \frac{327680}{81}n_f^2 C_A^4\Big\}L_z^9 + \mathcal{O}(L_z^8)\Big]$$

$$+ a_s^7\Big[\Big\{-\frac{262144}{45}C_A^7\Big\}L_z^{13} + \Big\{\frac{3309568}{27}C_A^7 - \frac{1703936}{135}C_A^6 n_f\Big\}L_z^{12} + \Big\{\Big(-\frac{449429504}{405} + \frac{1310720}{3}\zeta_2\Big)C_A^7 + \frac{11583488}{45}C_A^6 n_f - \frac{917504}{81}n_f^2 C_A^5\Big\}L_z^{11} + \mathcal{O}(L_z^{10})\Big] + \mathcal{O}(a_s^8).$$

Till 4-loop [Vogt, Moch et al.], [De Florian et al.], [Das et all]

INTEGRAL REPRESENTATION

- **Knowing the functional form of each building blocks one can derive the integral form as:**

$$\ln \Delta_{c\bar{c}}(q^2, \mu_R^2, \mu_F^2, z) = \ln C_0^c(q^2, \mu_R^2, \mu_F^2) + \left\{ \int_{\mu_F^2}^{q^2(1-z)^2} \frac{d\lambda^2}{\lambda^2} P'_{cc}(a_s(\lambda^2), z) + Q^c(a_s(q^2(1-z)^2), z) \right\}$$

- **Some Details:**
  - C_0^c **captures the** $\delta(1-z)$ **contribution from** $\hat{F}_c$ & S_c
  - **Finite contributions from cancellation between** Γ_{cc} & S_c

$$P'_{cc}(z) \propto \left[A^c \left(\frac{1}{1-z} \right)_+ + C^c \ln(1-z) + D^c \right]$$

  - **Finite contributions coming from** S_c

$$Q^c(a_s(q^2(1-z)^2), z) \propto \left(\frac{1}{1-z} \mathcal{G}_{sv}(a_s(q^2(1-z)^2)) \right)_+ + \mathcal{G}_{nsv}(a_s(q^2(1-z)^2), z)$$

MELLIN SPACE N

- **To study the all-order behaviour we need integral representation for $\Delta_{c\bar{c}}$.**

$$\Delta_N^{c\bar{c}}(q^2) = \int_0^1 dz\ z^{N-1}\ \Delta_{c\bar{c}}(q^2, z)$$

- **Threshold limit $z \to 1$ in z - space translates to $N \to \infty$ in N-space.**
- **Taking till $\frac{1}{N}$ corrections from SV and NSV terms :**

$$\left(\frac{\ln(1-z)}{1-z}\right)_+ \sim \frac{\ln^2 N}{2} - \frac{\ln N}{2N} + \frac{1}{2N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

$$\ln^k(1-z) \sim \frac{\ln^k N}{N} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

NSV RESUMMATION

- **Hence the inclusion of the NSV logarithms modifies the existing resumed expression as :**

$$\Delta_{c,N}(q^2,\mu_R^2,\mu_F^2) = \left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\tilde{g}_{0,i}(q^2,\mu_R^2,\mu_F^2)\right)\exp\left(\Psi^c_{SV,N}(q^2,\mu_F^2) + \Psi^c_{NSV,N}(q^2,\mu_F^2)\right)$$

- **where,**

$$\Psi^c_{\rm sv,N} = g_1^c(\omega)\ln(N) + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g^c_{i+2}(\omega)$$

[Sterman et al.]
[Catani et al.]

- **and,**

$$\omega = 2a_s\beta_0 \ln N$$

$$\Psi^c_{\rm NSV,N} = \frac{1}{N}\left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2) h_i^c(\omega,N)\right)$$

$$h_0^c(\omega,N) = h_{00}^c(\omega) + h_{01}^c(\omega)\ln(N), \quad h_i^c(\omega,N) = \sum_{k=0}^{i} h_{ik}^c(\omega)\ \ln^k(N)$$

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Known Result since 1989

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[Sterman et al.] [Catani et al.]

- **and,**

$$\omega = 2a_s\beta_0 \ln N$$

New Result !!

$$\Psi^c_{\mathrm{NSV,N}} = \frac{1}{N}\left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2)h_i^c(\omega,N)\right)$$

$$h_0^c(\omega,N) = h_{00}^c(\omega) + h_{01}^c(\omega)\ln(N), \quad h_i^c(\omega,N) = \sum_{k=0}^{i} h_{ik}^c(\omega)\ \ln^k(N)$$

$\ln N/N$ TOWERS

- The towers of $\ln N/N$ that we sum over :

$\Delta_N^c =$

Resumed terms :

Only 1-loop info	Only 2-loop info	…	Only n-loop info
$a_s \frac{\ln N}{N}$	$a_s^2 \frac{\ln^2 N}{N}$		$a_s^n \frac{\ln^n N}{N}$
$a_s^2 \frac{\ln^3 N}{N}$	$a_s^3 \frac{\ln^4 N}{N}$		
$\vdots$	$\vdots$		$\vdots$
$a_s^i \frac{\ln^{2i-1} N}{N}$	$a_s^i \frac{\ln^{2i-2} N}{N}$		$a_s^i \frac{\ln^{2i-n} N}{N}$
Exponents : g_1^c, h_0^c	g_2^c, h_1^c		g_{n+1}^c, h_n^c

CHECKS ON RESUMMATION

- Expansion of the resummed result matches with the fixed order till 4-loop.

$$\Delta_{c,N}(q^2,\mu_R^2,\mu_F^2) = \left(\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\tilde{g}_{0,i}(q^2,\mu_R^2,\mu_F^2)\right)\exp\left(\Psi^c_{SV,N}(q^2,\mu_F^2) + \Psi^c_{NSV,N}(q^2,\mu_F^2)\right)$$

- The leading logarithm for SV+ NSV matches with the existing result :

$$\Delta^{DY}_{LL} = g_0 \exp\left[\ln N g_1(\omega) + \frac{1}{N} h_0(\omega, N)\right] = \exp\left[8C_F a_s\left(\ln^2 N + \frac{\ln N}{N}\right)\right]$$

[Beneke et al.]

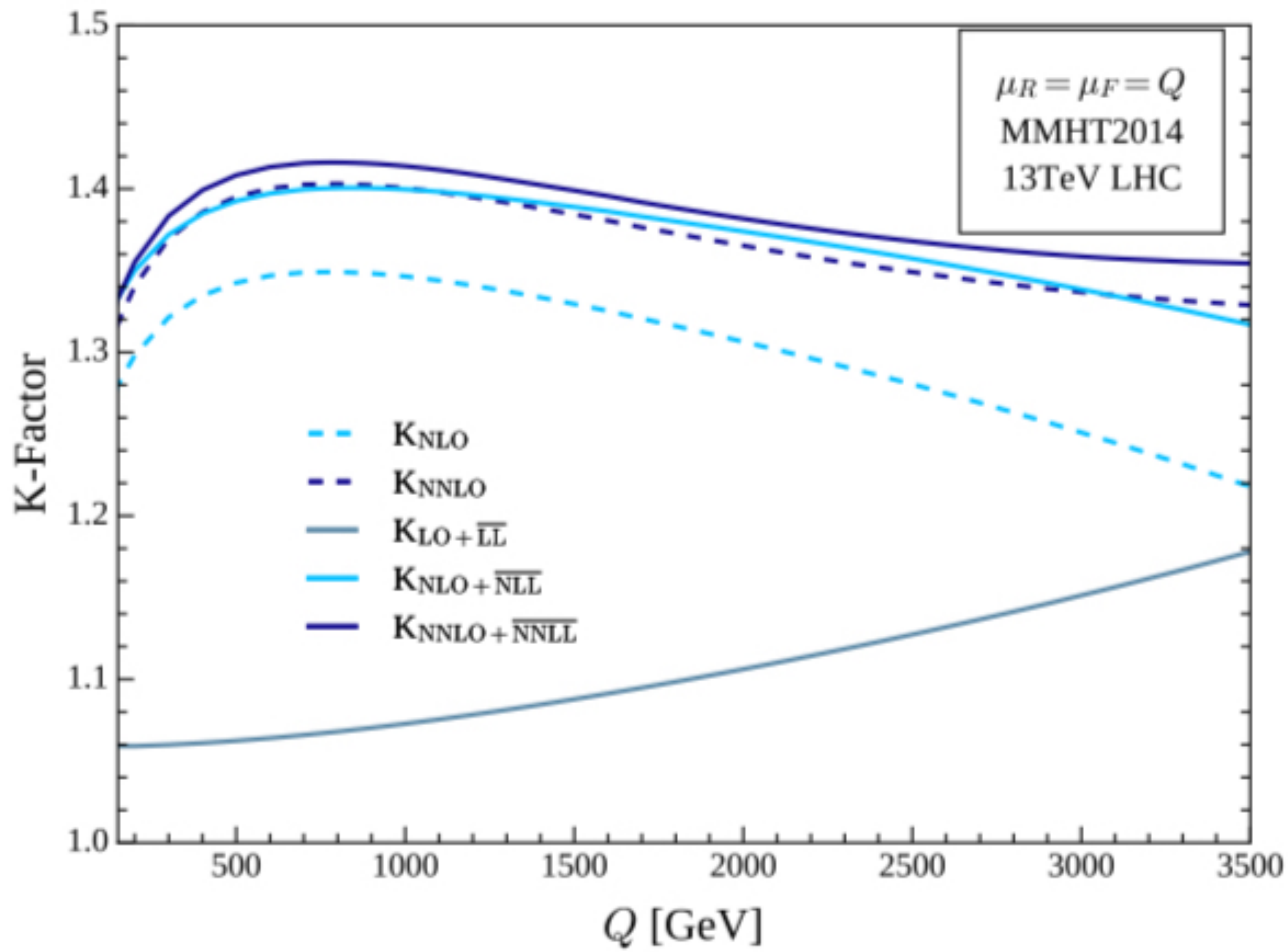
[Laenen et al.]

- Now we perform Mellin Inversion of the resummed result to study the numerical impact.

DY PHENOMENOLOGY

$\mu_R = \mu_F = Q$(GeV)	LO + $\overline{\text{LL}}$	NLO	NLO + $\overline{\text{NLL}}$	NNLO	NNLO + $\overline{\text{NNLL}}$
500	1.0624	1.3425	1.3925	1.3950	1.4082
1000	1.0728	1.3464	1.3995	1.4004	1.4138
2000	1.1062	1.3064	1.3739	1.3652	1.3818

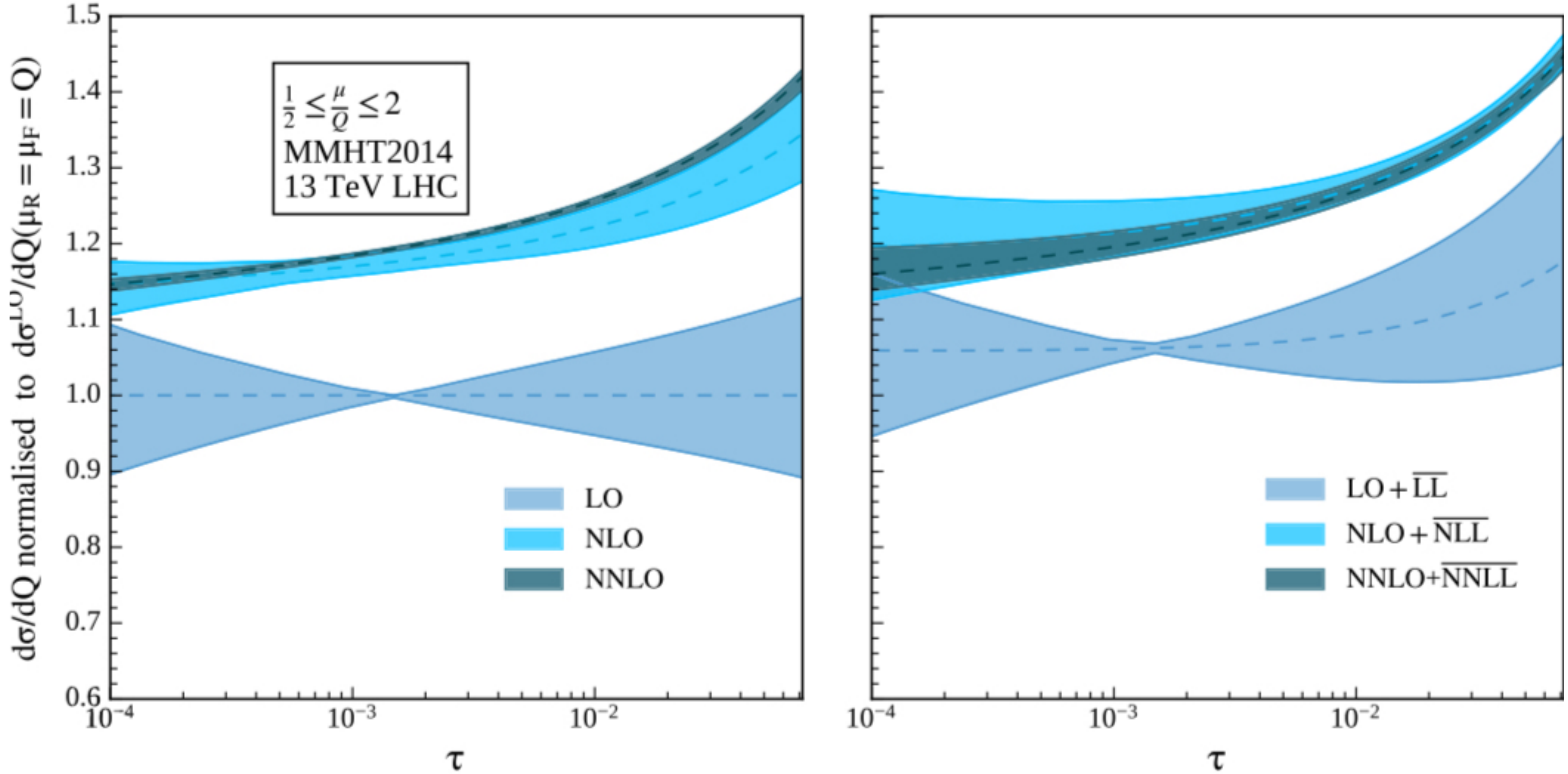
K-factor values for resummed results in comparison to the fixed order ones.



DY PHENOMENOLOGY

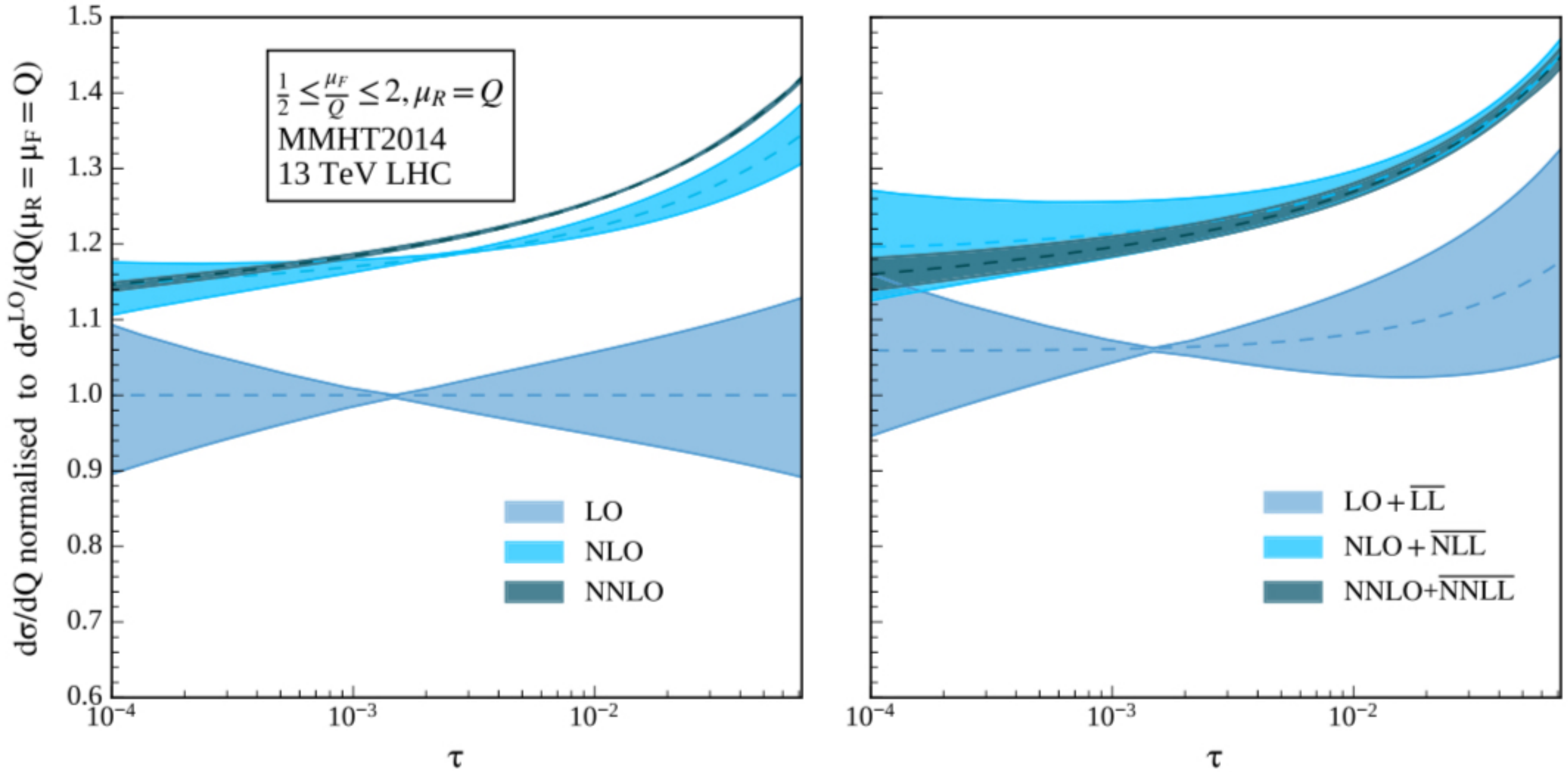
Q	LO	LO + $\overline{\text{LL}}$	NLO	NLO + $\overline{\text{NLL}}$	NNLO	NNLO + $\overline{\text{NNLL}}$
1000	$2.3476^{+4.10\%}_{-3.92\%}$	$2.5184^{+4.49\%}_{-4.25\%}$	$3.1609^{+1.79\%}_{-1.69\%}$	$3.2857^{+2.08\%}_{-1.18\%}$	$3.2876^{+0.20\%}_{-0.31\%}$	$3.3191^{+1.13\%}_{-0.86\%}$
2000	$0.0501^{+8.50\%}_{-7.46\%}$	$0.0554^{+9.10\%}_{-7.91\%}$	$0.0654^{+2.83\%}_{-2.98\%}$	$0.0688^{+1.43\%}_{-1.23\%}$	$0.0684^{+0.37\%}_{-0.62\%}$	$0.0692^{+0.89\%}_{-0.78\%}$

Values of resummed cross section in 10^{-5} pb/GeV at various orders in comparison to Fixed Order



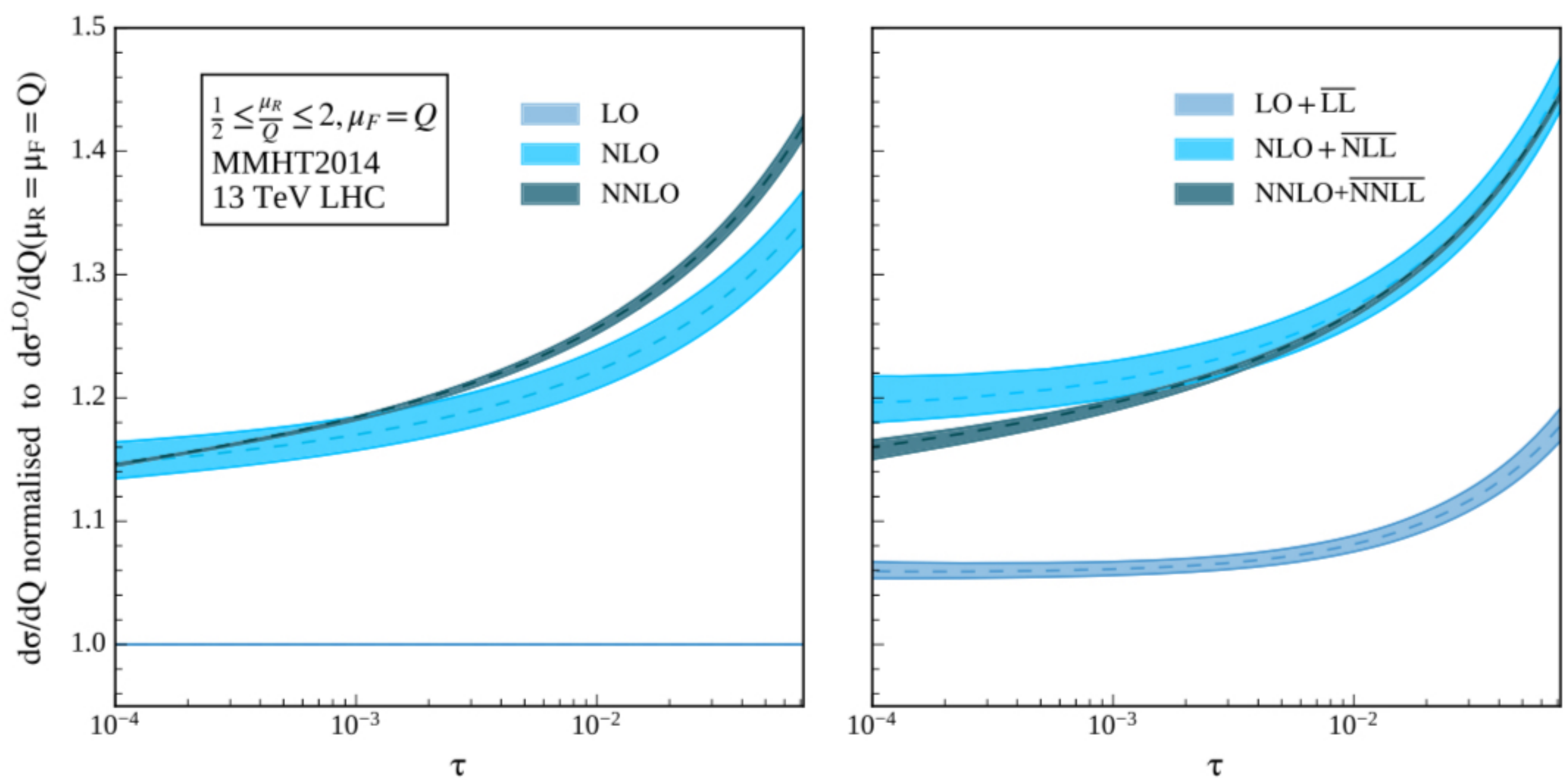
7-point scale variation of the resummed result against fixed order result around the central scale

DY PHENOMENOLOGY



- The main source of uncertainties in 7-point variation comes from μ_F .
- μ_F uncertainty is more at NNLO than at NLO.
- At one-loop , $q\bar{q}$ channel contributes to 22.09% of the NLO cross- section, whereas qg contributes to -5.04% of the NLO cross-section.

- At two-loop, $q\bar{q}$ channel contributes to 4.86% of the NNLO cross-section, whereas qg contributes to -2.47% of the NNLO cross-section.
- Different partonic channels do not mix under μ_R variation.
- μ_R uncertainty is significantly decreased for resummed result as compared to the fixed order ones.



CONCLUSIONS

- What has been studied so far ?
  - Using collinear factorisation and RG invariance and exploiting fixed order results, we propose an all order formula.
  - We propose an integral representation which can resume both SV and NSV logarithms to all orders.
  - Hence we have extended the Resummation of the NSV logarithms till N^2LL accuracy.
  - We find the SV + NSV resummed results give significant contributions owing to the large coefficients of the NSV terms.
- What more to do ?
  - Study the Functional form of the soft collinear function at the level of the amplitude.
  - Modify the existing formalism for off-Diagonal Channels.

THANK YOU !!