Pion and kaon structure from Mellin moments of PDFs and GPDs

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Thomas Jefferson National Accelerator Facility
Theory seminar

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Collaborators

- **C. Lauer** (Temple University)
- **J. Delmar** (Temple University)
- **C. Alexandrou** (Univ. of Cyprus/Cyprus Institute)
- **S. Bacchio** (Cyprus Institute)
- **I. Cloet** (Argonne National Lab)
- **K. Hadjiyiannakou** (Univ. of Cyprus/Cyprus Institute)
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Extended Twisted Mass Collaboration

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Extended Twisted Mass Collaboration

Relevant publications

- *The Mellin moments $\langle x \rangle$ and $\langle x^2 \rangle$ for the pion and kaon from lattice QCD,*
  C. Alexandrou, S. Bacchio, I. Cloët, M. Constantinou, K. Hadjiyiannakou, G. Koutsou, C. Lauer
  PRD 103, 014508 (2021), [arXiv:2010.03495]

- *The pion and kaon $\langle x^3 \rangle$ from lattice QCD and PDF reconstruction from Mellin moments,*
  C. Alexandrou, S. Bacchio, I. Cloët, M. Constantinou, K. Hadjiyiannakou, G. Koutsou, C. Lauer
  PRD 104 (2021) 5, 054504, [arXiv:2104.02247]

- *The scalar, vector and tensor form factors for the pion and kaon from lattice QCD,*
  C. Alexandrou, S. Bacchio, I. Cloët, M. Constantinou, J. Delmar, K. Hadjiyiannakou, G. Koutsou, C. Lauer
  [arXiv:2111.08135]

OUTLINE

A. Motivation

B. Mellin moments in lattice QCD

C. Reconstruction of PDFs

D. SU(3) flavor symmetry breaking

E. Form factors

F. Summary

Pions and Kaons

★ Non-perturbative nature of QCD leads to emergent phenomena such as massive hadrons even at the chiral limit

★ QCD exhibits dynamical chiral symmetry breaking (DCSB) gives rise to Nambu-Goldstone boson (e.g., pions and kaons)

★ Exploring the quark and gluon structure of pions and kaons can shed light in the interplay and connections between the trace anomaly and DCSB

★ Experimental data only for the pion (pion induced Drell-Yan reaction) and for the limited region $x \in [0.21 - 0.99]$ [J. S. Conway et al., PRD 39, 92 (1989)]

★ Contradictory conclusions on the large-x behavior of pion PDF:
- initial E615 data show a $(1-x)^1$ behavior [R. Holt et al., RMP 82, 2991 (2010)], [M. Aicher et al., PRL 105, 252003 (2010)]
- reanalysis of E615 data shows a $(1-x)^2$ fall
- DSE predict $(1-x)^2$ fall [K. Bednar et al. PRL 124, 042002 (2020)]
- Lattice QCD calculations do not reach to a consensus [M. Constantinou, EPJA 57, 77 (2021), arXiv:2010.02445]

★ Direct interest in JLab 12 GeV

★ EIC will address pion and kaon structure [EIC Yellow Report, arXiv:2103.05419], [Aguilar et al., EPJA 55, 190 (2019)]

More on the PDF reconstruction

★ **Reconstruction of the light-cone PDFs** *not realistic?*

- increased statistical noise for high moments
- operator mixing
- need for boosted frame for $\langle x^2 \rangle$ and higher to avoid mixing

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★ No recent lattice QCD results for high moments using local operators

Reference	Method	Renorm.	mixing	m_π (MeV)	N_f	$\langle x^3\rangle^u_\pi$ (2GeV)	initial scale
This work	local operator	non-perturb.	not present	260	2+1+1	0.024(18)	2 GeV
Ref. [5]	local operator	perturb.	present	chiral extrap.	0	0.051(21)	2.4 GeV
Ref. [41]	local operator	perturb.	present	chiral extrap.	0	0.046(16)	2.4 GeV
Ref. [7]	local operator	non-perturb.	present	chiral extrap.	2	0.074(10)	2 GeV

Reference	Method	Renorm.	mixing	m_π (MeV)	N_f	$\langle x^3\rangle^u_K$ (2GeV)	$\langle x^3\rangle^s_K$ (2GeV)	initial scale
This work	local operator	non-perturb.	not present	260	2+1+1	0.033(6)	0.073(5)	2 GeV

[5]. C. Best et al., PRD 56, 2743 (1997)

[41]. W. Detmold et al., PRD 68, 034025 (2003)

[7]. D. Brommel, Ph.D. thesis (2007)

Parametrization of matrix elements

Euclidean space:

$$\langle M(p')\,|\,\overline{q}\gamma^{\{\mu}D^{\nu\}}q\,|\,M(p)\rangle = C\,\left[2P^{\{\mu}P^{\nu\}}A_{20} + 2\Delta^{\{\mu}\Delta^{\nu\}}B_{20}\right]$$

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Forward limit (avoiding mixing)

$$\langle M(p)\,|\,\overline{q}\gamma^{\{0}D^{0\}}q\,|\,M(p)\rangle = \frac{1}{4E_M(p)}\left(m_M^2 - 4E_M^2(p)\right)\langle x\rangle_M^q$$

$$\langle M(p)\,|\,\overline{q}\gamma^{\mu}D^{\nu}D^{4}q\,|\,M(p)\rangle = -\,p_\mu\,p_\nu\,\langle x^2\rangle_M^q$$

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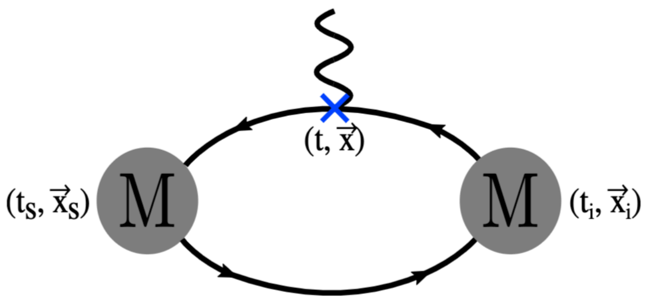
★ Avoiding mixing increases the computational cost!

Technical Aspects



★ Nf=2+1+1 twisted mass fermions & clover term

★ Ensemble parameters:

Pion mass:	260 MeV
Kaon mass:	530 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm

★ Kinematical setup:

$\vec{p}$	$T_{\rm sink}/a$	$N_{\rm confs}$	$N_{\rm src}$	Total statistics
(0,0,0)	12, 14, 16, 18, 20, 24	122	16	1,952
$(\pm1,\pm1,\pm1)$	12	122	16	15,616
$(\pm1,\pm1,\pm1)$	14, 16, 18	122	72	70,272

★ Excited states: single-state & two-state fits

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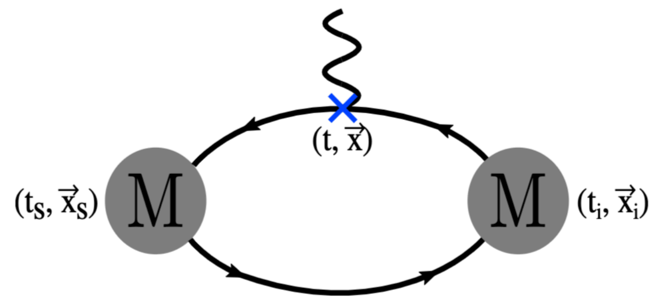

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Rest frame: signal constant with Tsink increase

[Lepage, "*The Analysis of Algorithms for Lattice Field Theory*" (1989)]

Boosted frame: signal decays with Tsink increase

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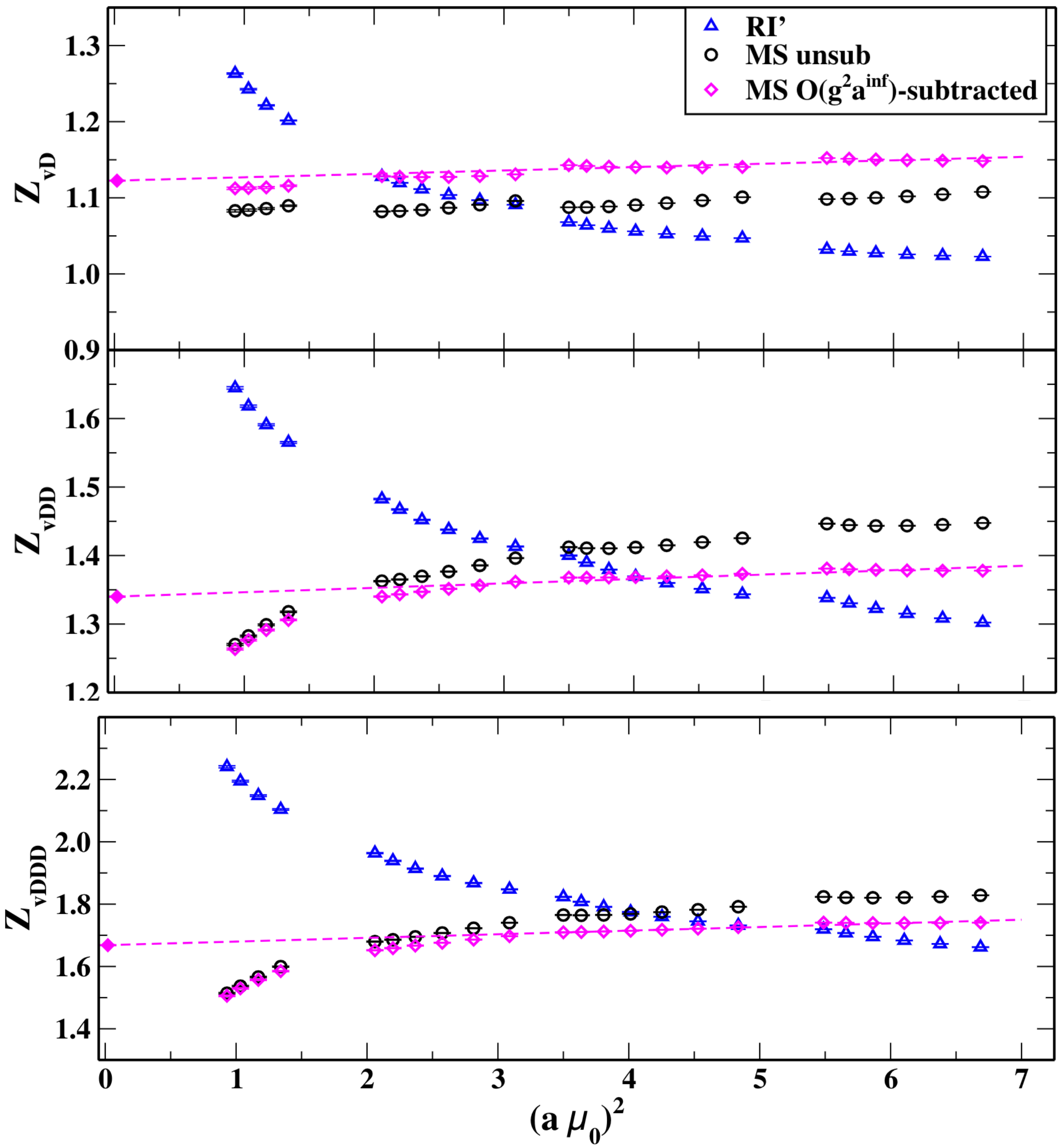
Non-perturbative Renormalization



$$Z^{\overline{\rm MS}}_{\rm vD}(2\,{\rm GeV}) = 1.123(1)(5)$$
$$Z^{\overline{\rm MS}}_{\rm vDD}(2\,{\rm GeV}) = 1.340(1)(15)$$
$$Z^{\overline{\rm MS}}_{\rm vDDD}(2\,{\rm GeV}) = 1.668(1)(26)$$

★ RI' scheme (democratic momenta)

$$Z_q^{-1}\, Z_{\mathcal{O}}\, \frac{1}{12} {\rm Tr} \left[\Gamma^L_{\mathcal{O}}(p) \, \left(\Gamma^{\rm Born}_{\mathcal{O}}(p)\right)^{-1}\right] \Big|_{p^2=\mu_0^2} = 1$$

$$Z_q = \frac{1}{12} {\rm Tr} \left[(S^L(p))^{-1}\, S^{\rm Born}(p)\right] \Big|_{p^2=\mu_0^2}$$

$$(ap) \equiv 2\pi \left(\frac{n_t}{L_t} + \frac{1}{2\,L_t}, \frac{n_x}{L_s}, \frac{n_x}{L_s}, \frac{n_x}{L_s}\right) \qquad \sum_i p_i^4/(\sum_i p_i^2)^2 < 0.3$$

[M. Constantinou et al., JHEP 08, 068 (2010), arXiv:1004.1115]

★ Chiral extrapolation (negligible)

$\beta = 1.726, \quad a = 0.093$ fm		
$a\mu$	am_{PS}	lattice size
0.0060	0.1680	$24^3 \times 48$
0.0080	0.1916	$24^3 \times 48$
0.0100	0.2129	$24^3 \times 48$
0.0115	0.2293	$24^3 \times 48$
0.0130	0.2432	$24^3 \times 48$

★ Subtraction of $\mathcal{O}(g^2\, a^\infty)$

[M. Constantinou et al., PRD 91, 014502 (2015), arXiv:1408.6047]

★ Conversion & evolution to $\overline{\rm MS}(2\,{\rm GeV})$

$$Z^{\overline{\rm MS}}_{\mathcal{O}}(a\mu_0) = Z^{\overline{\rm MS}}_{\mathcal{O}}(2\,{\rm GeV}) + Z^{(1)}_{\mathcal{O}} \cdot (a\,\mu_0)^2$$

Recapitulation

★ Matrix elements of pion and kaon coupled with local operators

★ Isolation of ground state

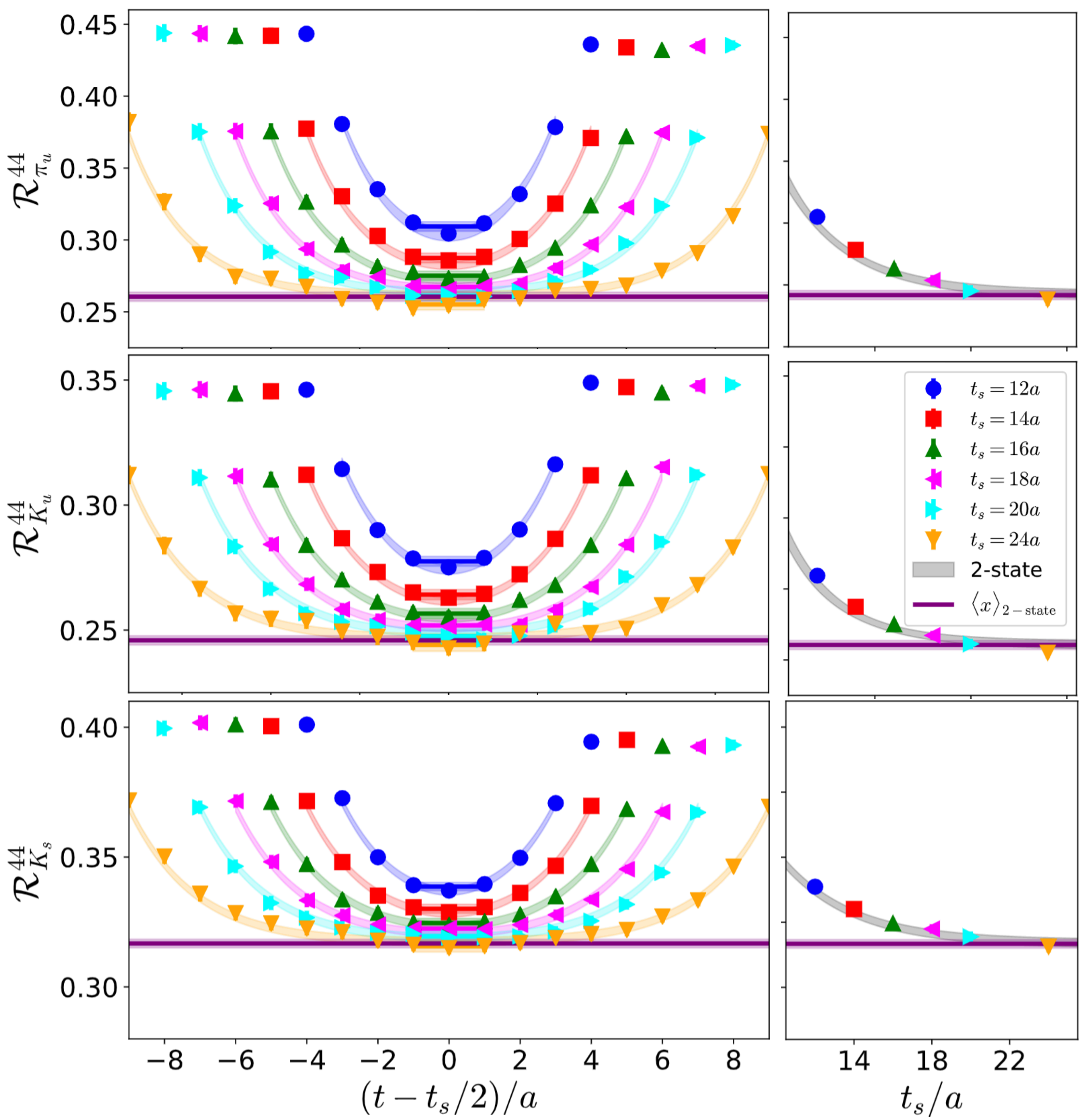
★ Renormalization

★ Extraction of Mellin moments

Mellin Moments

Excited-states contamination

Rest frame



- ★ **Signal does not decay with Tsink increase in rest frame**

  [G. P. Lepage, "*The Analysis of Algorithms for Lattice Field Theory*" (1989)]

- ★ **Excited-states contamination sizable in $\langle x \rangle$**

- ★ **Convergence found for Tsink > 1.65 fm**

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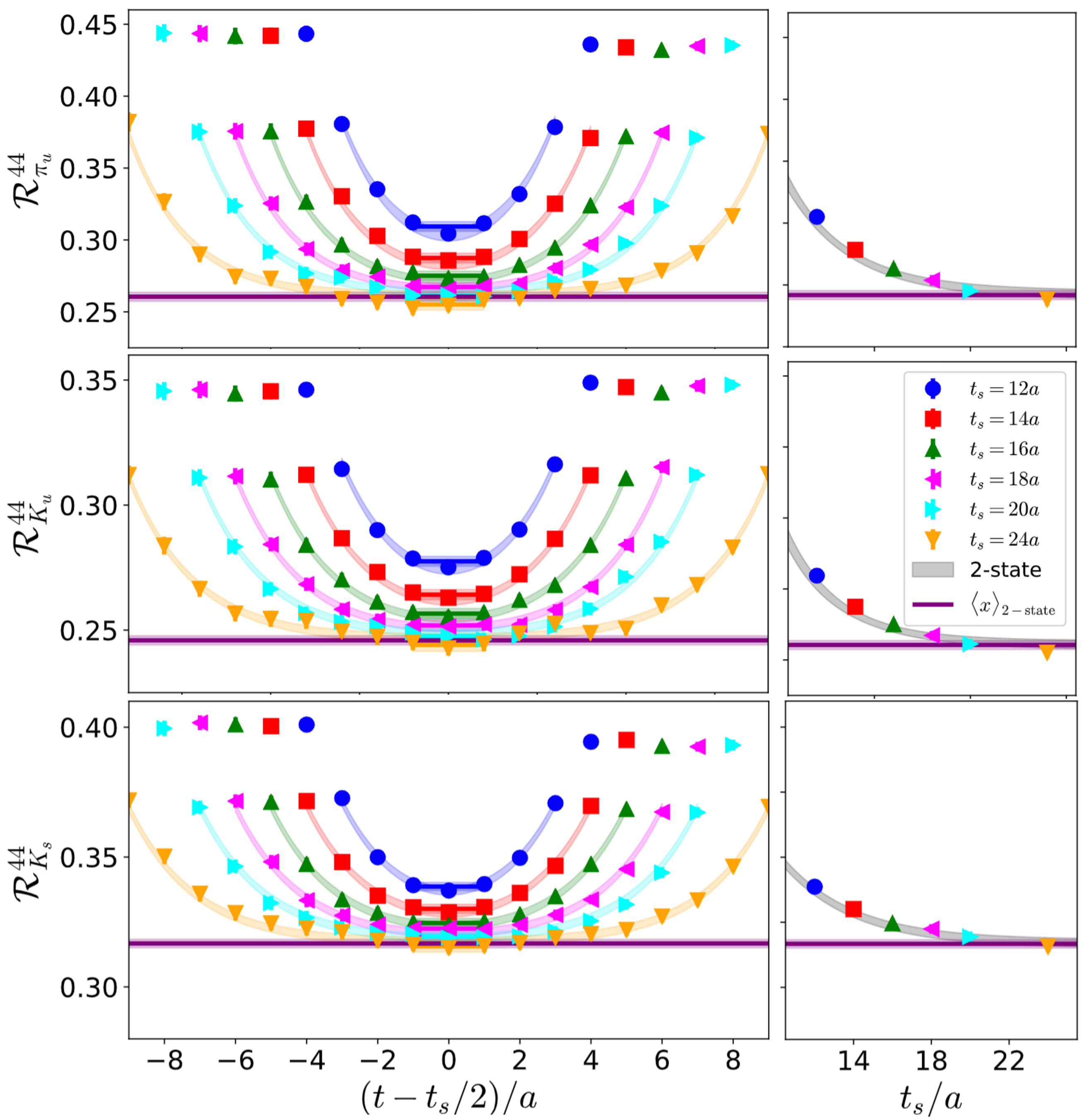

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t_s/a	$\langle x \rangle^{\pi}_{u^+}$	$\langle x \rangle^{k}_{u}$	$\langle x \rangle^{k}_{s}$
12	0.309(3)	0.278(2)	0.339(2)
14	0.287(3)	0.264(2)	0.330(2)
16	0.275(3)	0.257(2)	0.325(2)
18	0.267(3)	0.252(2)	0.322(2)
20	0.261(4)	0.248(2)	0.319(2)
24	0.255(4)	0.244(3)	0.316(2)
2-state (a)	0.261(3)	0.246(2)	0.317(2)
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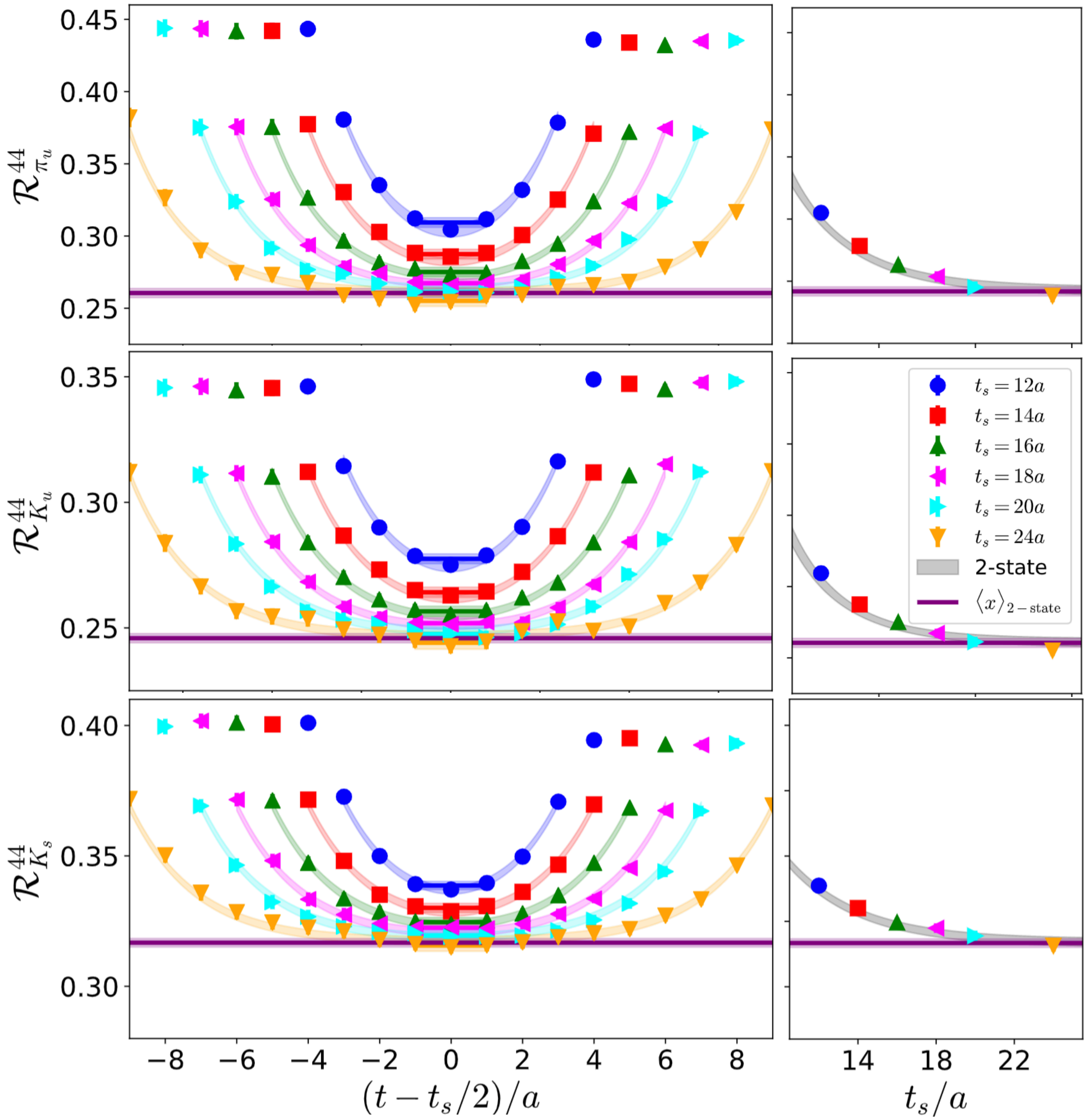

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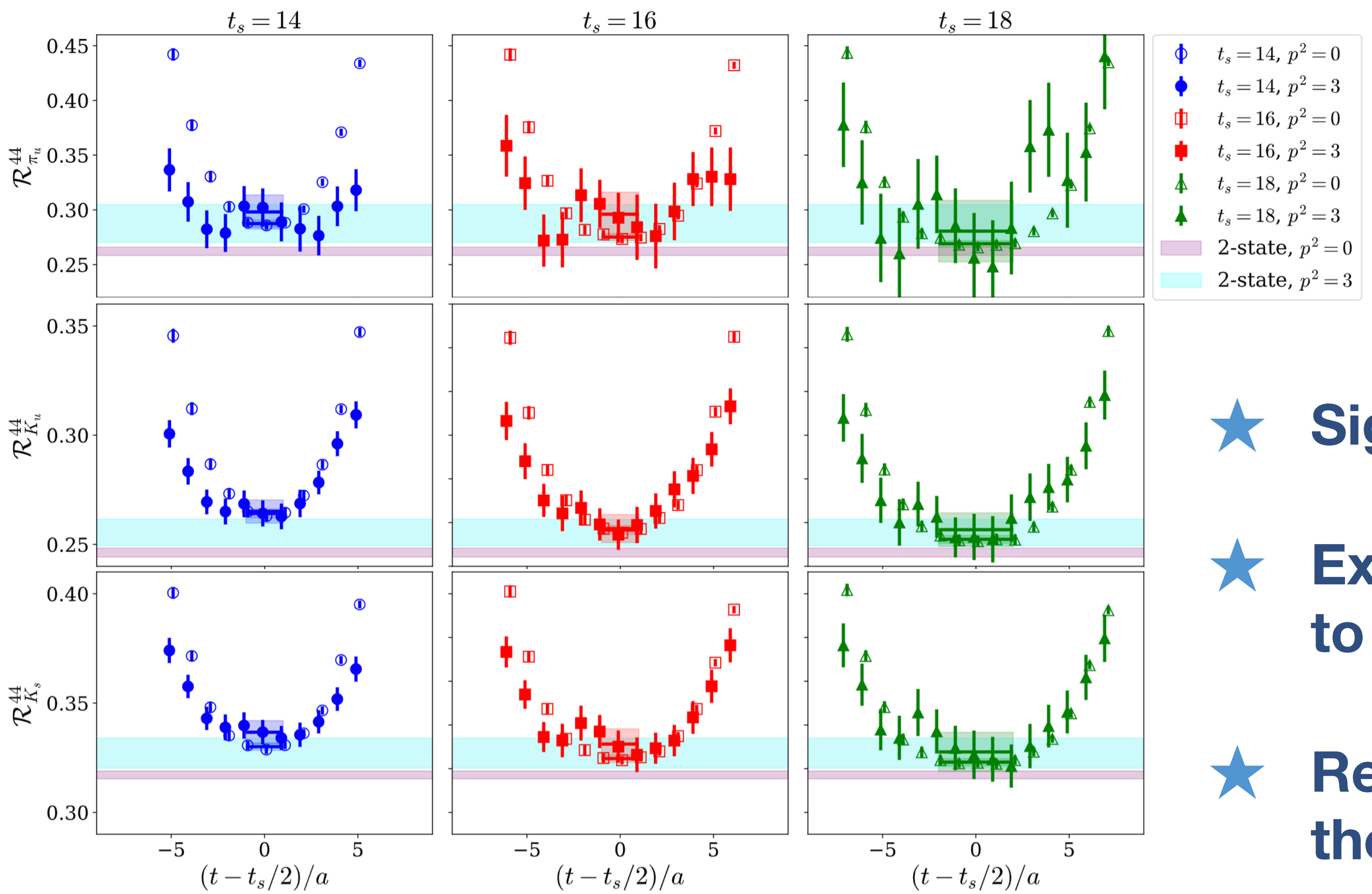
Rest frame vs boosted frame



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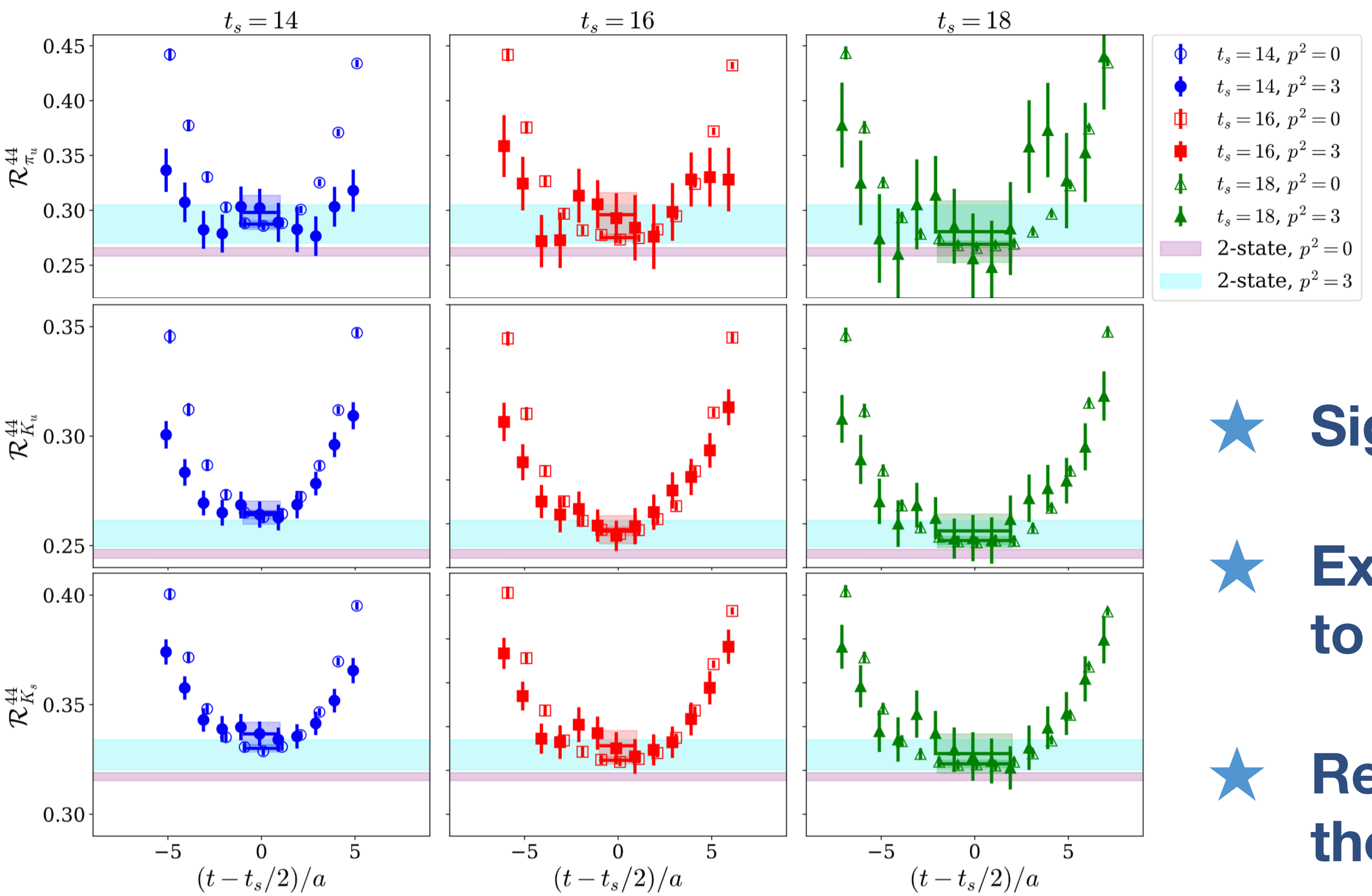

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Conclusions:

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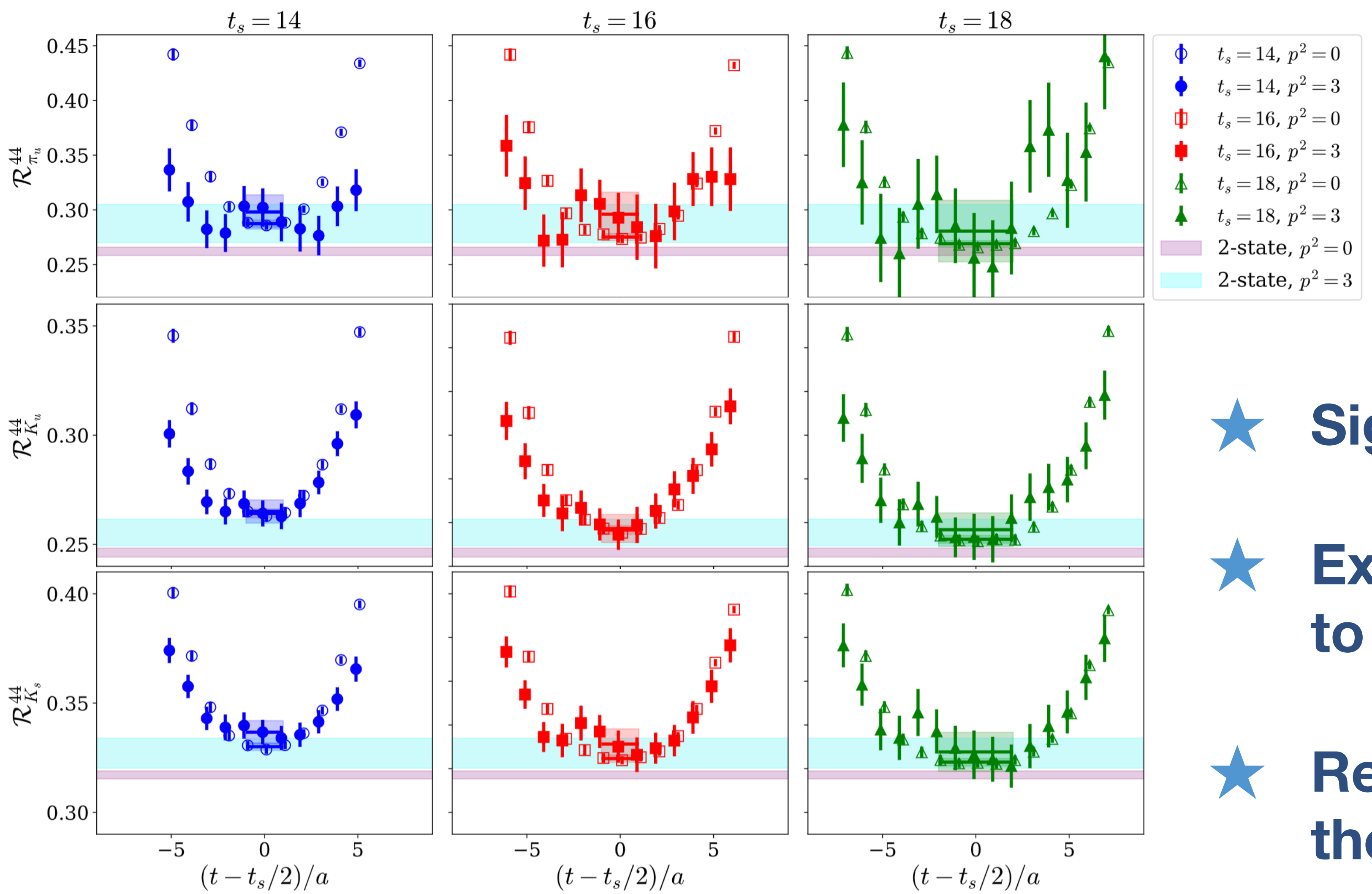
Rest frame vs boosted frame



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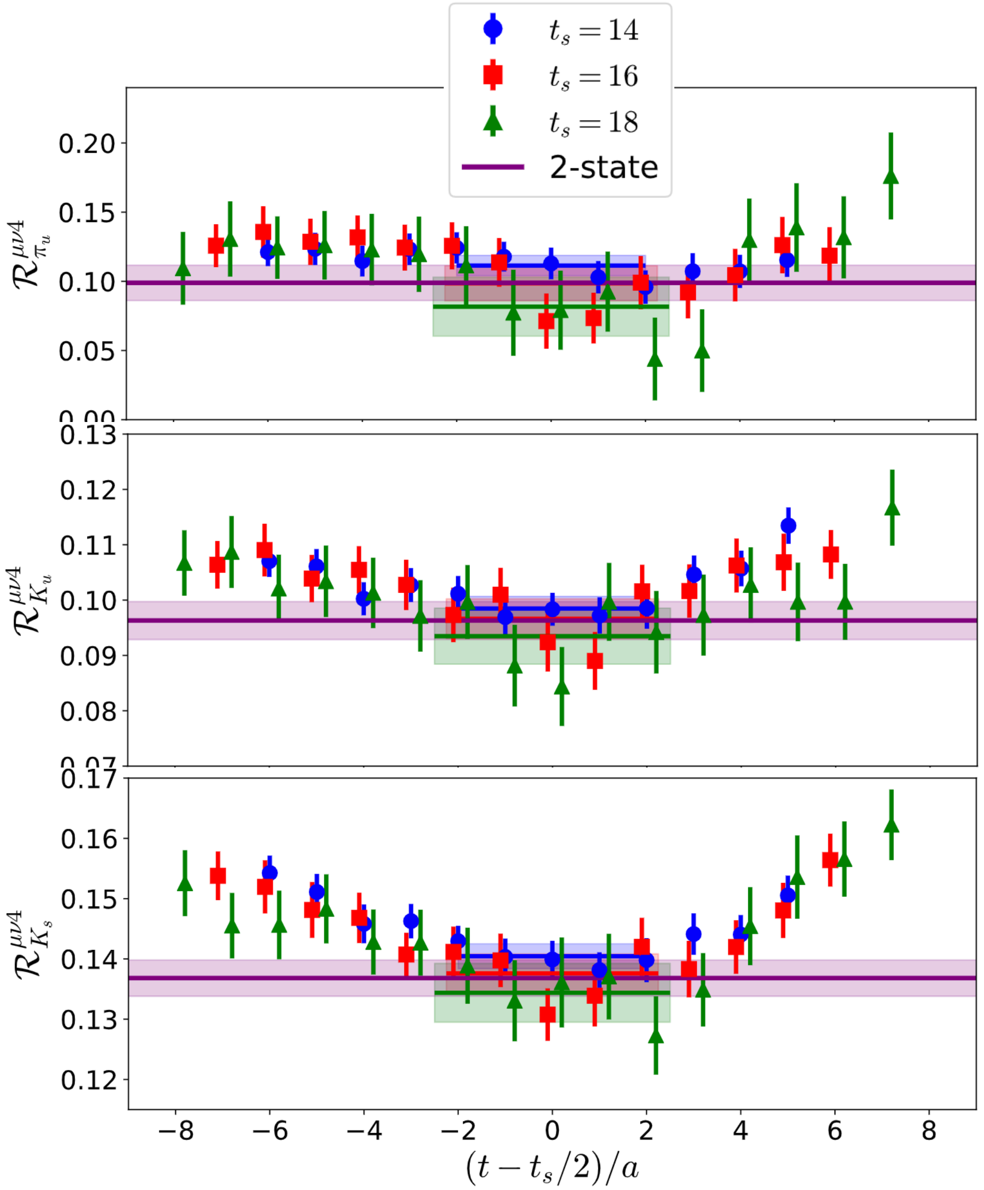
Calculations of $\langle x^2 \rangle$ and $\langle x^3 \rangle$ can be combined without increase in computational cost

Higher moments



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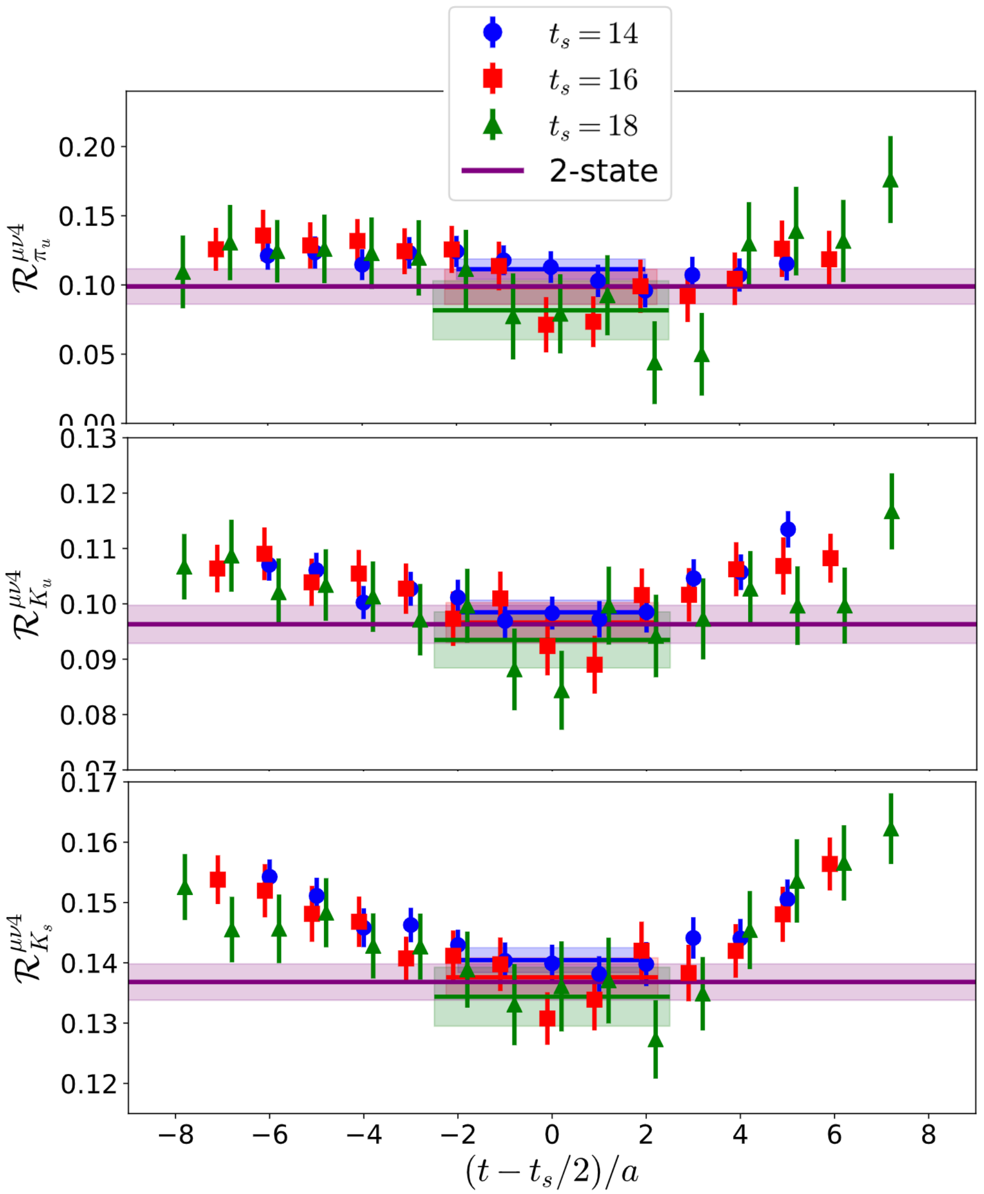

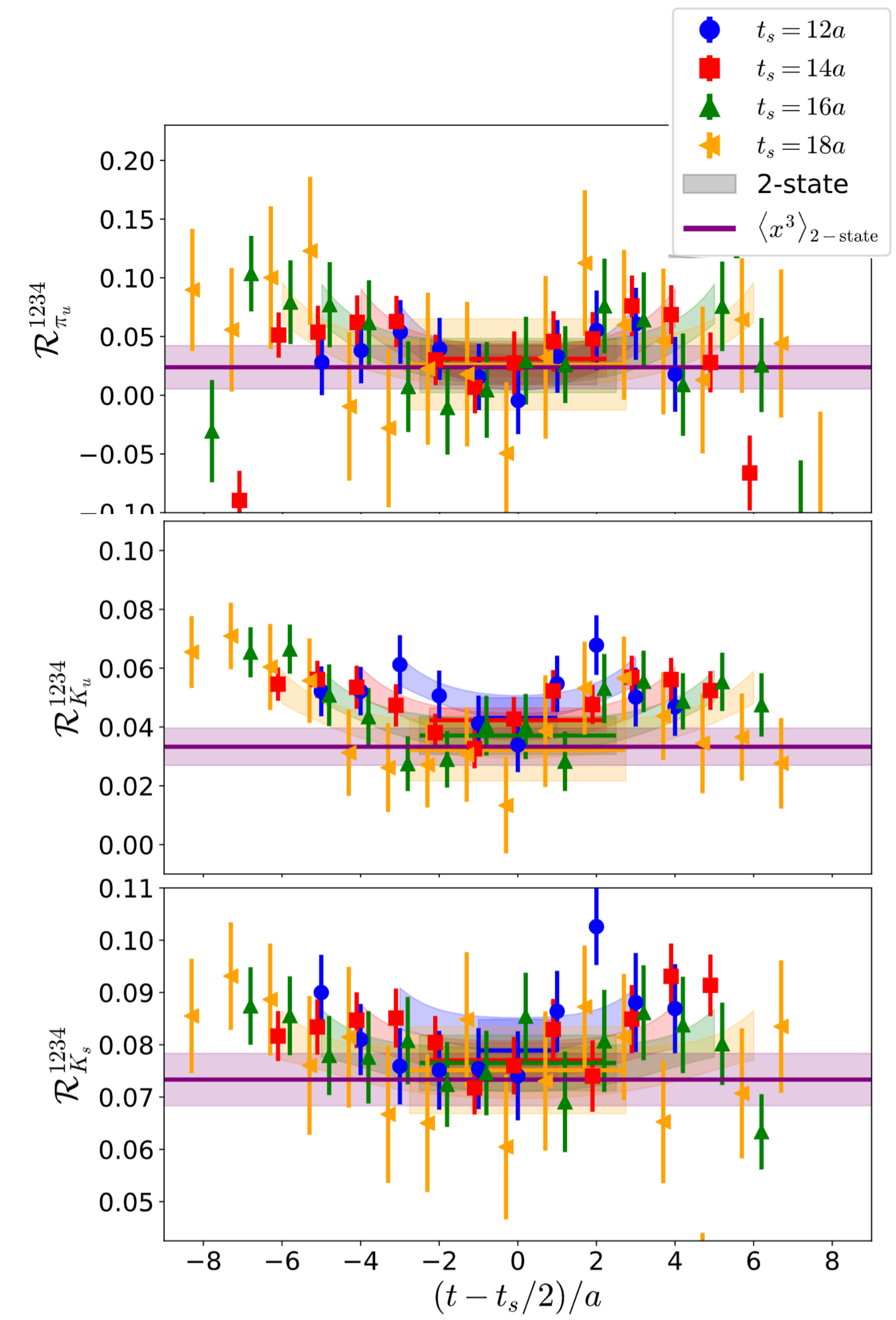

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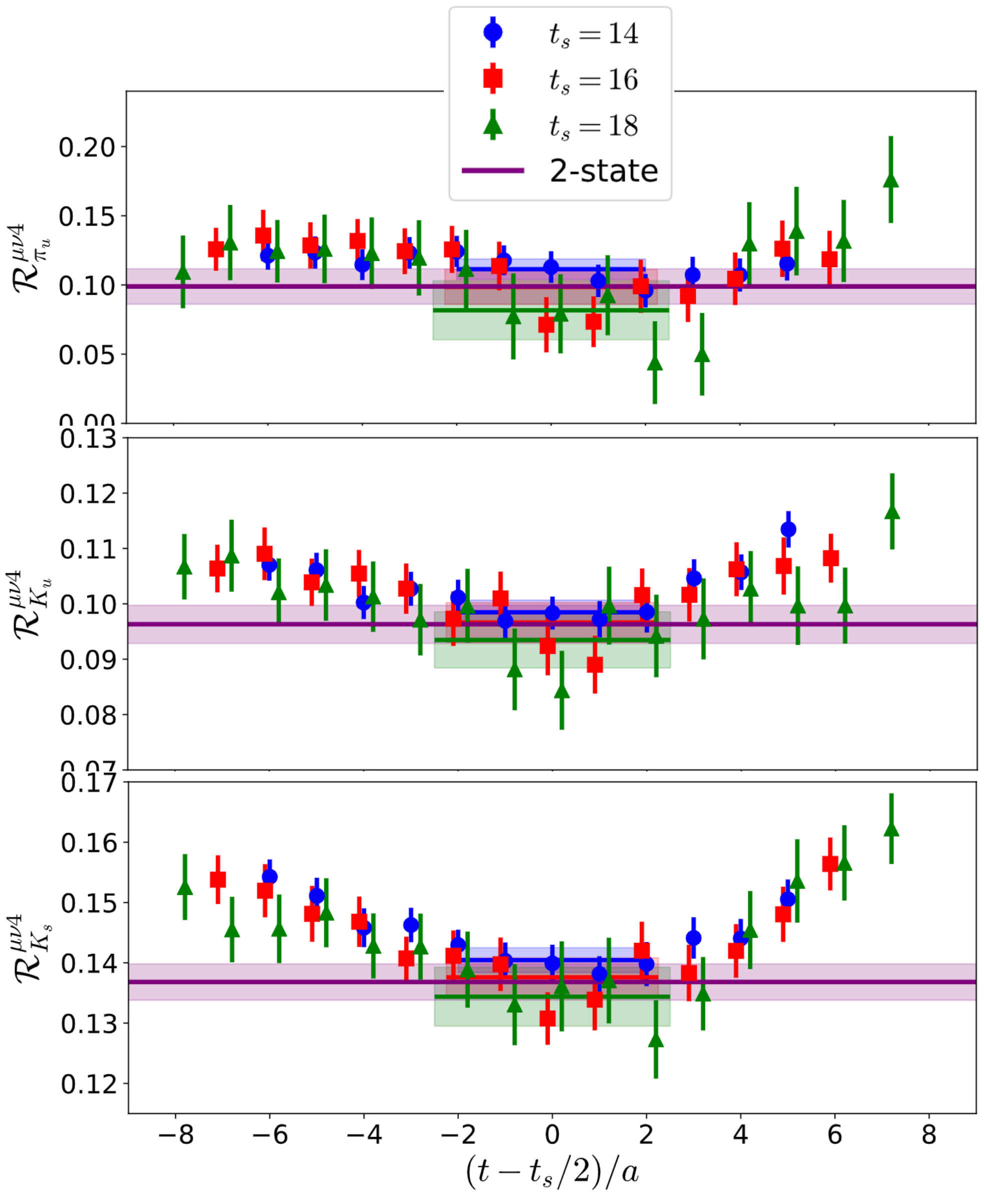

Pion

t_s/a	$\langle x^2 \rangle^u_\pi$	$\langle x^3 \rangle^u_\pi$
12	0.110(6)	0.026(17)
14	0.114(5)	0.031(15)
16	0.105(9)	0.025(23)
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2-state	0.110(7)	0.024(18)

Kaon

t_s/a	$\langle x^2 \rangle^u_K$	$\langle x^2 \rangle^s_K$	$\langle x^3 \rangle^u_K$	$\langle x^3 \rangle^s_K$
12	0.101(2)	0.146(2)	0.043(7)	00.079(6)
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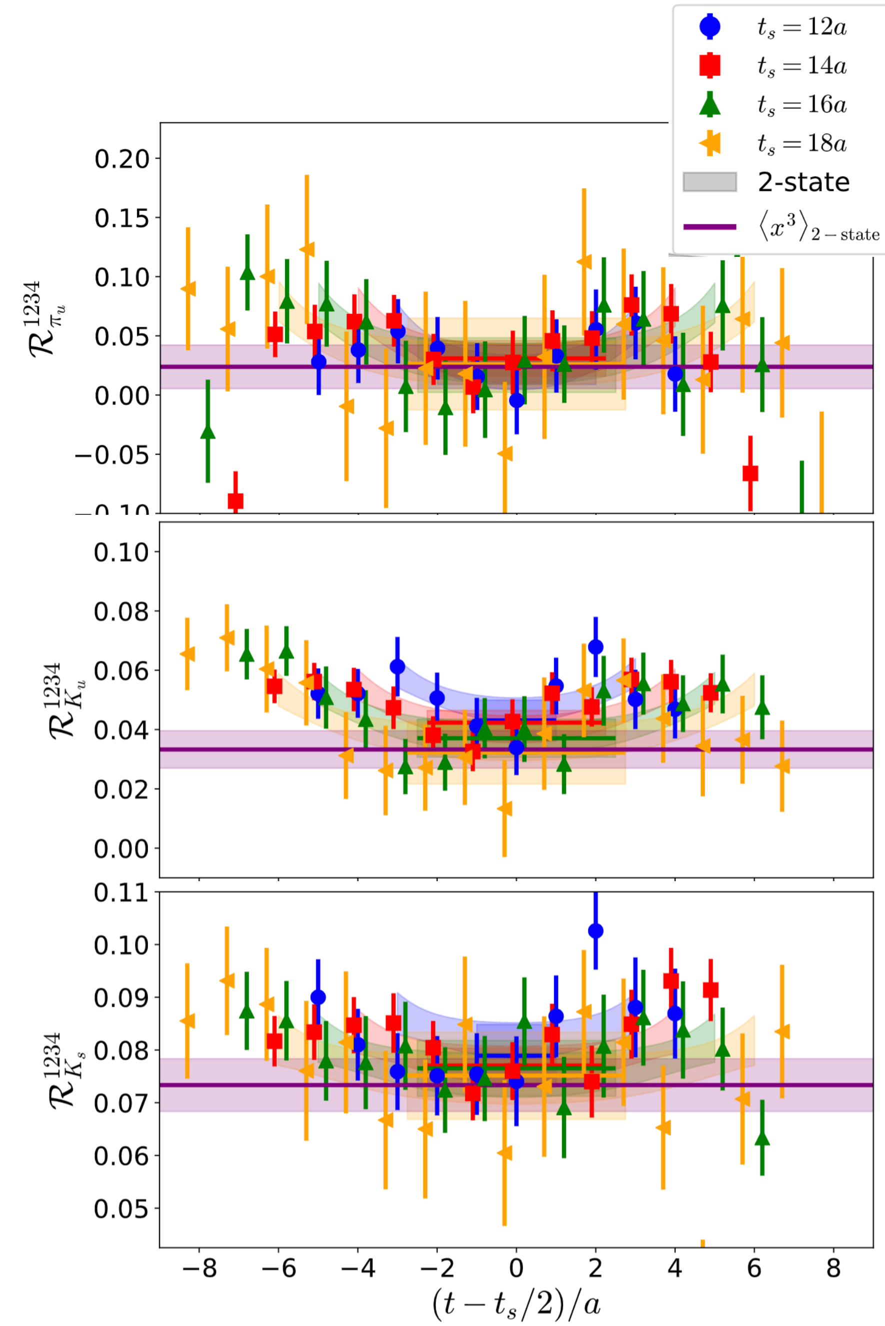

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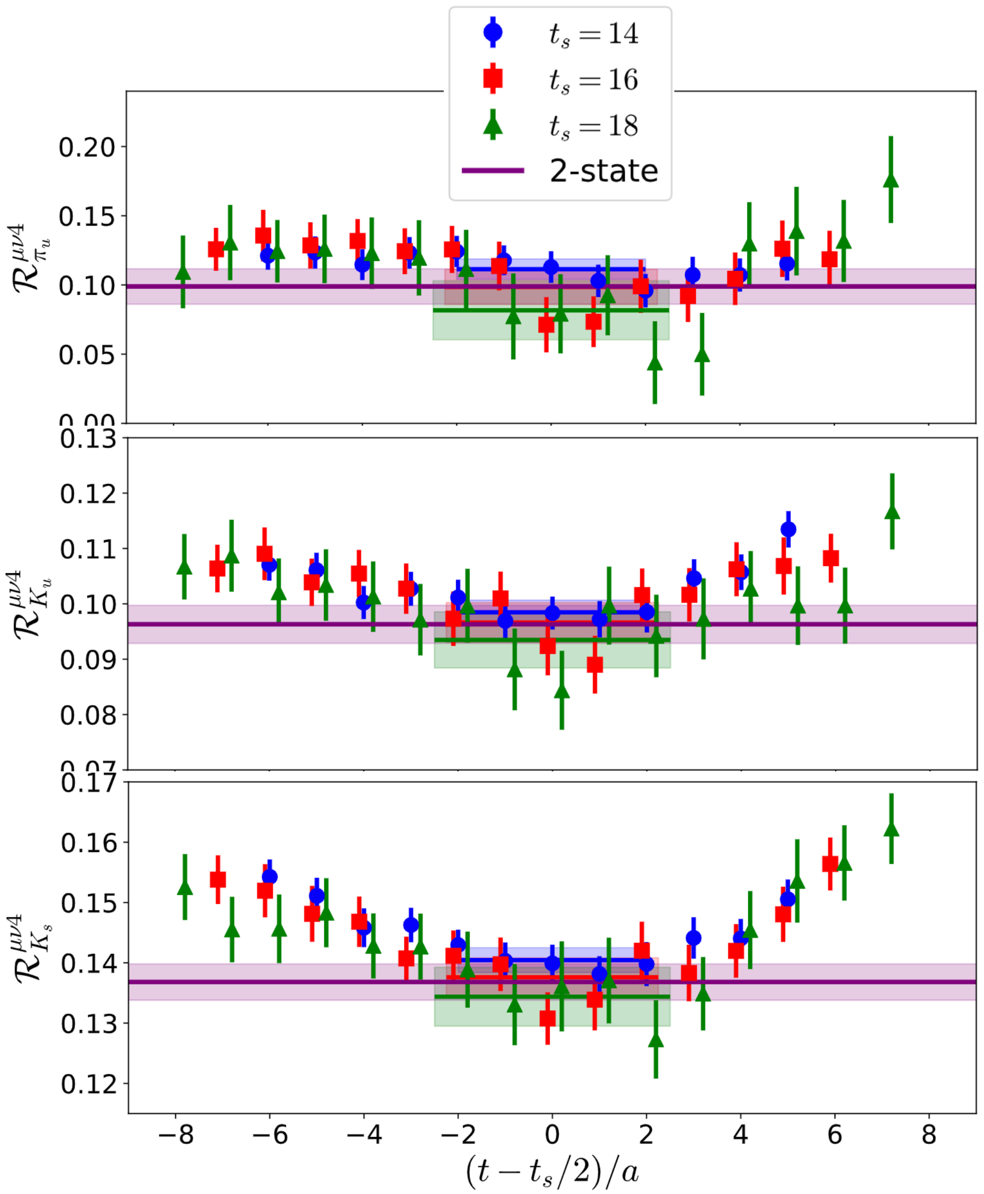

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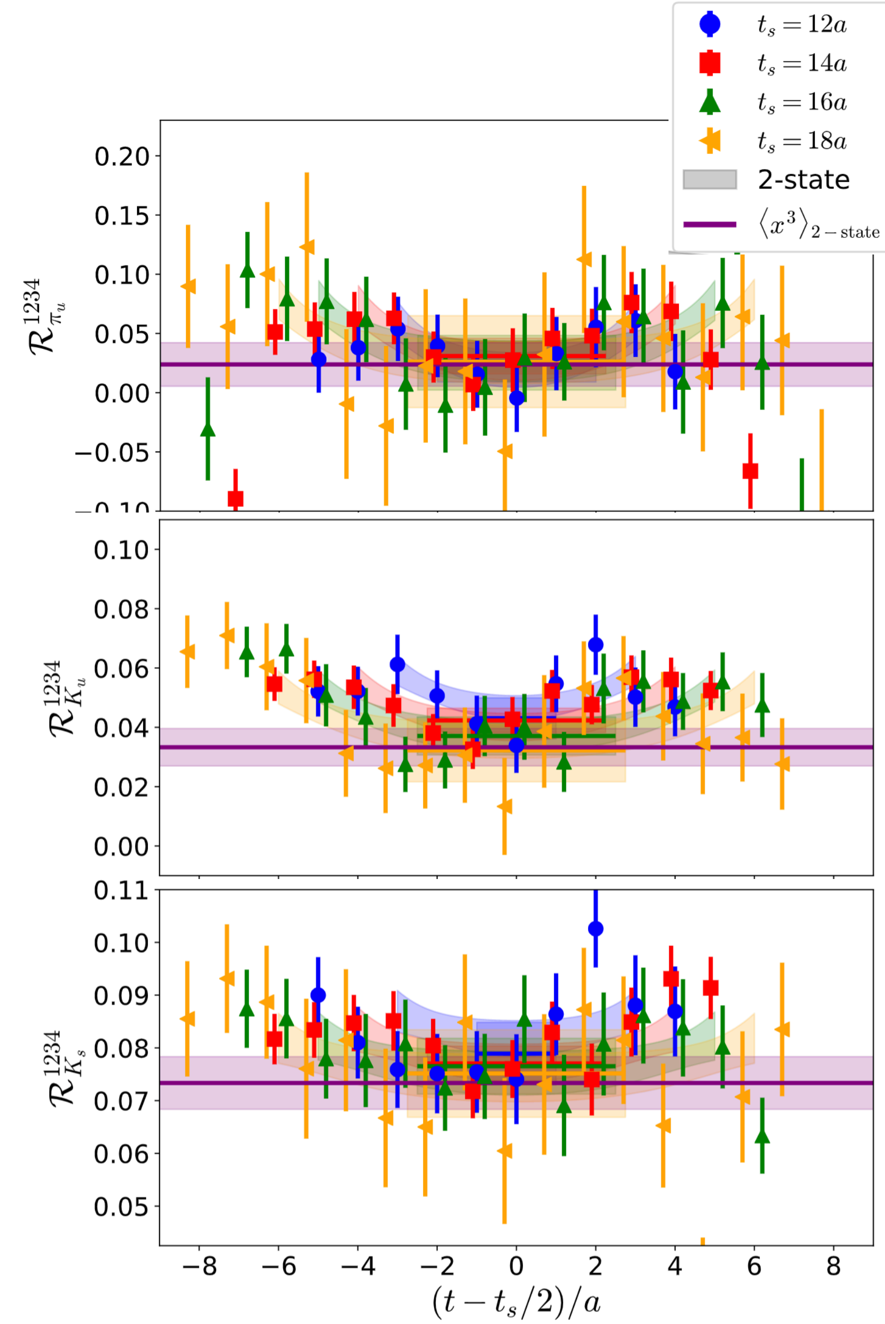
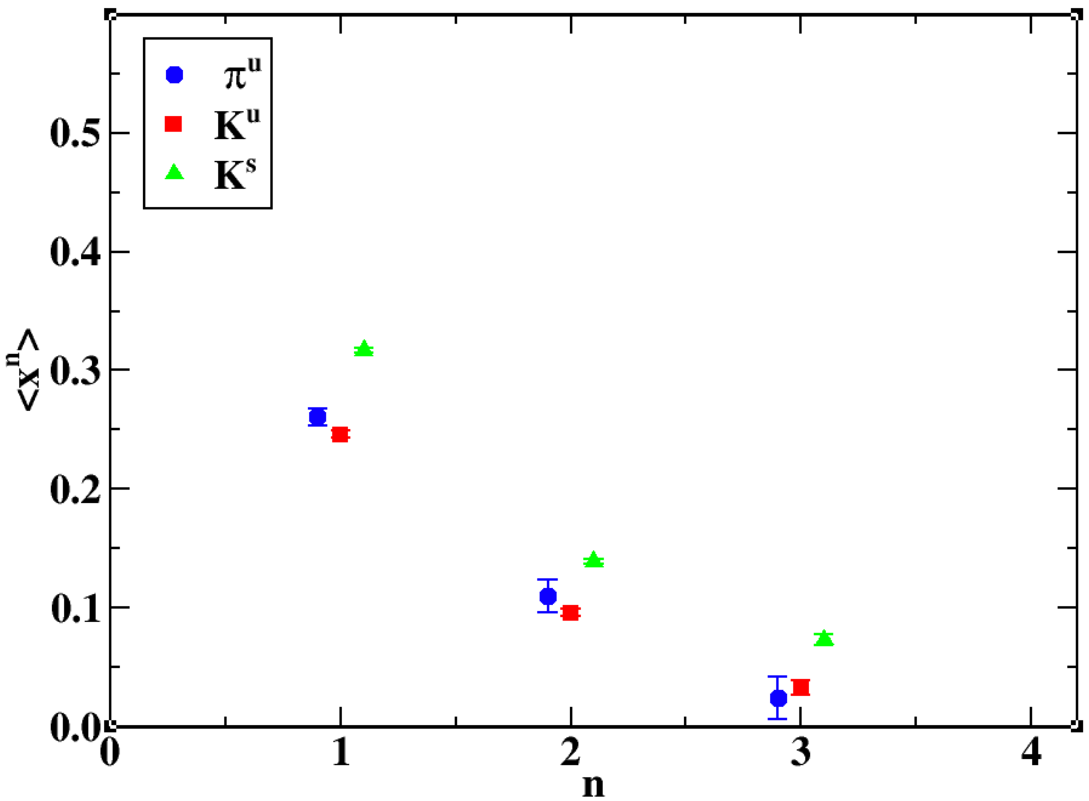


★ **Excited-states contamination not as prominent as for $\langle x\rangle$**

★ **Effect of excited states non negligible for PDF analysis**

Moments summary



- Expected decay as Mellin moment increases
- Up contribution to pion and kaon is similar
- Strange contribution to kaon dominant

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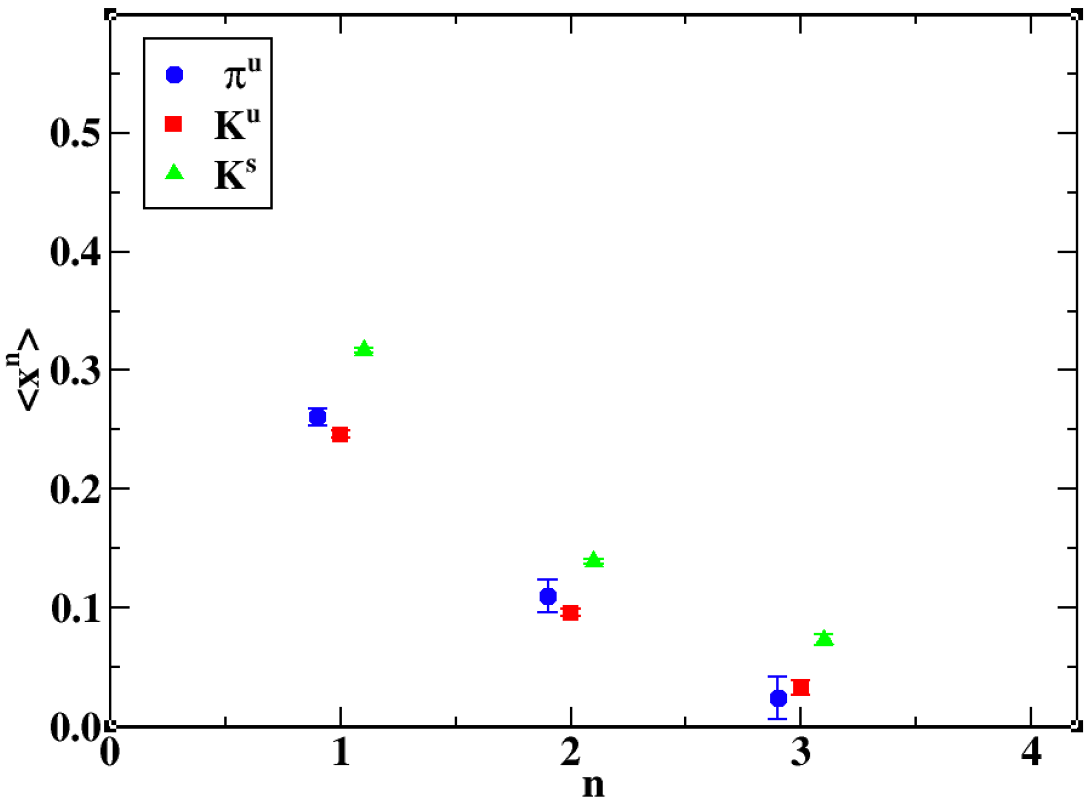

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What can we learn for PDFs from their moments

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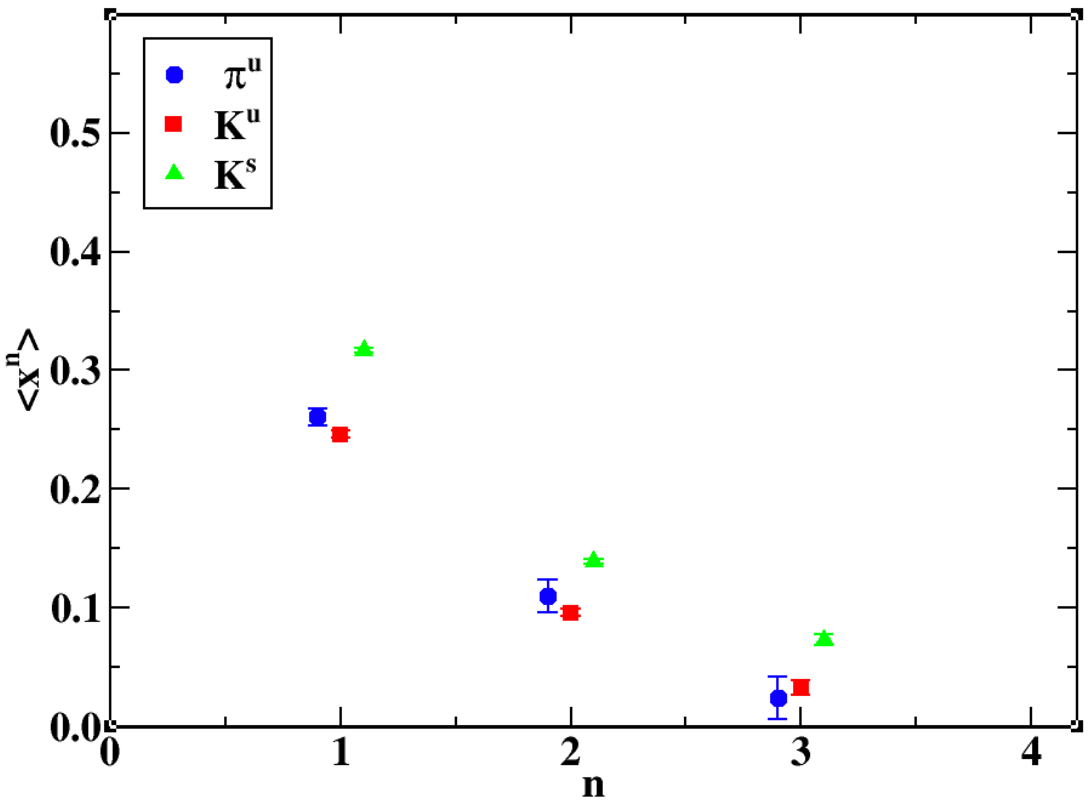

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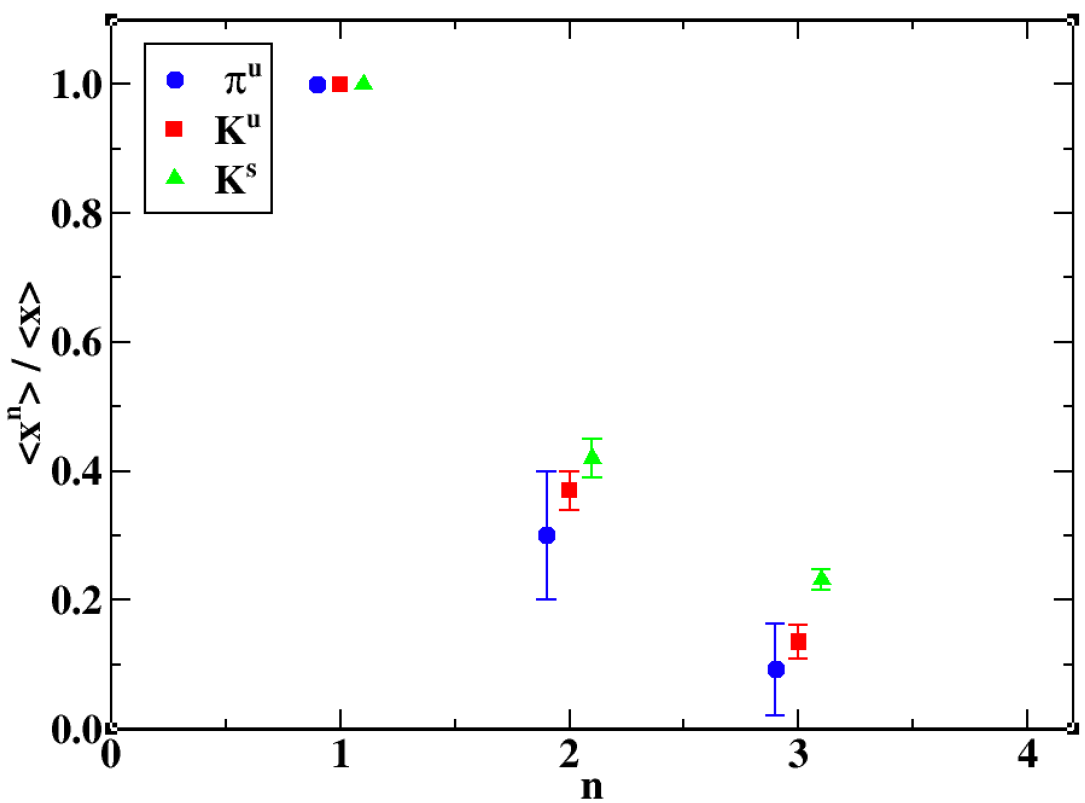
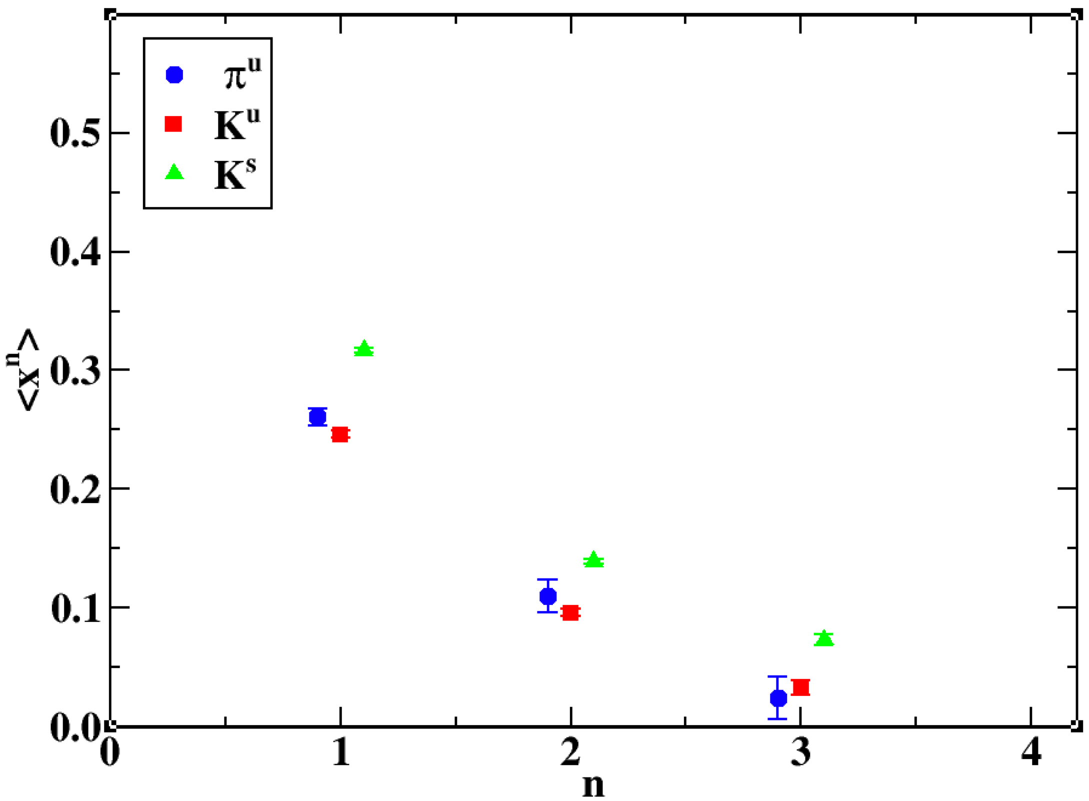


★ **Larger moments have support at higher x**

- $\langle x^2 \rangle^u_\pi \sim 20 - 40\,\%\, \langle x \rangle^u_\pi$ $\quad \langle x^3 \rangle^u_\pi \sim 5 - 20\,\%\, \langle x \rangle^u_\pi$
- $\langle x^2 \rangle^u_K \sim 35 - 40\,\%\, \langle x \rangle^u_K$ $\quad \langle x^3 \rangle^u_K \sim 10 - 15\,\%\, \langle x \rangle^u_K$
- $\langle x^2 \rangle^s_K \sim 40 - 45\,\%\, \langle x \rangle^s_K$ $\quad \langle x^3 \rangle^s_K \sim 20 - 25\,\%\, \langle x \rangle^s_K$

Moments summary



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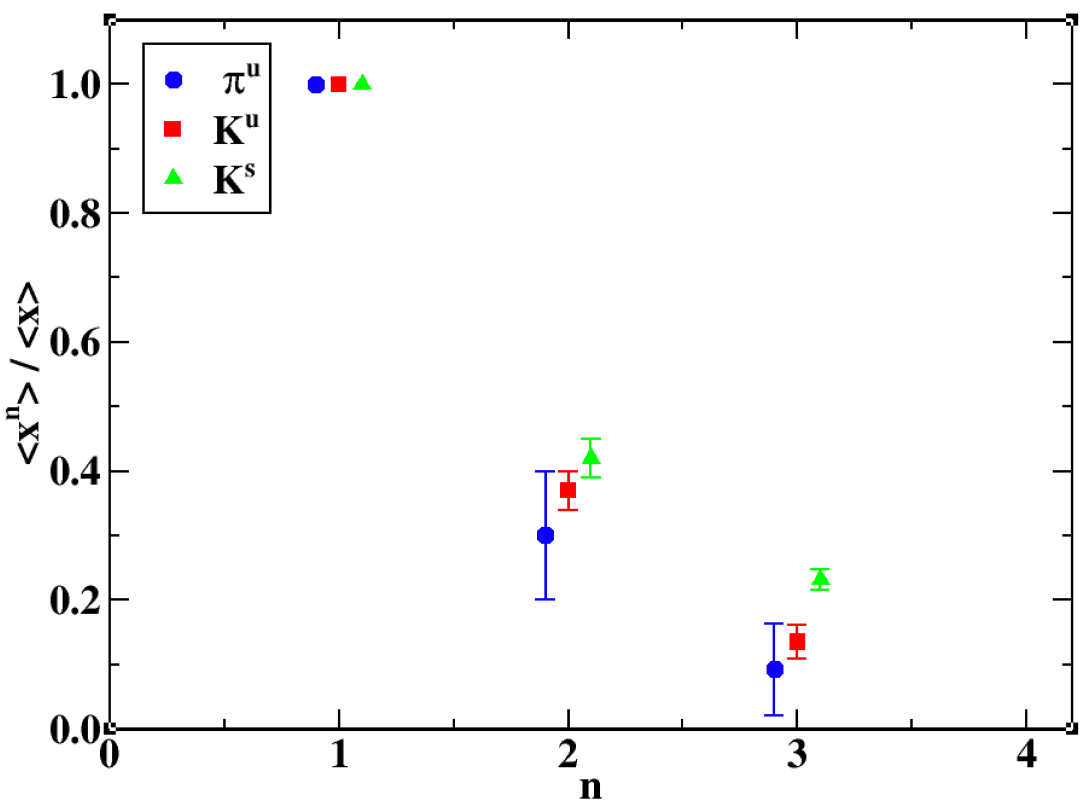

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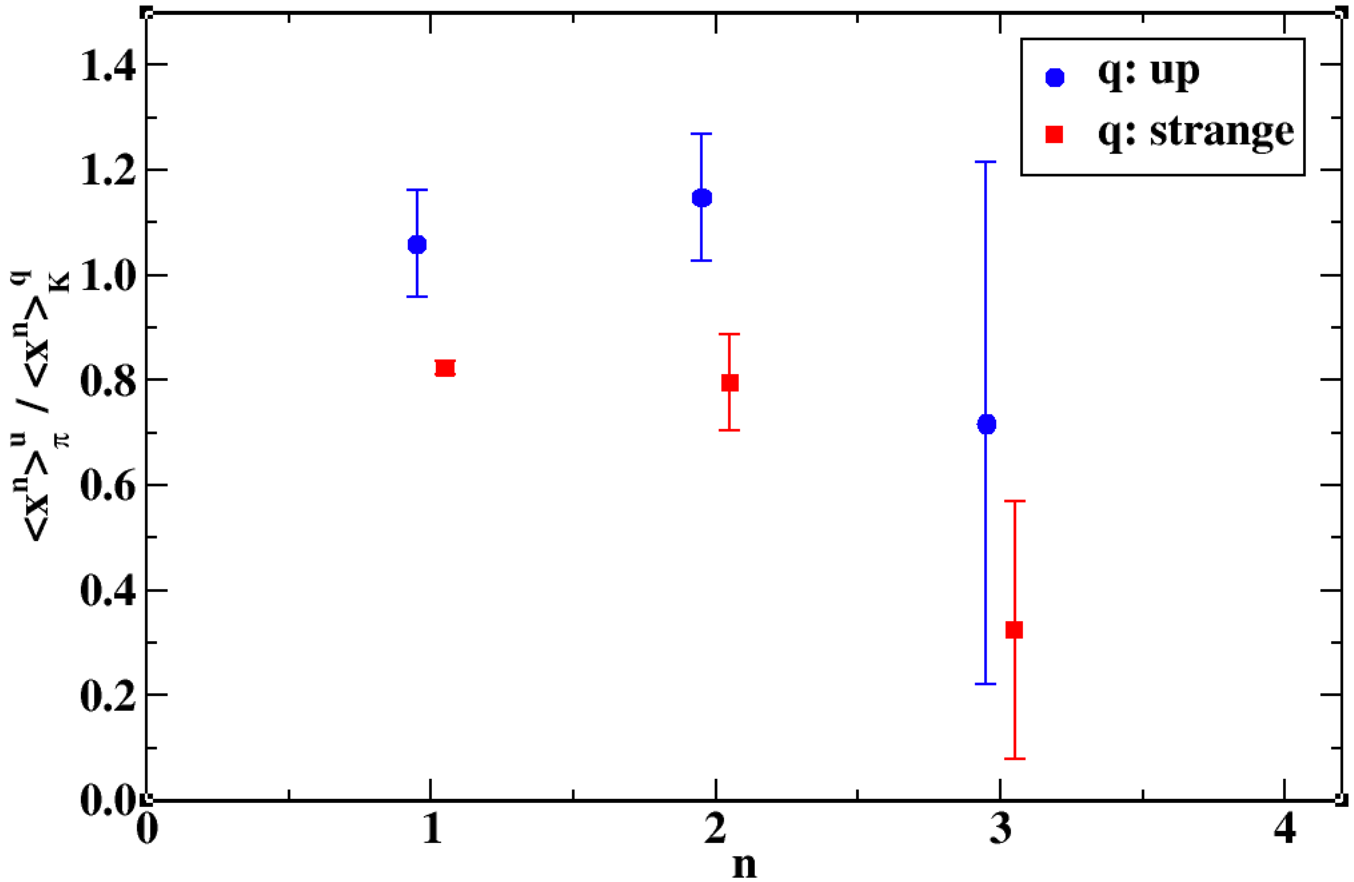
- $\langle x^2\rangle^s_K \sim 40-45\,\%\,\langle x\rangle^s_K$ $\langle x^3\rangle^s_K \sim 20-25\,\%\,\langle x\rangle^s_K$

What can we learn for SU(3) flavor symmetry breaking

SU(3) flavor symmetry breaking

★ Shape of up-quark pion and kaon PDFs expected to be similar

★ Strange-quark kaon expected to have support at higher-x than up-quark



★ Qualitative picture confirms expectations from quark mass effects

SU(3) flavor symmetry breaking

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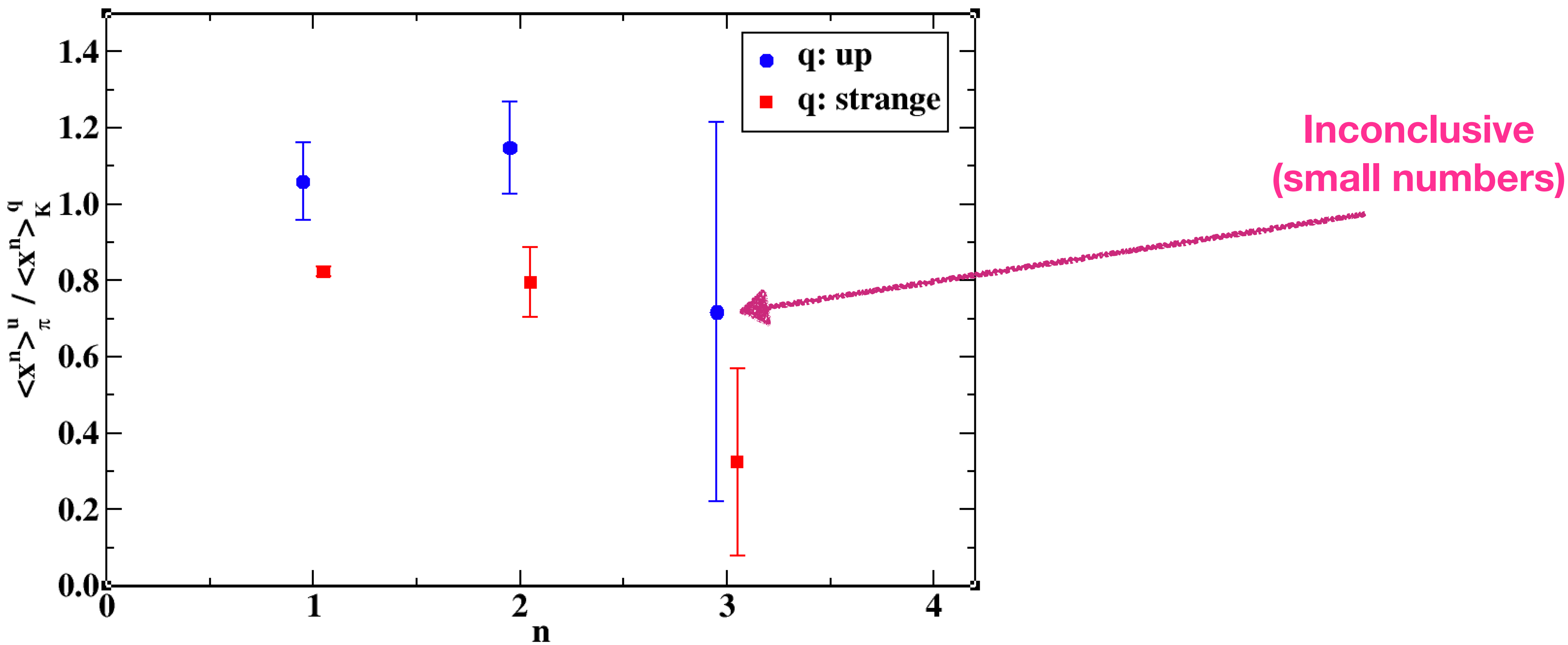
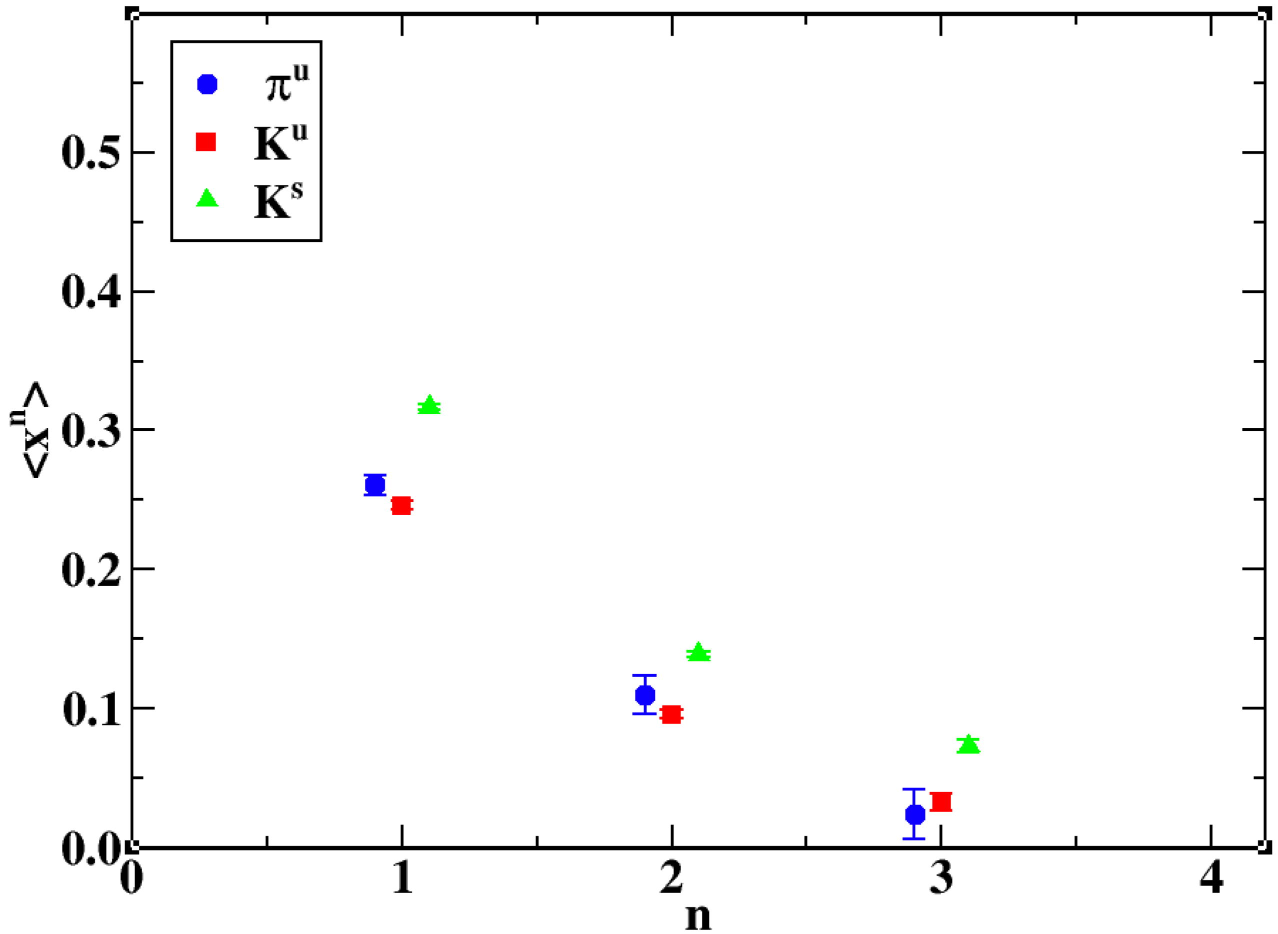


★ **Qualitative picture confirms expectations from quark mass effects**

Recapitulation



$$\langle x\rangle^u_{\pi^+} = 0.261(3)(6) \qquad \langle x^2\rangle^u_{\pi^+} = 0.110(7)(12) \qquad \langle x^3\rangle^u_{\pi^+} = 0.024(18)(2)$$

$$\langle x\rangle^u_{K^+} = 0.246(2)(2) \qquad \langle x^2\rangle^u_{K^+} = 0.096(2)(2) \qquad \langle x^3\rangle^u_{K^+} = 0.033(6)(1)$$

$$\langle x\rangle^s_{K^+} = 0.317(2)(1) \qquad \langle x^2\rangle^s_{K^+} = 0.139(2)(1) \qquad \langle x^3\rangle^s_{K^+} = 0.073(5)(2)$$

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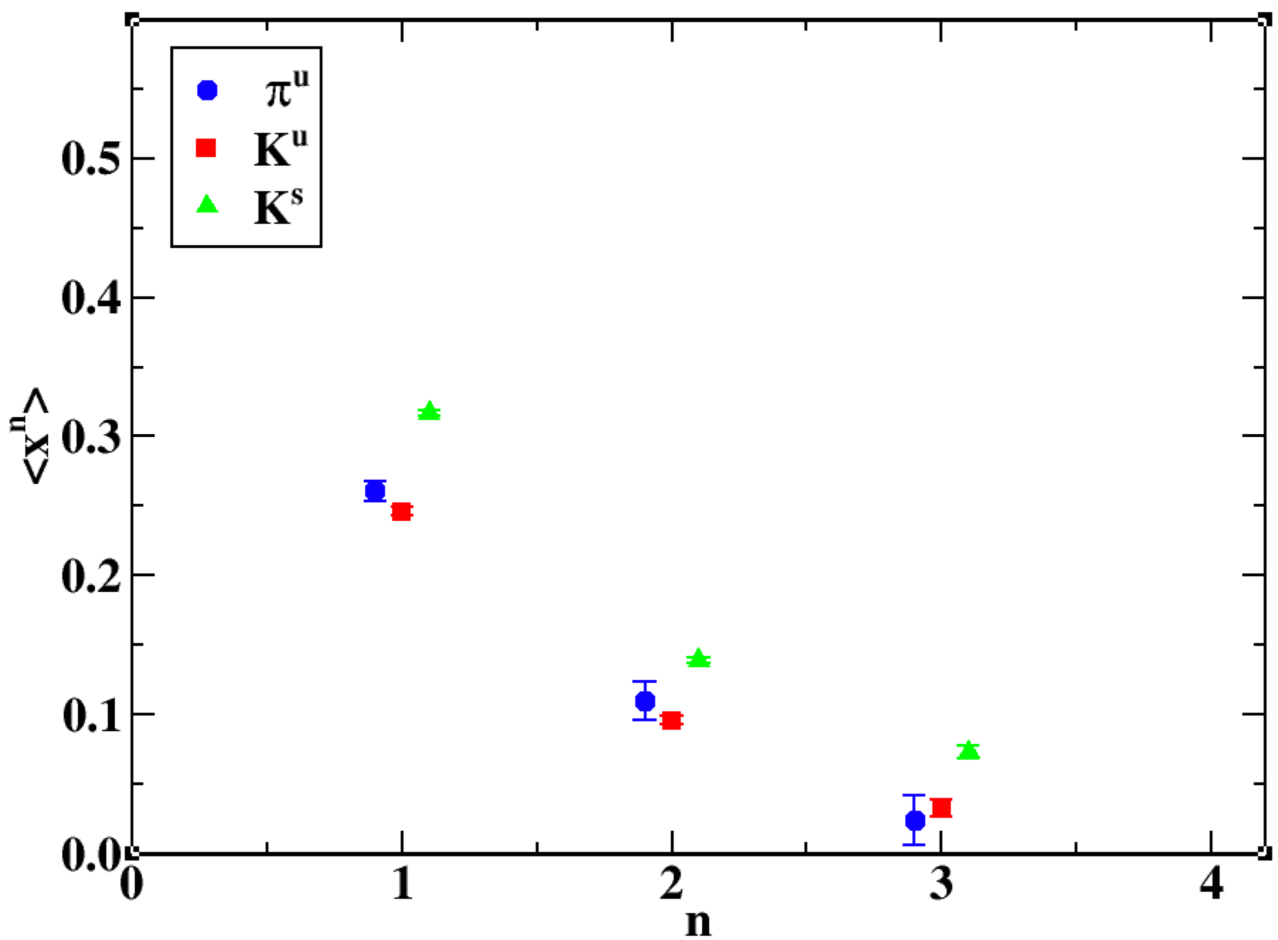

Increase of moment →

Pion (u)	$\langle x\rangle^u_{\pi^+} = 0.261(3)(6)$	$\langle x^2\rangle^u_{\pi^+} = 0.110(7)(12)$	$\langle x^3\rangle^u_{\pi^+} = 0.024(18)(2)$
Kaon (u)	$\langle x\rangle^u_{K^+} = 0.246(2)(2)$	$\langle x^2\rangle^u_{K^+} = 0.096(2)(2)$	$\langle x^3\rangle^u_{K^+} = 0.033(6)(1)$
Kaon (s)	$\langle x\rangle^s_{K^+} = 0.317(2)(1)$	$\langle x^2\rangle^s_{K^+} = 0.139(2)(1)$	$\langle x^3\rangle^s_{K^+} = 0.073(5)(2)$

PDF reconstruction

Fit functions for PDFs

$$q_M^f(x) = N x^\alpha (1-x)^\beta (1 + \rho\sqrt{x} + \gamma x)$$

$$N = \frac{1}{B(\alpha+1,\beta+1) + \gamma B(2+\alpha,\beta+1)}$$

$$\langle x^n \rangle = \frac{\left(\prod_{i=1}^n (i+\alpha)\right)\left(n+2+\alpha+\beta+(i+1+\alpha)\gamma\right)}{\left(\prod_{i=1}^n (i+2+\alpha+\beta)\right)\left(2+\alpha+\beta+(1+\alpha)\gamma\right)}, \quad n>0$$

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Lattice data

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Lattice data

$\overline{\mathrm{MS}}(5.2\,\mathrm{GeV})$

fit type	α_π^u	β_π^u	γ_π^u
2-parameter	-0.04(20)	2.23(65)	0
3-parameter	-0.54(22)	2.76(64)	22.17(17.87)
fit type	α_K^u	β_K^u	γ_K^u
2-parameter	-0.05(7)	2.42(24)	0
3-parameter	-0.56(72)	3.01(23)	25.11(5.23)
fit type	α_K^s	β_K^s	γ_K^s
2-parameter	0.21(8)	2.13(20)	0
3-parameter	0.18(95)	2.051(3.46)	0.347(16.10)

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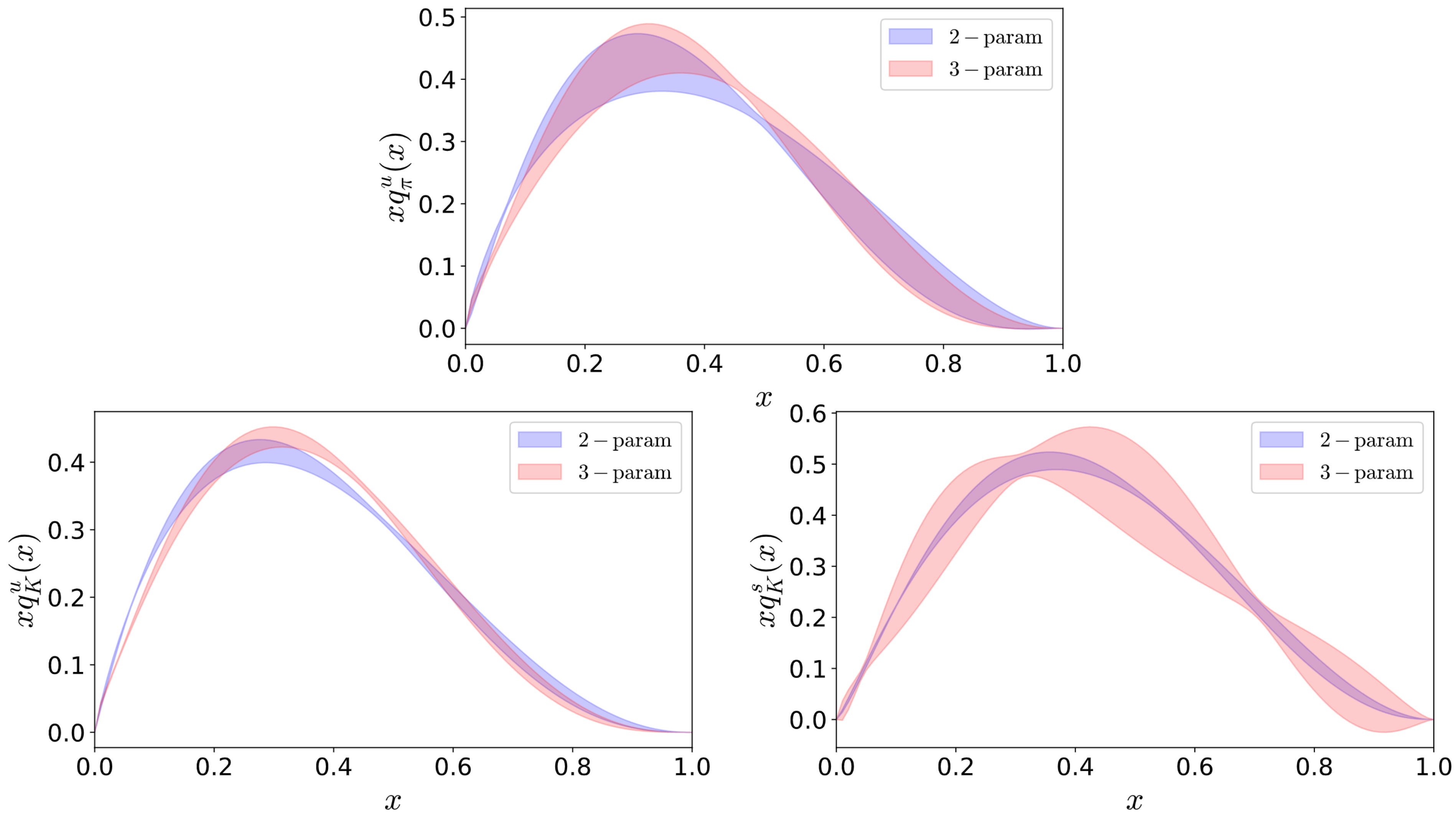
- ★ **3-parameter fit not very stable**
- ★ β **governs large-x behavior**
- ★ **Lattice data favor $(1-x)^2$ decay**

PDFs dependence on fits



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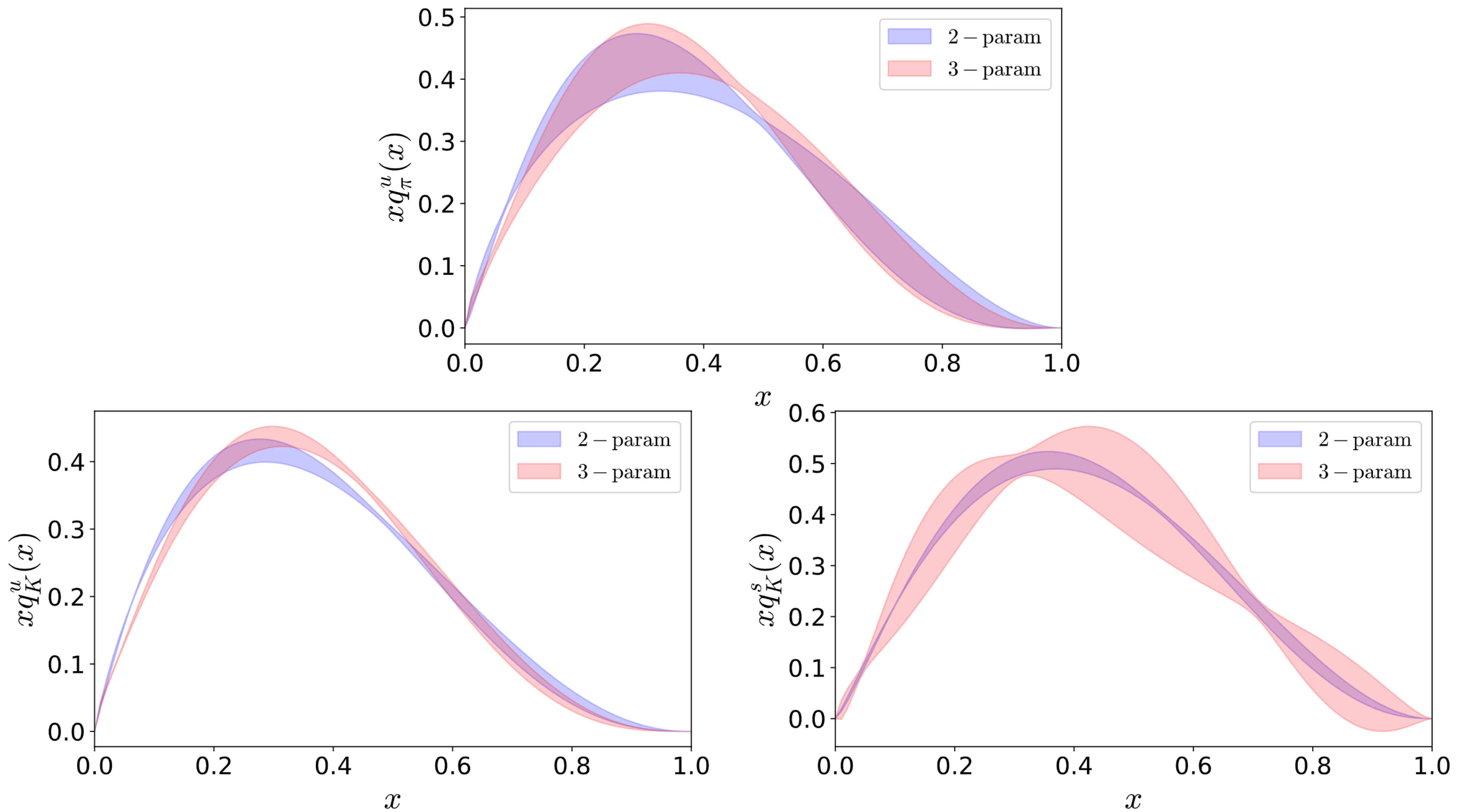

★ **Estimating γ is competing with other parameters**
(information up to $\langle x^3 \rangle$)

★ **PDFs shape compatible for both fits**

★ **2-parameter fit has smaller uncertainties**

Excited-states effects

★ **Excited-states effect more prominent for $\langle x \rangle$**

Excited-states effects

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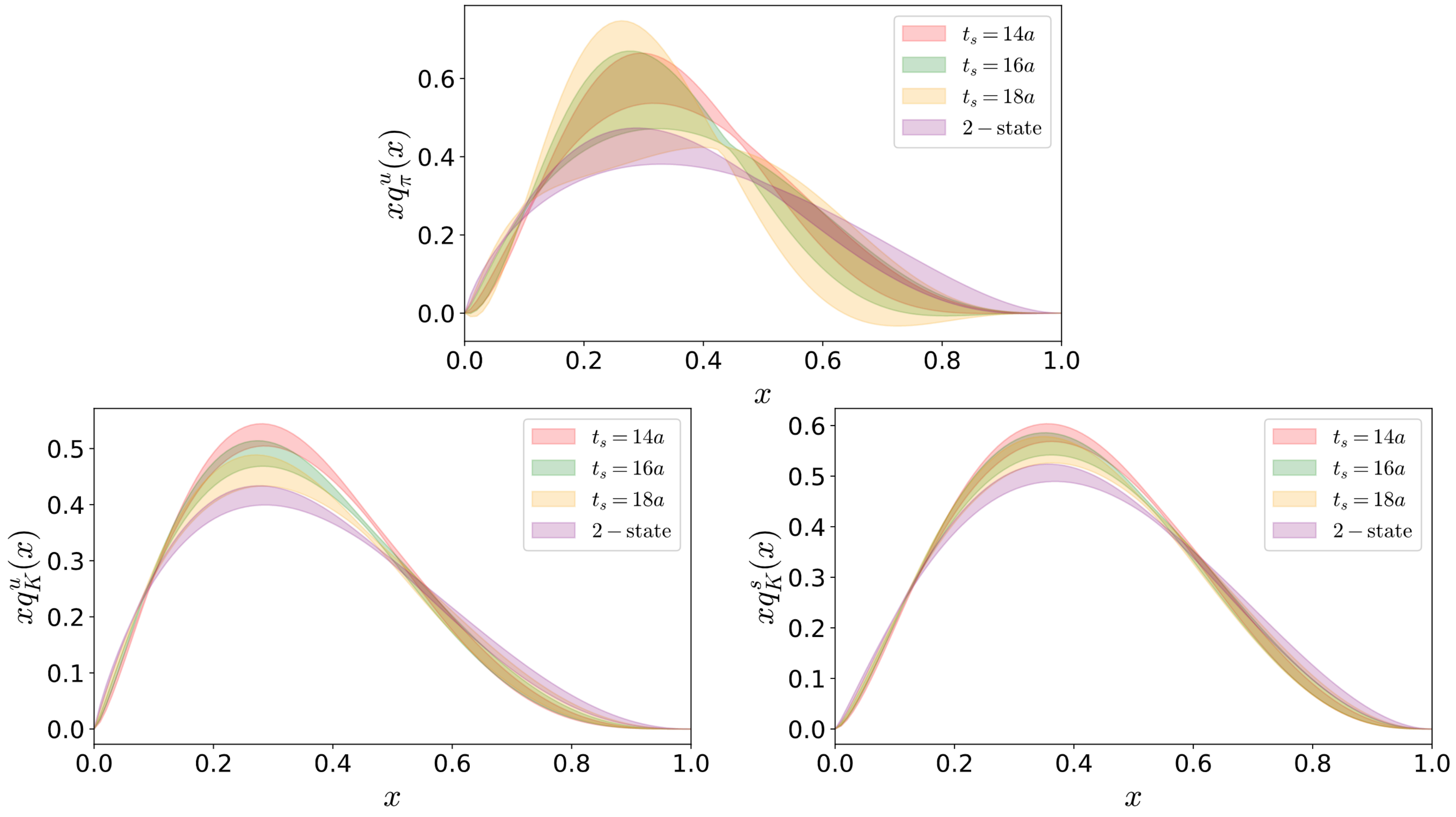

★ **Small-x region insensitive to excited-states effects**

★ **Large-x region: 2-state fit higher than small Tsink values**

★ **Peak: susceptible to excited-states effect**
(Elimination of excited states bring the peak to the expected value)

Quality of reconstruction

★ **How much information do higher moments contain?**

Quality of reconstruction

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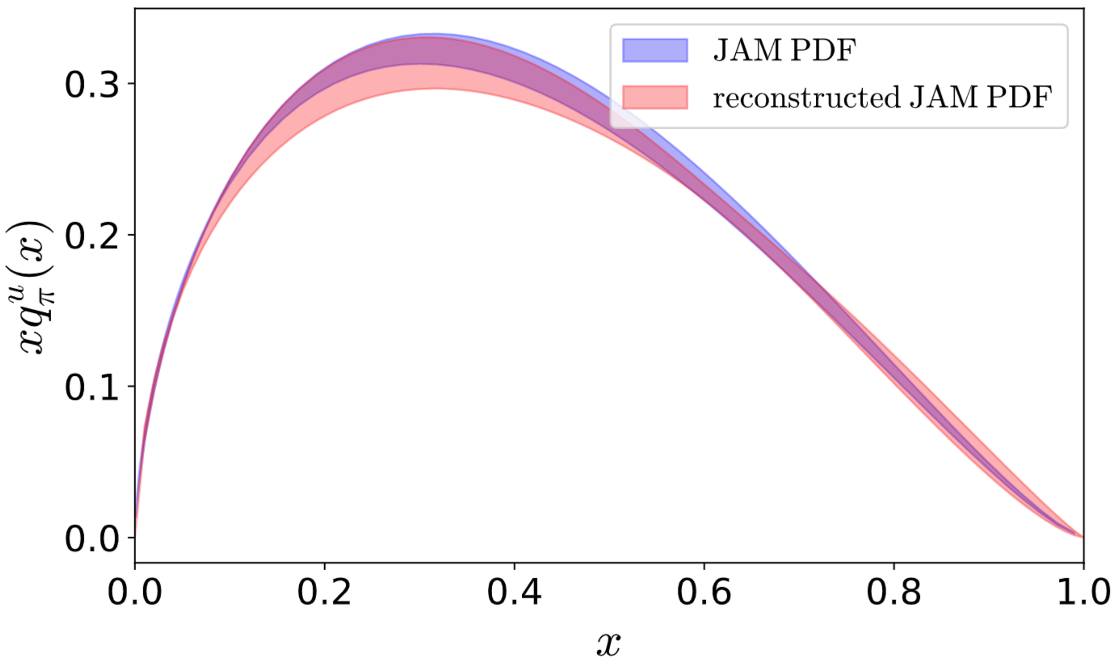

Quality of reconstruction

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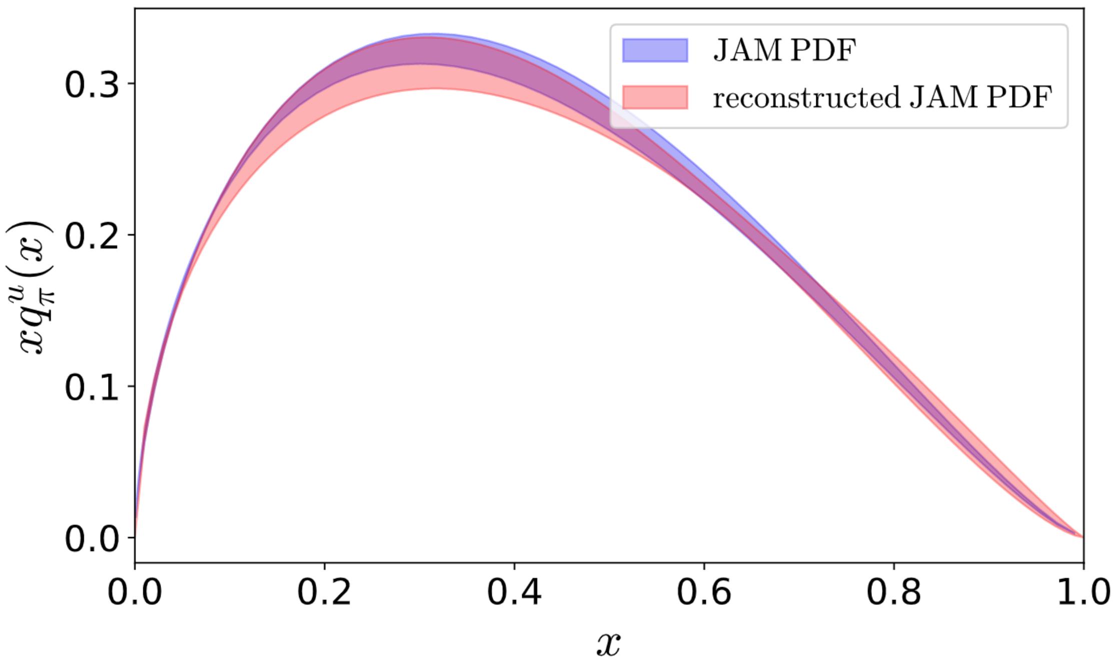

JAM PDF reconstructed correctly using the first 3 nontrivial moments

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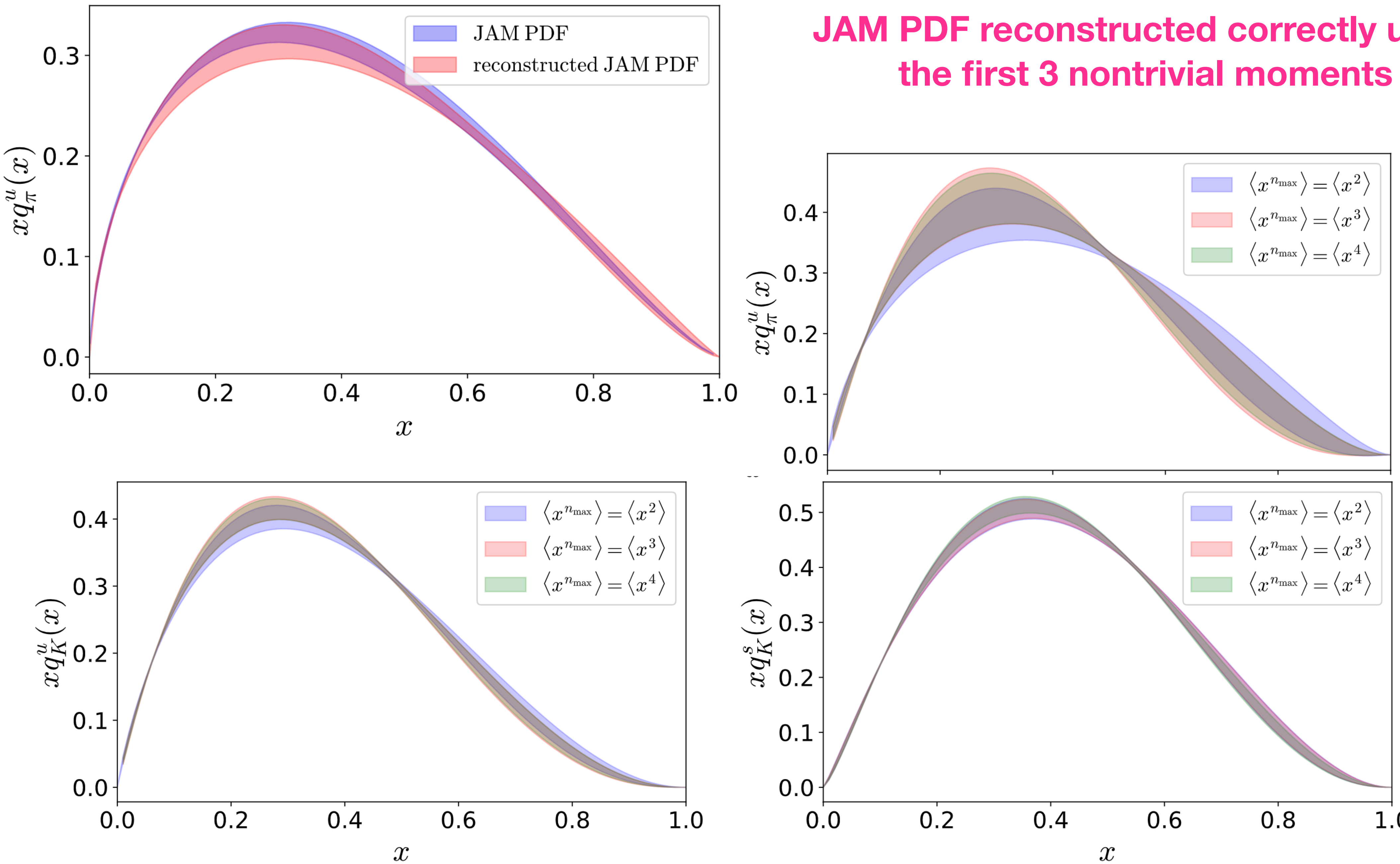

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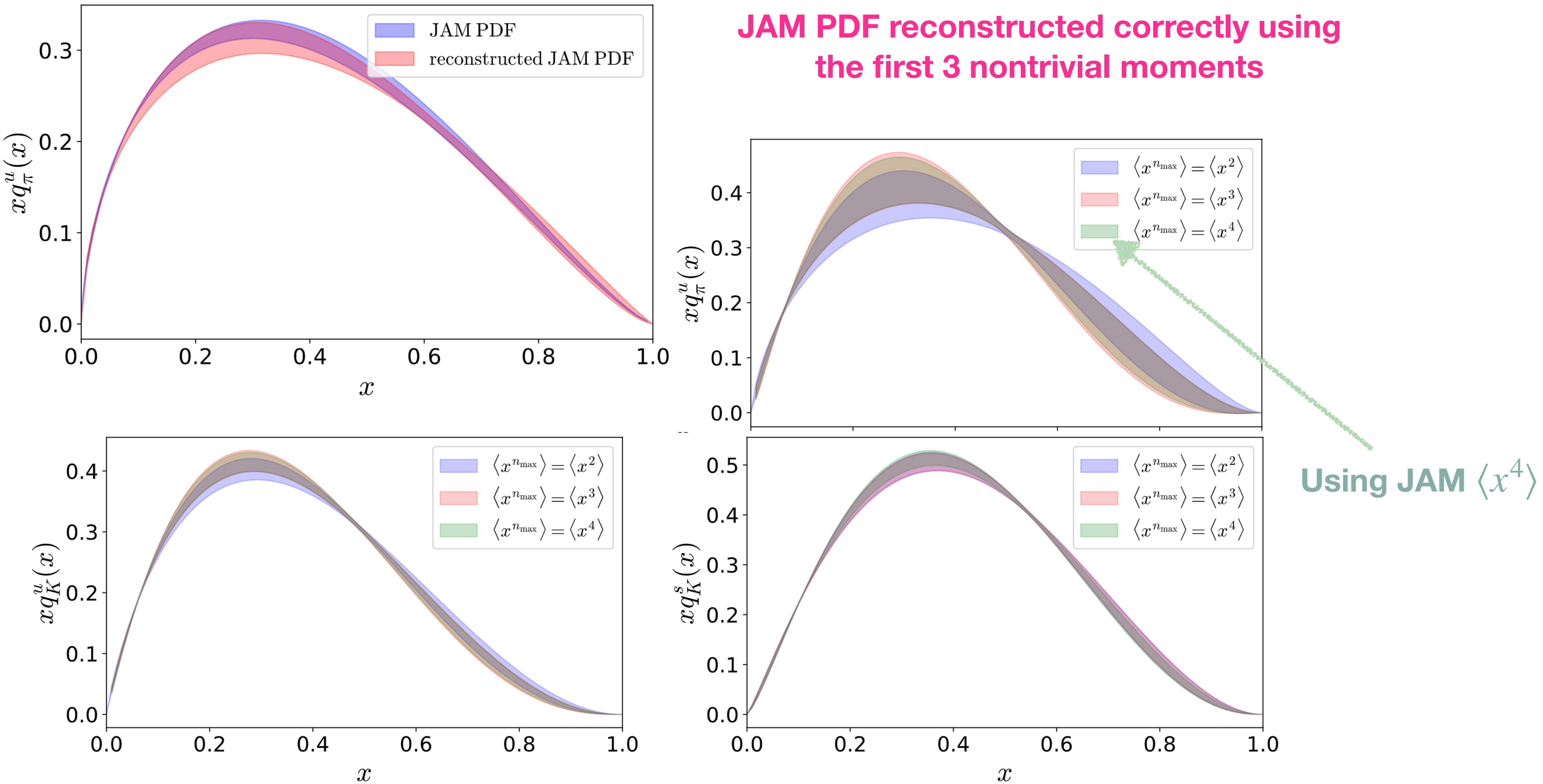

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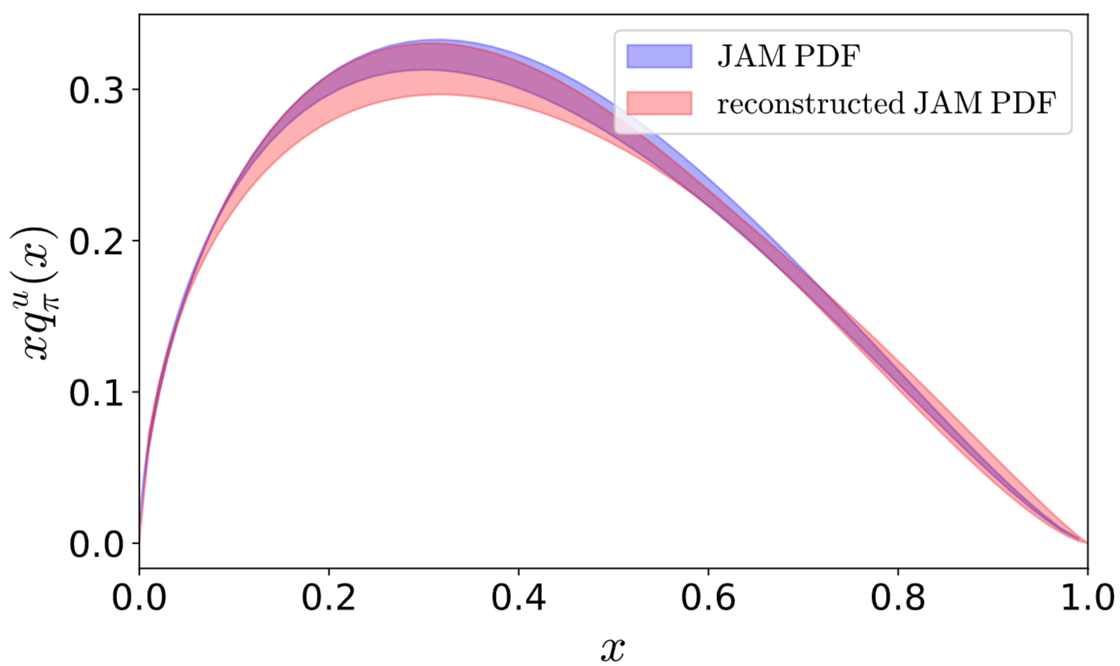

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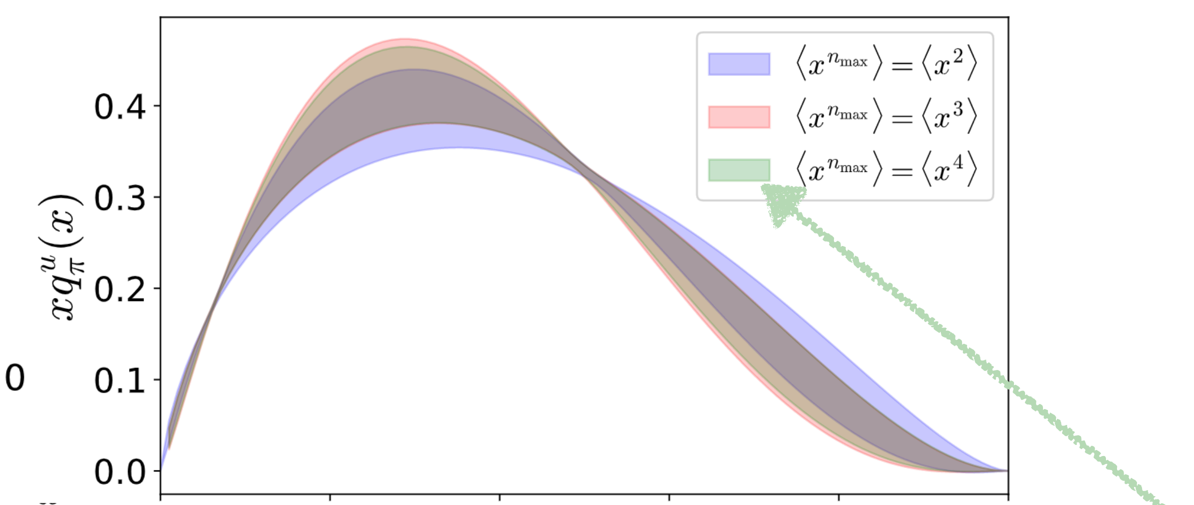
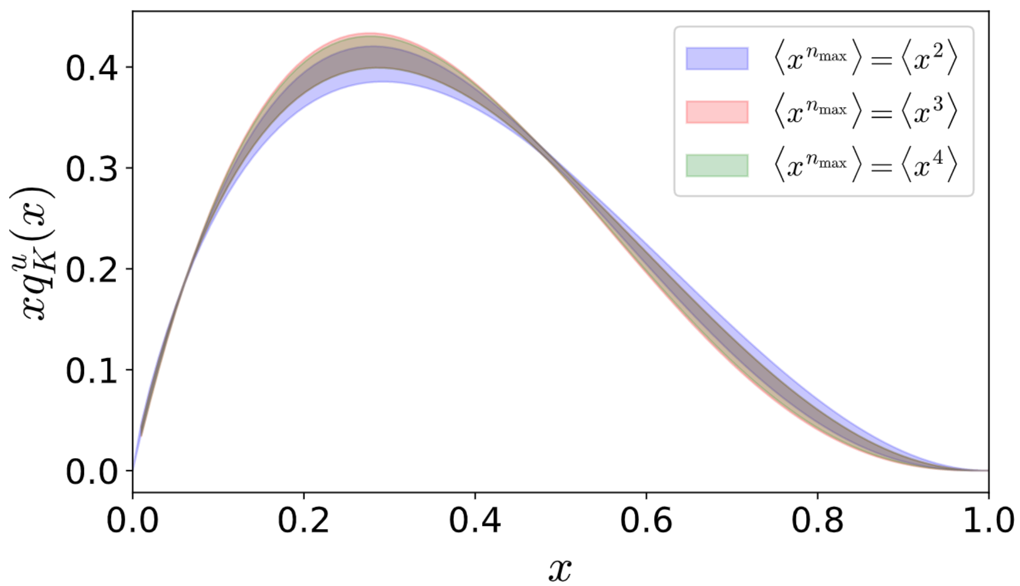
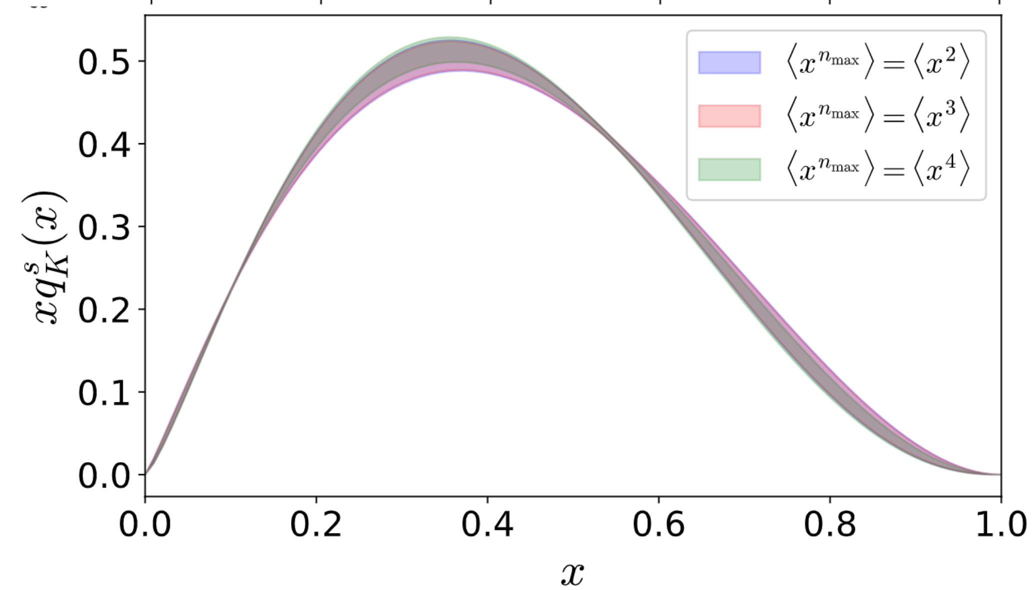


Using JAM $\langle x^4\rangle$

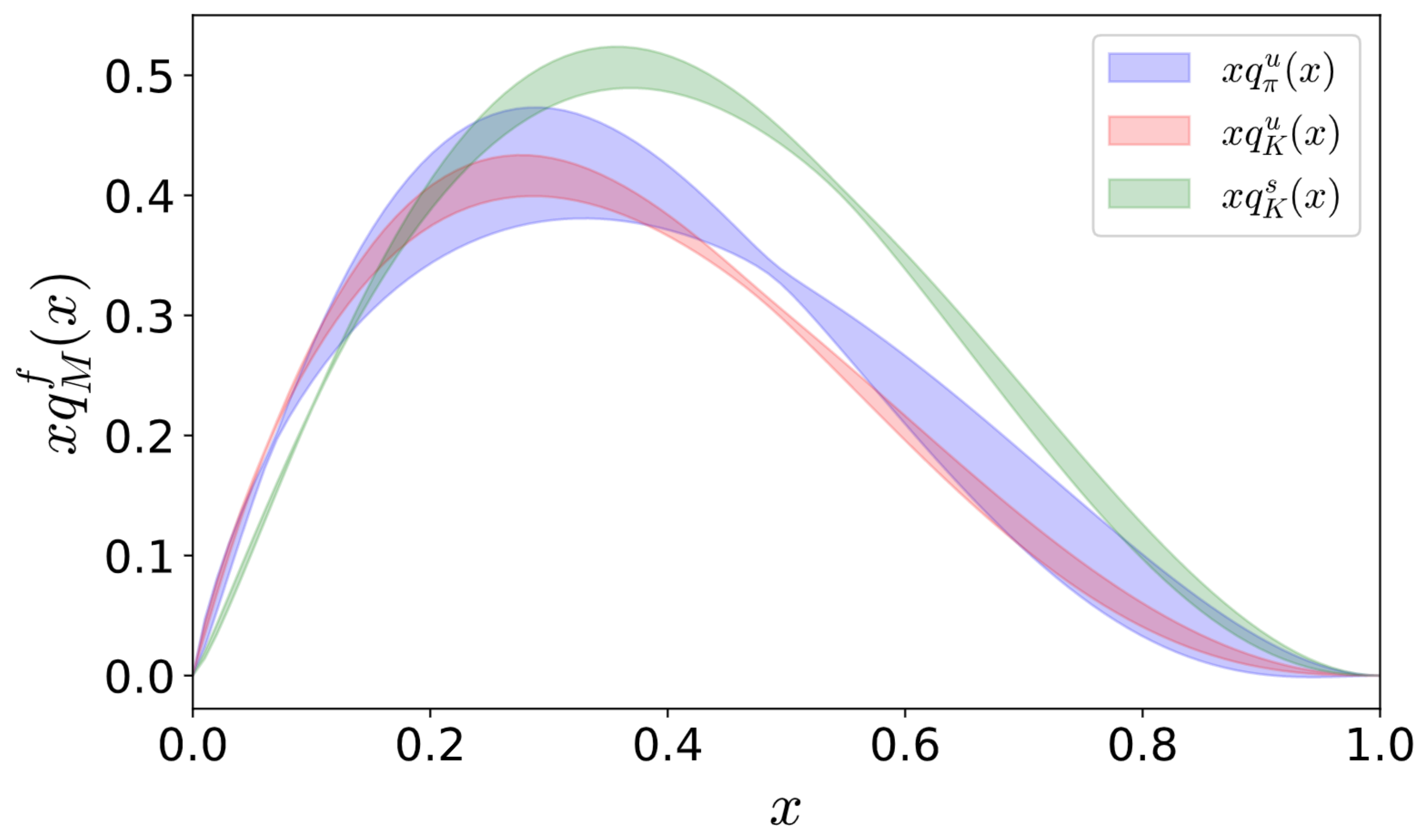




★ **Most of the information is contained in the moments up to $\langle x^3\rangle$**

$\langle x^3\rangle_{\max}$ **fully compatible with** $\langle x^4\rangle_{\max}$

SU(3) flavor symmetry breaking



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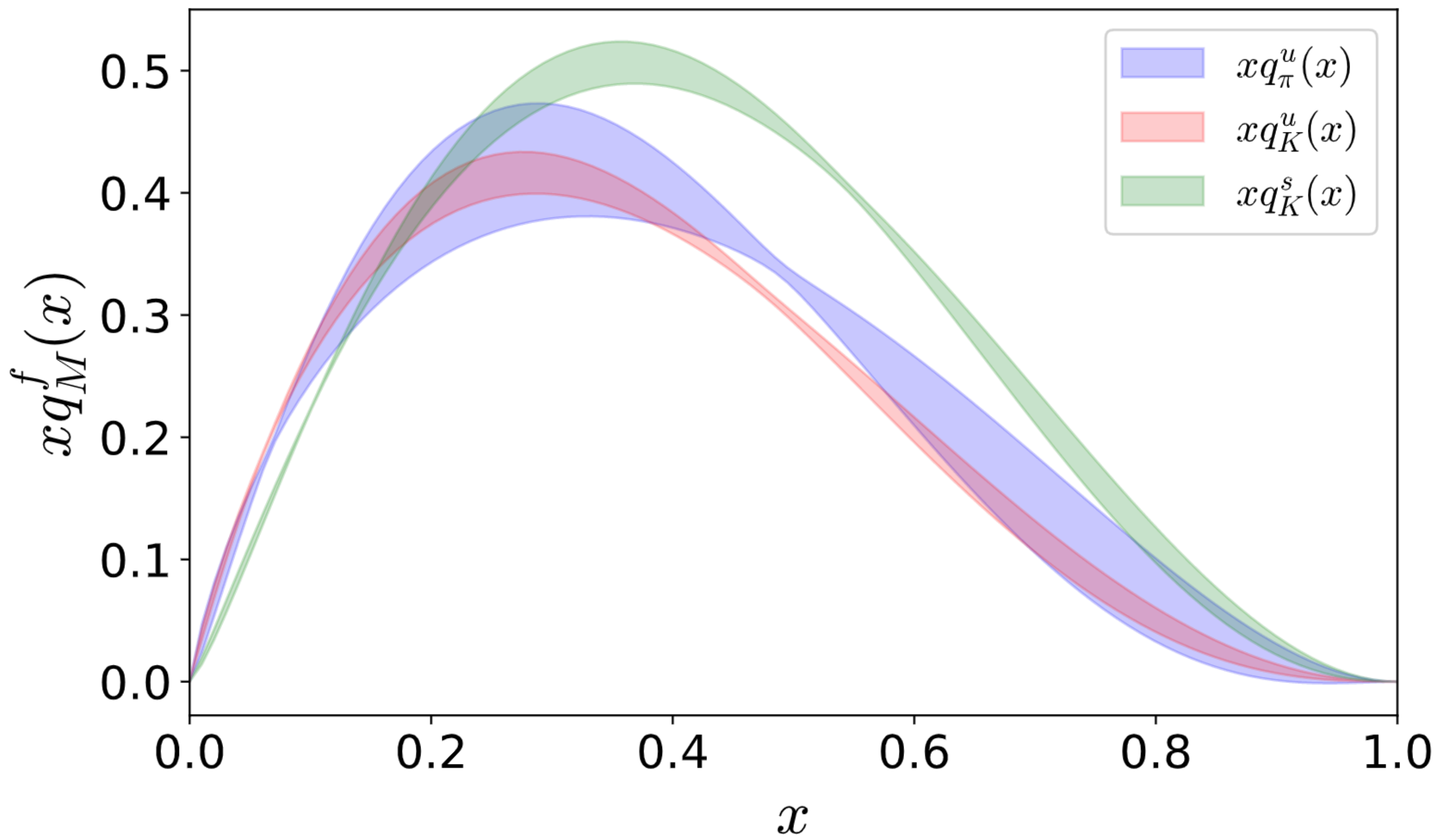

- ★ **Up-quark seems to have a similar role in pion and kaon.**
  $xq^u_\pi(x)$ **compatible with** $xq^u_K(x)$ **(small difference in** $x \in [0.45 - 0.55]$**)**

- ★ **Up-quark contribution support at small and intermediate x.**
  Peak of $xq^u_\pi(x)$ **and** $xq^u_K(x)$ **around** $x = 0.3$

- ★ **Strange-quark contribution support at intermediate and large x.**
  Peak of $xq^s_K(x)$ **around** $x = 0.36$

x-dependent PDFs from lattice QCD

★ **Alternative approaches proposed, e.g.:**

Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
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https://doi.org/10.1140/epja/s10050-021-00353-7
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Review

The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD

Martha Constantinou[a]

Temple University, Philadelphia, USA

x-dependent PDFs from lattice QCD

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THE EUROPEAN PHYSICAL JOURNAL A
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Review

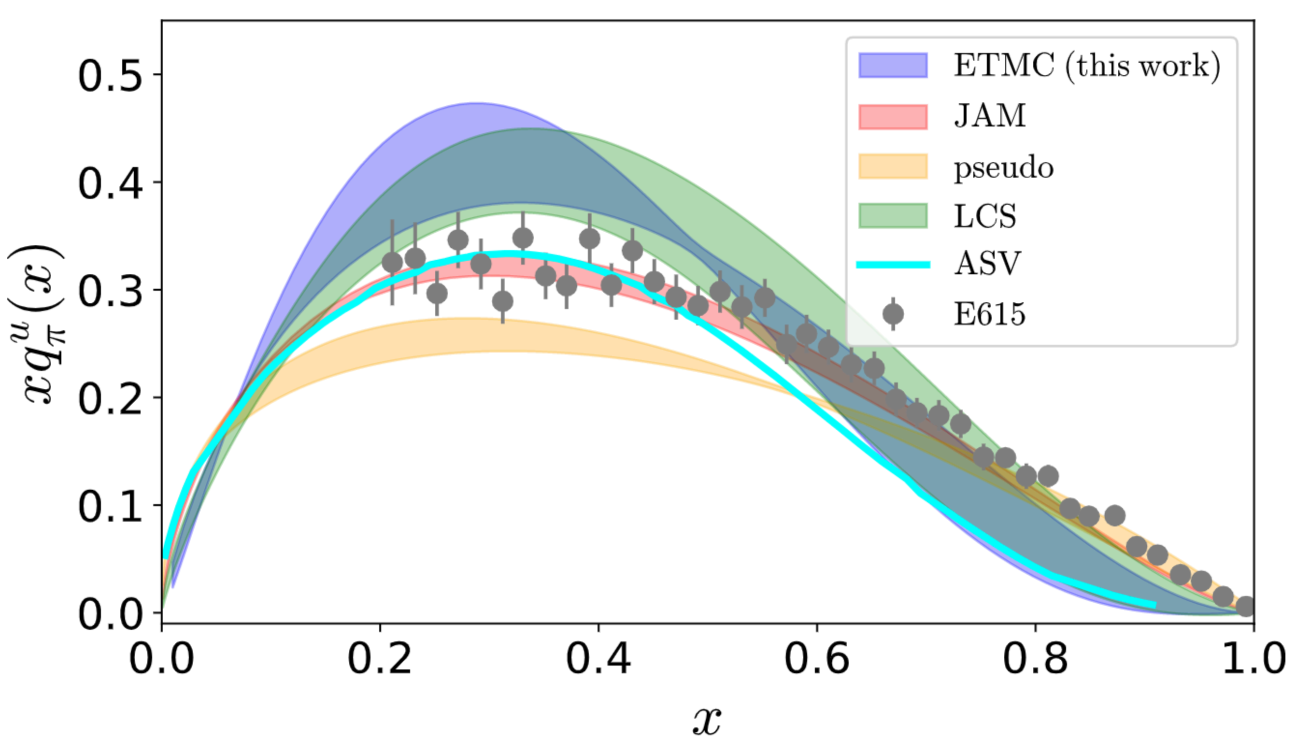
The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD

Martha Constantinou[a]
Temple University, Philadelphia, USA

Other Reviews:

[K. Cichy, M. Constantinou, Adv. in HEP, Volume 2019, 3036904, arXiv:1811.07248]
[X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543]

Pion: Comparison with other studies



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- ★ **Very good agreement with PDF from LCS**
- ★ **Tension with E615 data in region** $x \in [0.2 - 0.55]$
- ★ **Large-x behavior compatible with rescaled ASV**

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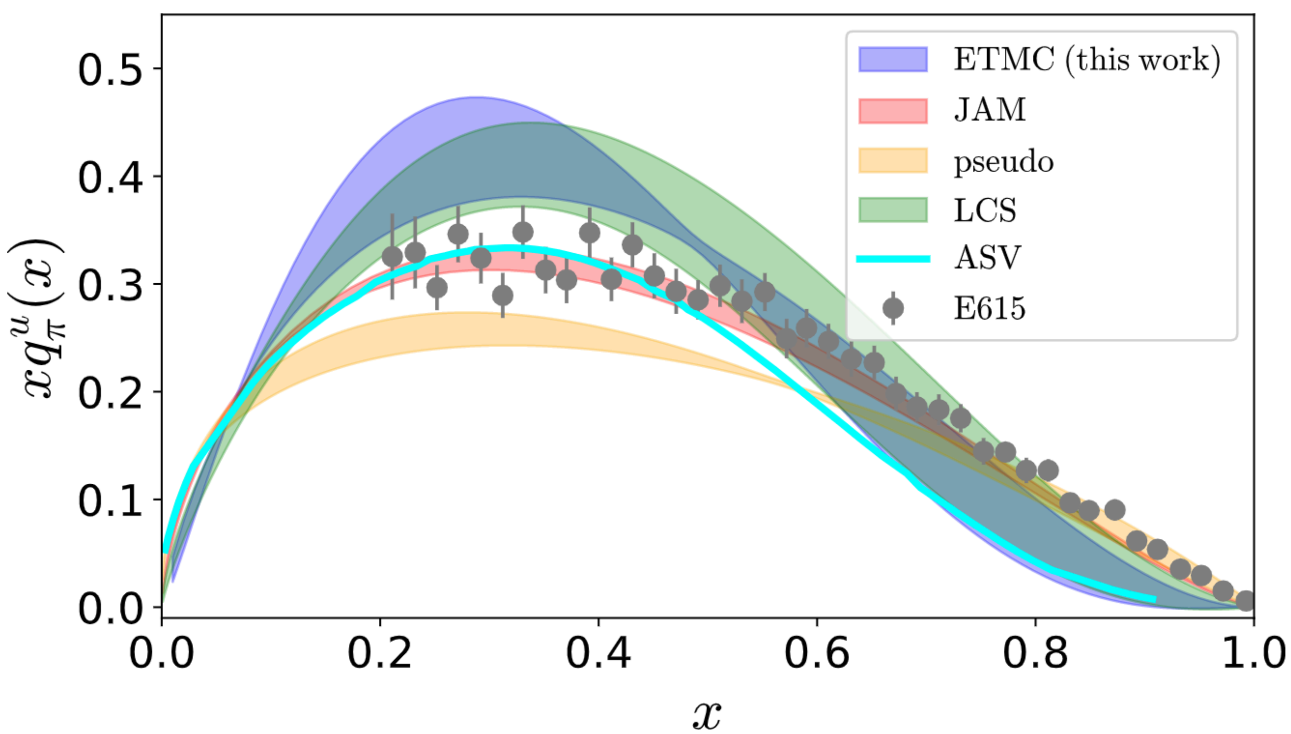

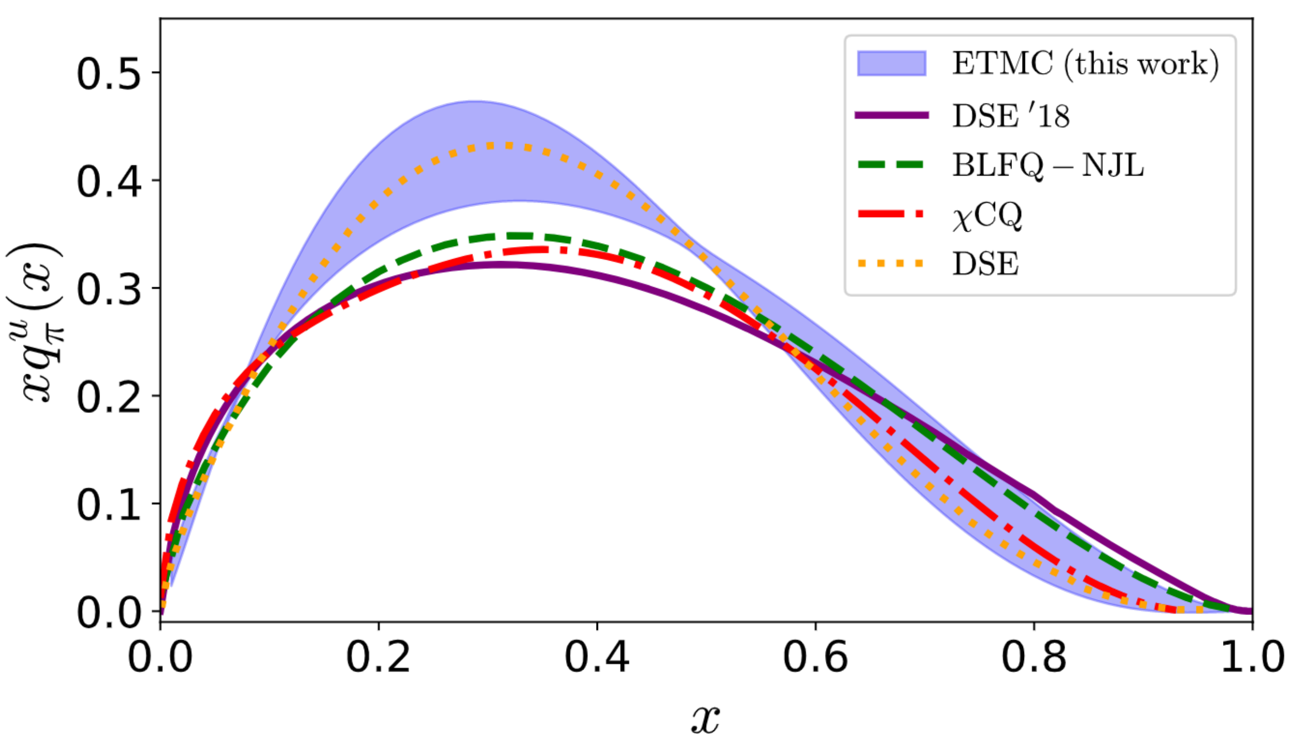

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- ★ **Small- and large-x regions compatible with models**

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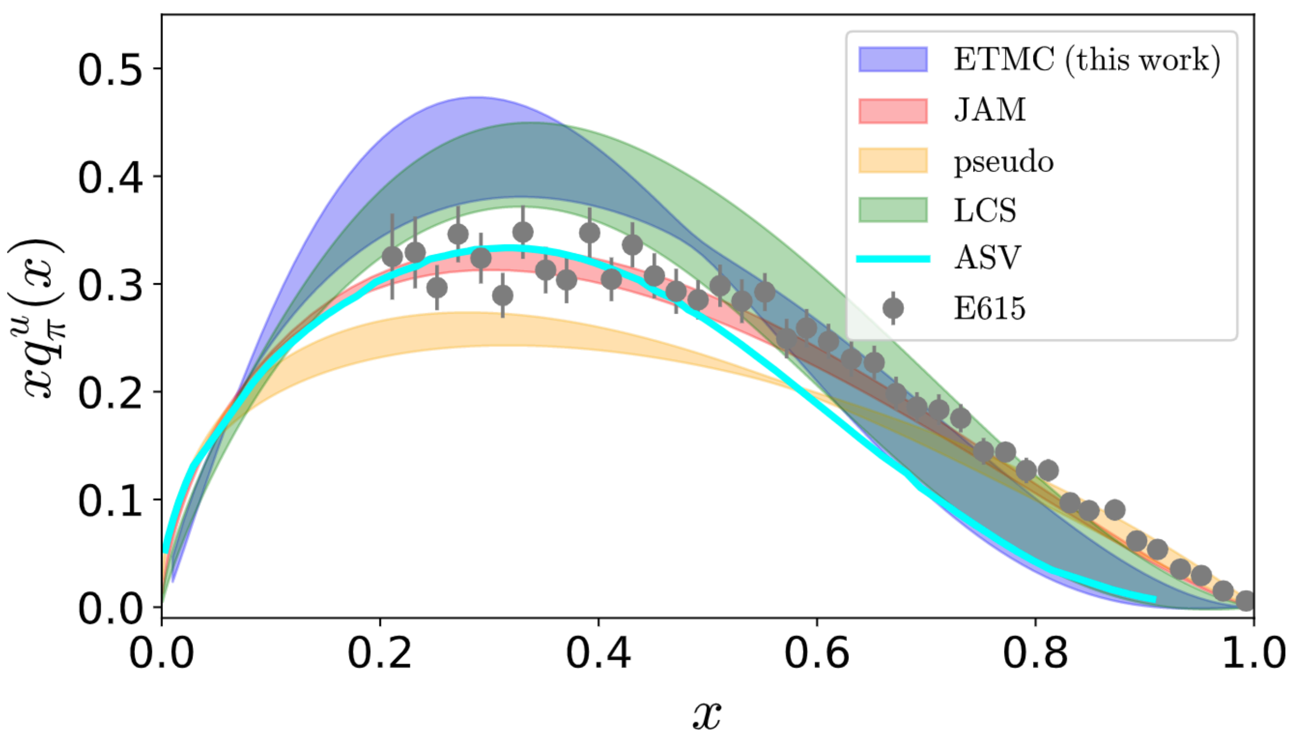

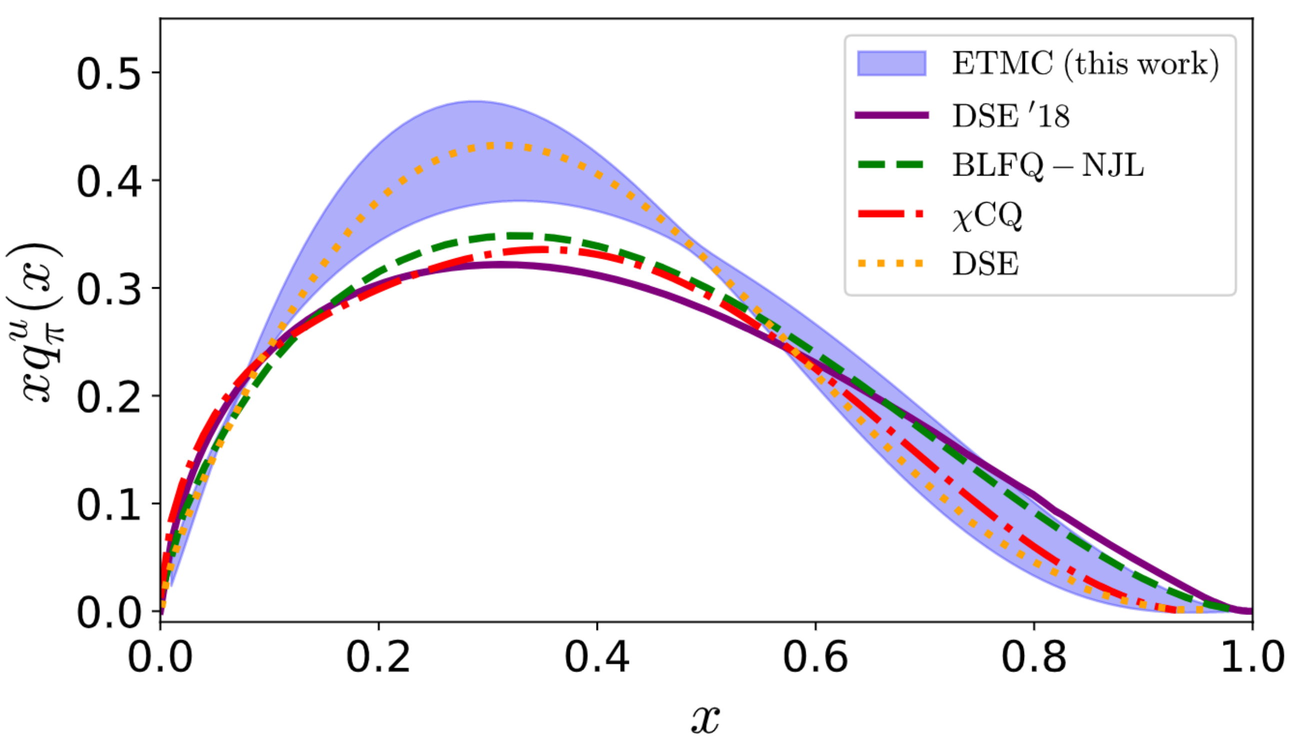

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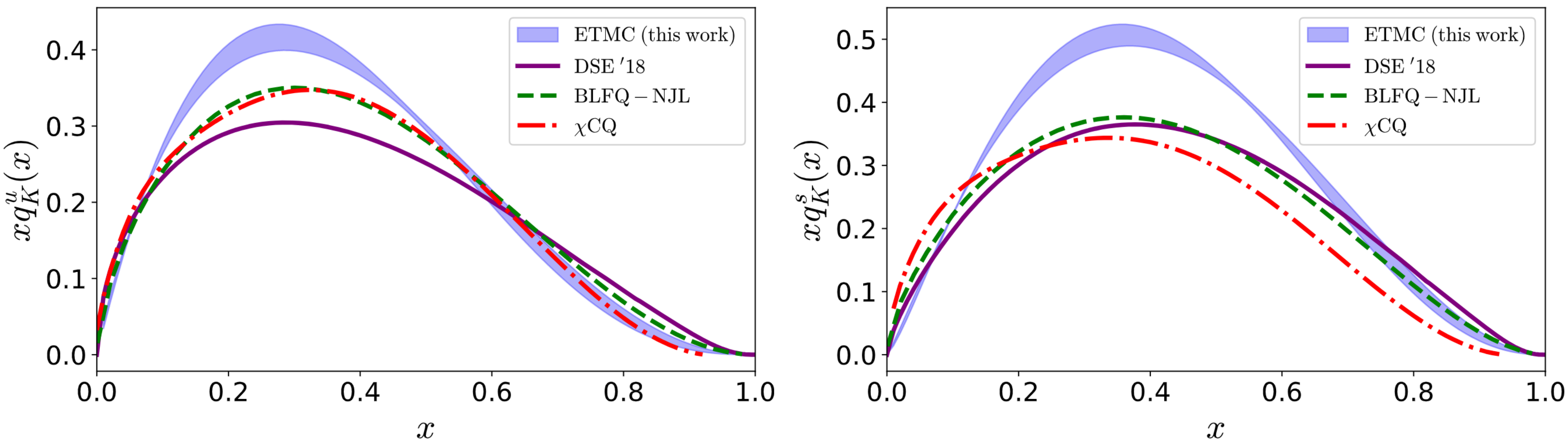
Comparison qualitative!

Kaon: Comparison with other studies



★ **Very limited studies**

★ **Peak of lattice data higher than models**

★ **Mellin moment $\langle x^4 \rangle_K^{u,s}$ compatible with lattice data**

Mellin moments from PDFs

$$\langle x^n \rangle = \int x^n f(x)\, dx$$

Mellin moments from PDFs

$$\langle x^n \rangle = \int x^n f(x)\, dx$$

q_M^f	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$	$\langle x^5 \rangle$	$\langle x^6 \rangle$
q_π^u	0.230(3)(7)	0.087(5)(8)	0.041(5)(9)	0.023(5)(6)	0.014(4)(5)	0.009(3)(3)
q_K^u	0.217(2)(5)	0.079(2)(1)	0.036(2)(2)	0.019(1)(2)	0.011(1)(2)	0.007(1)(1)
q_K^s	0.279(1)(5)	0.115(2)(6)	0.058(2)(2)	0.033(2)(2)	0.021(1)(2)	0.014(1)(2)

Mellin moments from PDFs

$$\langle x^n \rangle = \int x^n f(x)\, dx$$

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For comparison

JAM: $\langle x^4 \rangle_\pi^u = 0.027(2)$

BLFQ-NJL

$\langle x^4 \rangle_K^u = 0.021(3)$

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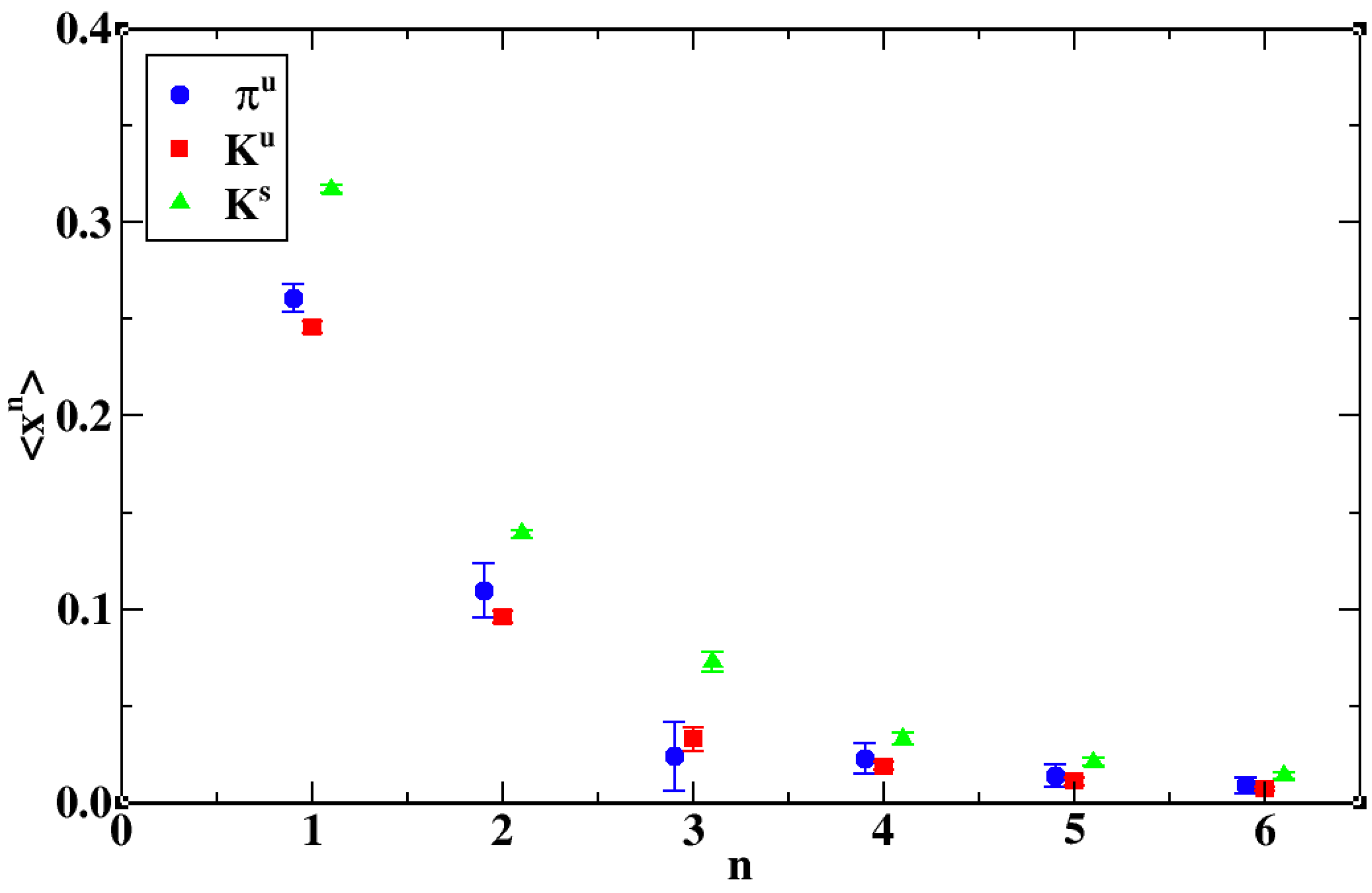

Scalar, Vector, Tensor

Form Factors

Pion and kaon form factors

$$\langle M(p')|\mathcal{O}^f_S|M(p)\rangle = \frac{1}{\sqrt{4E(p)E(p')}} A^{M^f}_{S10}\,,$$

$$\langle M(p')|\mathcal{O}^f_{V^\mu}|M(p)\rangle = -i\,\frac{2\,P^\mu}{\sqrt{4E(p)E(p')}}\,A^{M^f}_{10}\;,$$

$$\langle M(p')|\mathcal{O}^f_{T^{\mu\nu}}|M(p)\rangle = i\,\frac{(P^\mu q^\nu - P^\nu q^\mu)}{m_M\sqrt{4E(p)E(p')}}\,B^{M^f}_{T10}\,.$$

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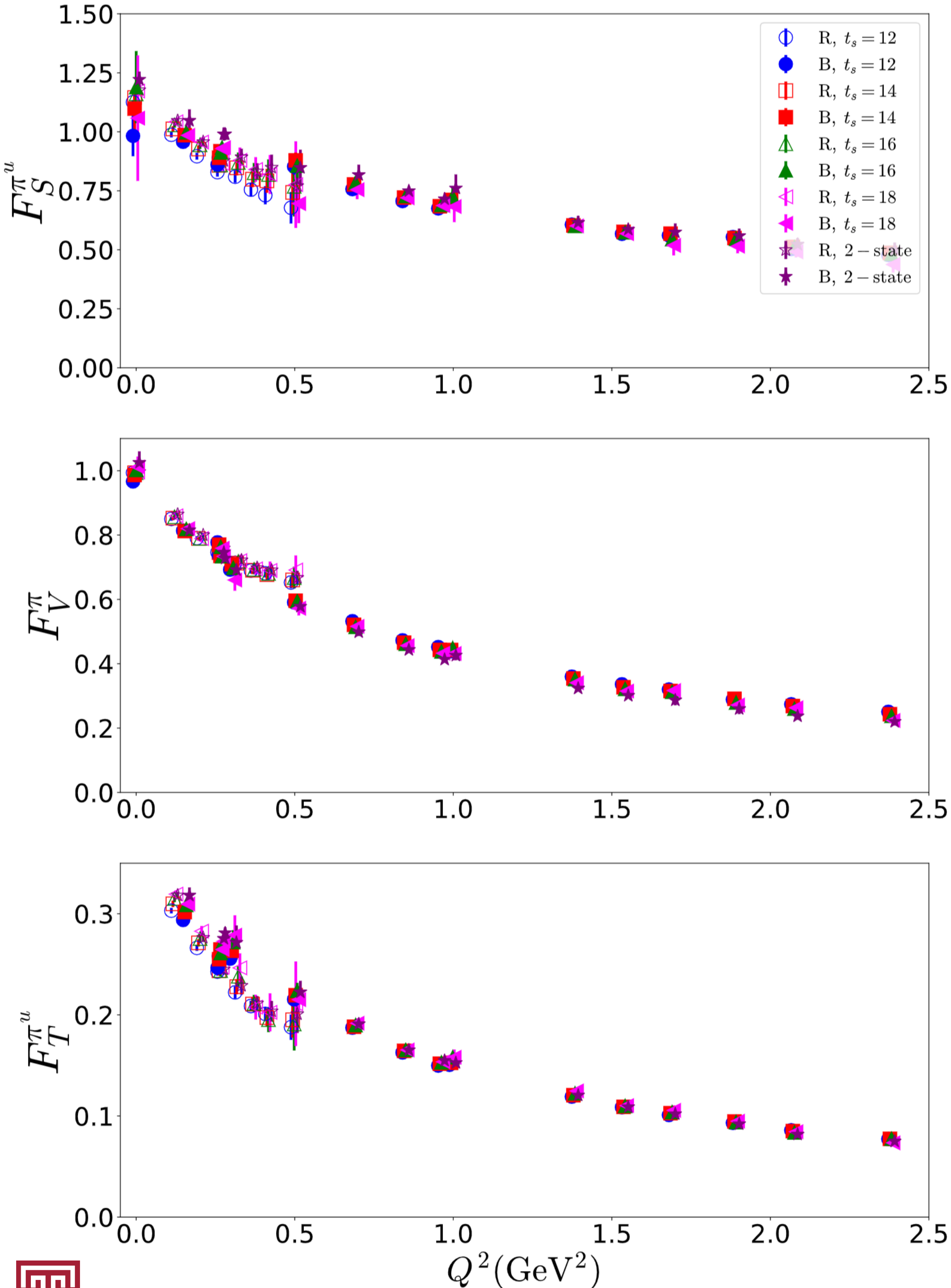

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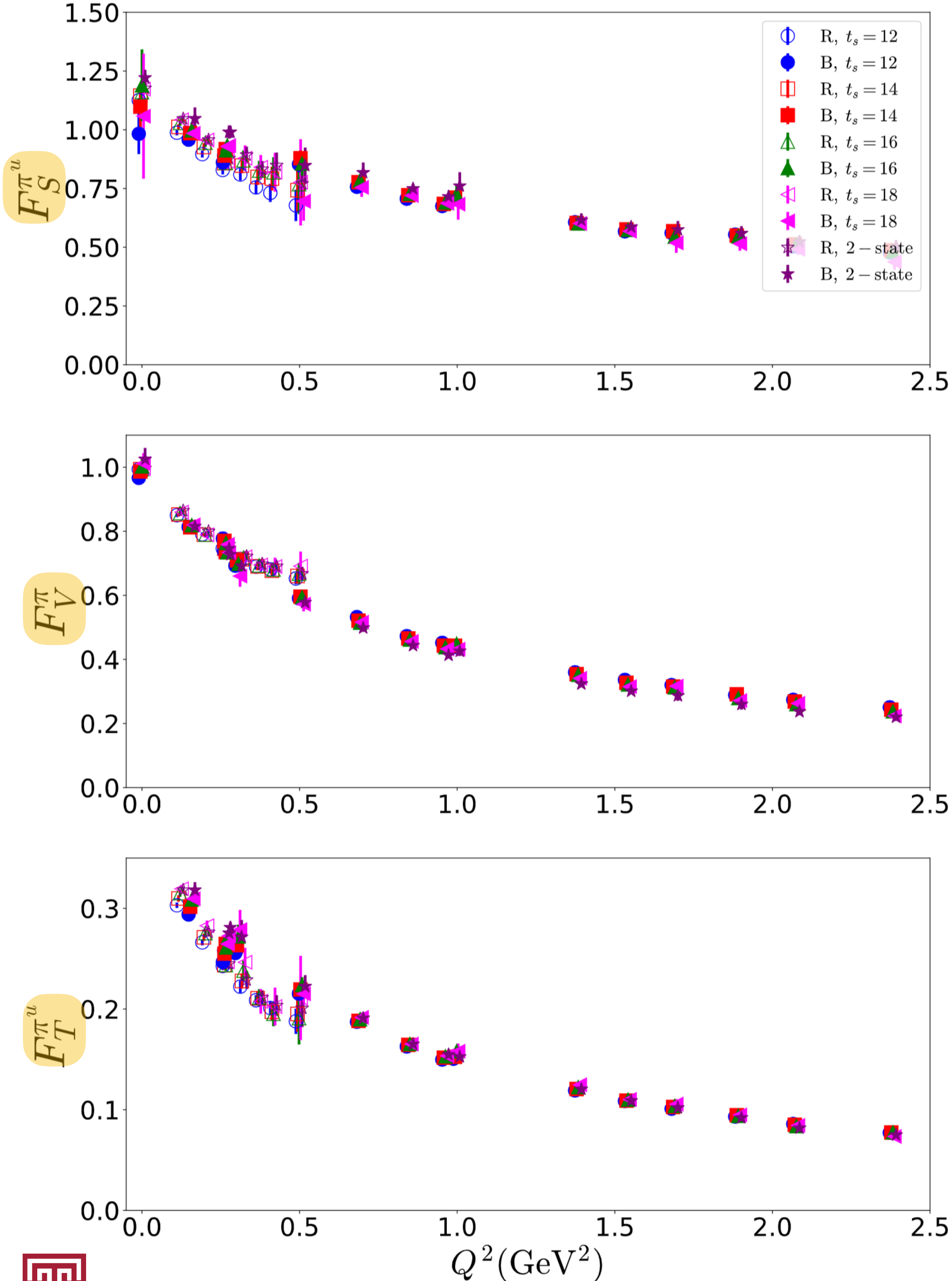

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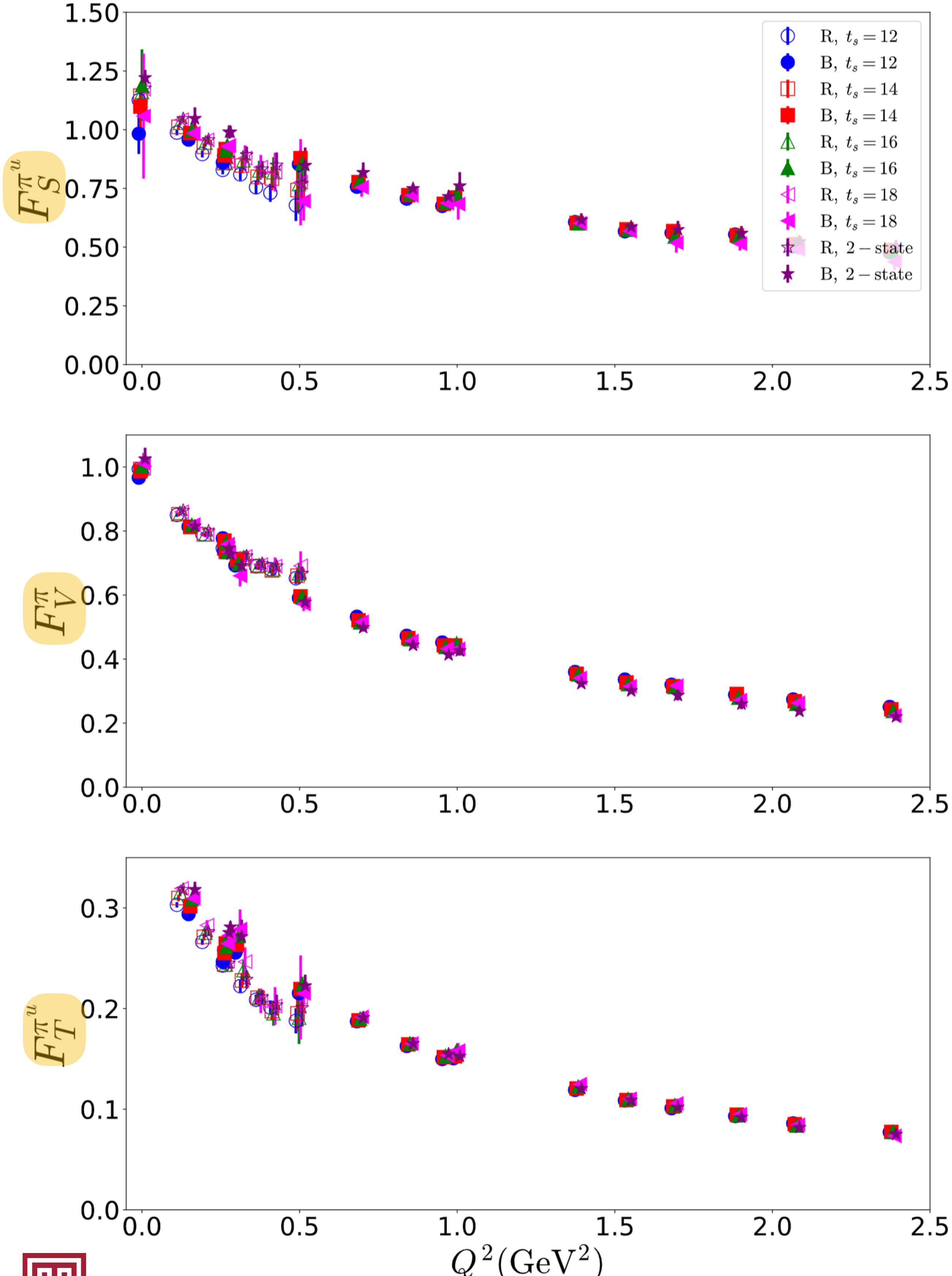

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★ **Rest and boosted frames give wide range of Q^2 values**

$$Q^2_{\rm rest} = 2m(E(q) - m)\,,$$

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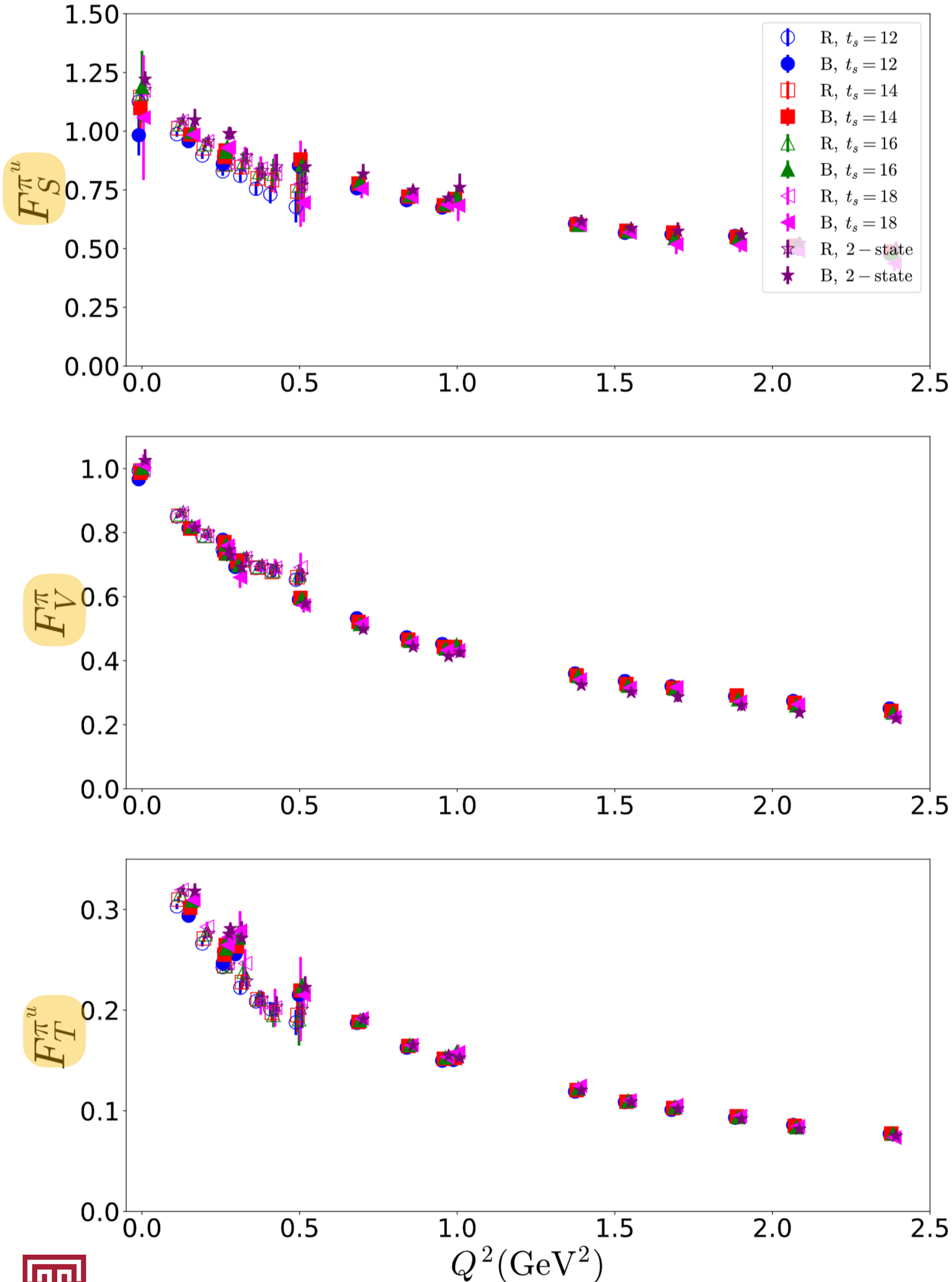
★ **For both frames and particles: $|Q^2_{max}|/E \sim 2 - 2.5$**

Pion and kaon form factors



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★ **Excited-states effects mostly in scalar and tensor FFs**

★ **R vs B frame: Evidence of cutoff effects**

Tensor anomalous magnetic moment κ_T

★ **$\kappa_T = F_T(0)$ extracted from parametrizations of lattice data**

$$F_\Gamma(Q^2) = \frac{F_\Gamma(0)}{1 + \frac{Q^2}{\mathcal{M}_\Gamma^2}}$$

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Alternatively

★ Q^2-dependence of F_V and F_T expected to be the same due to the elastic unitarity relation

[M. Hoferichter et al., PRL122 , 122001 (2019),
[Erratum: PRL 124, 199901 (2020), arXiv:1811.11181]

★ Argument holds for $Q^2 < 1$ GeV^2 (corrections needed beyond that due to inelastic states)

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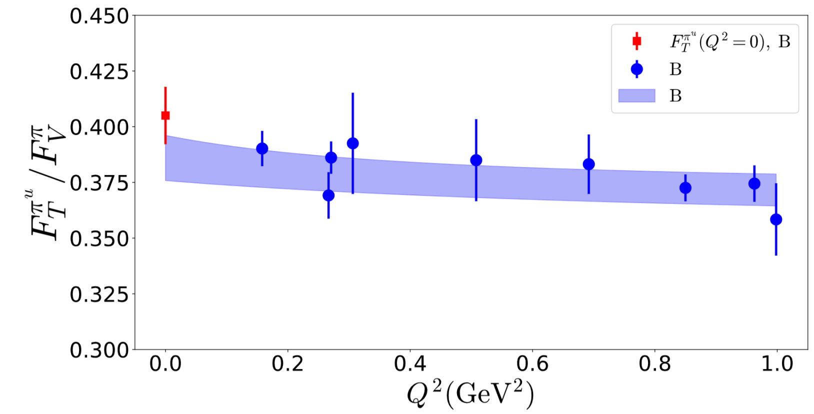

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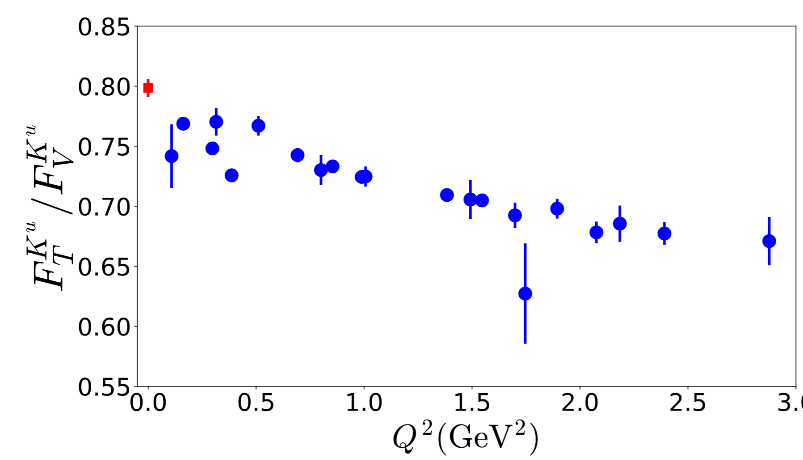

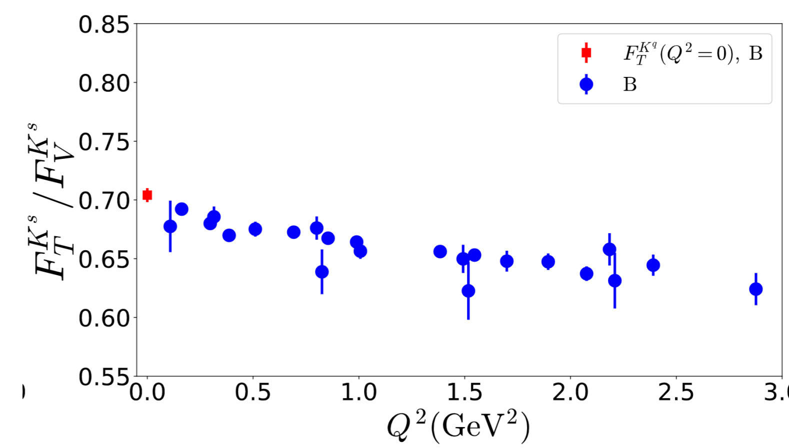

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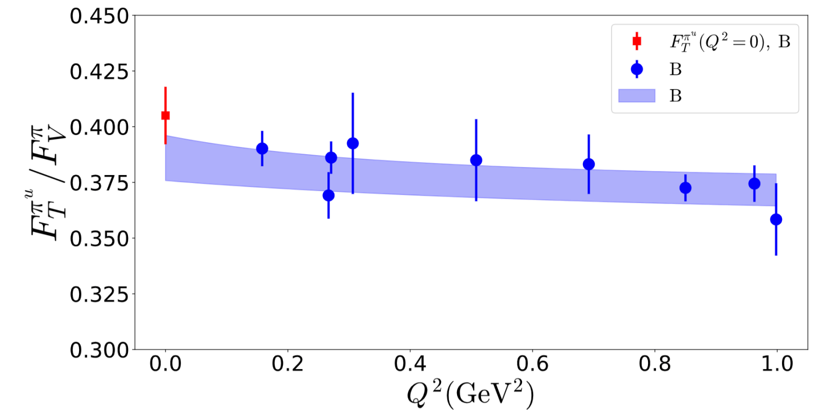

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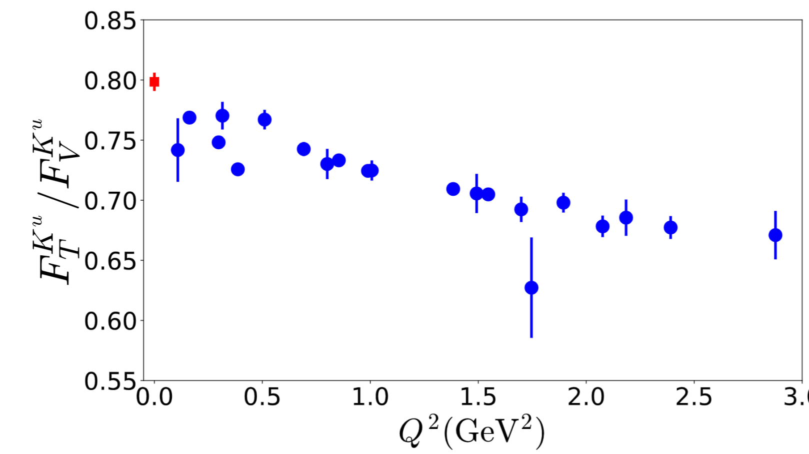
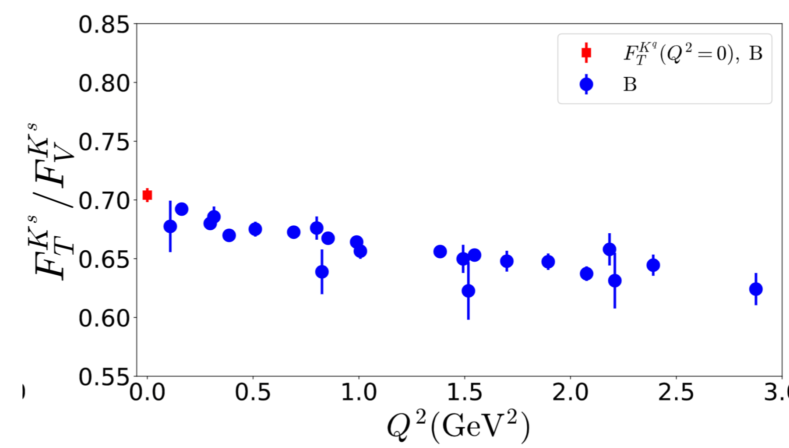
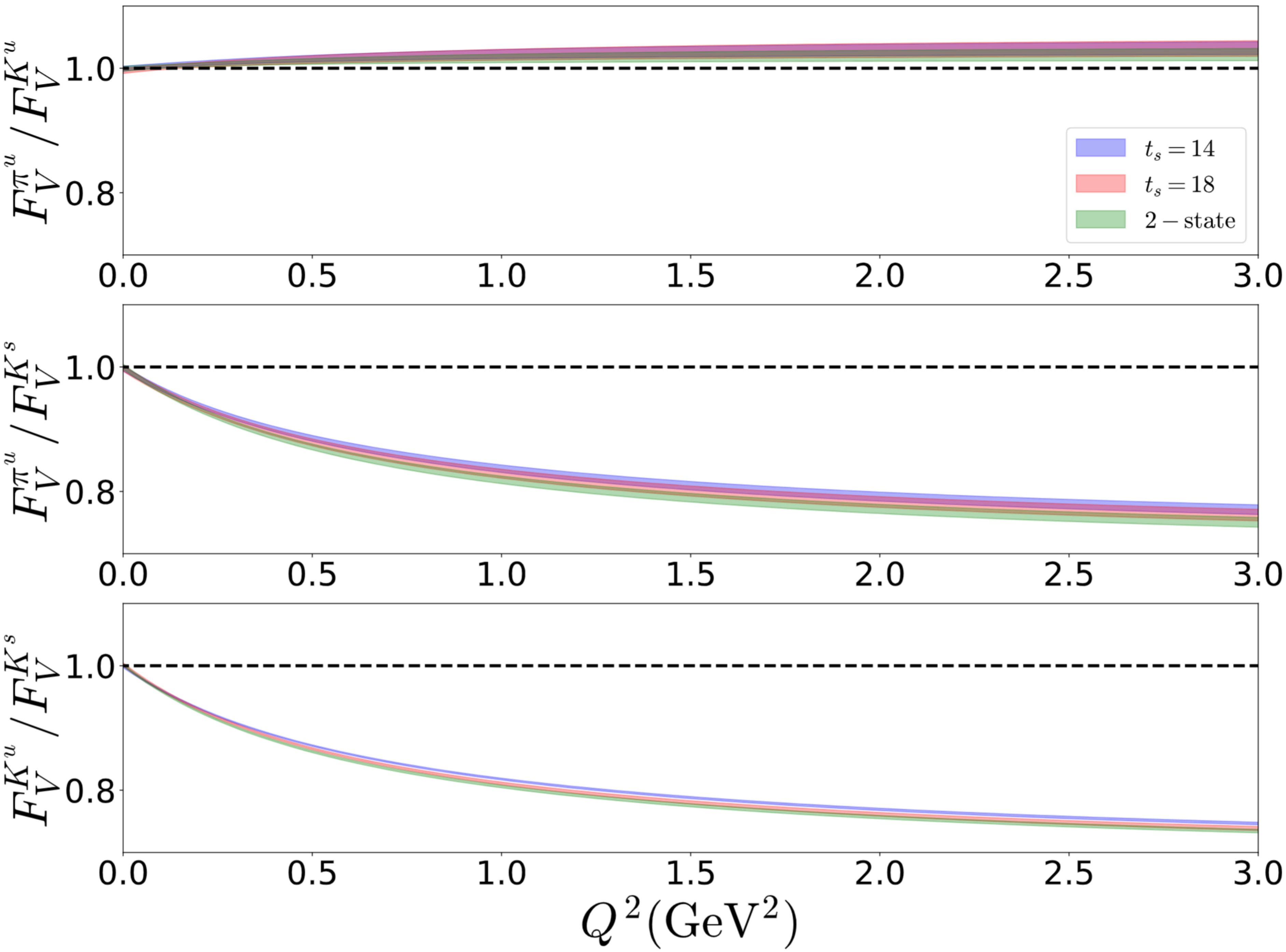
★ Q^2-behavior of ratio is mild compared to F_V and F_T (at Q^2 ~1 GeV^2: 5% vs 60-70%)





SU(3) flavor symmetry breaking



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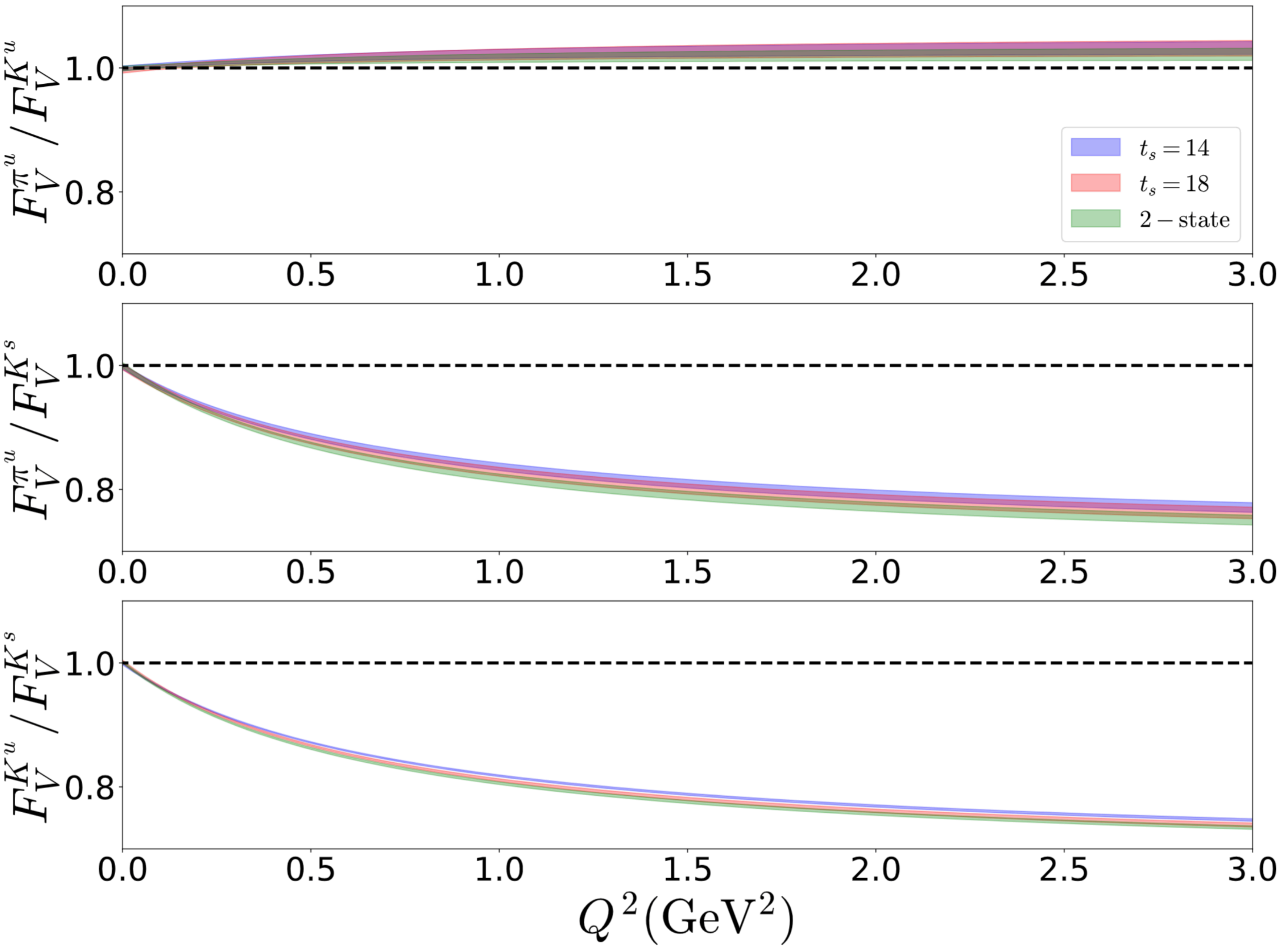
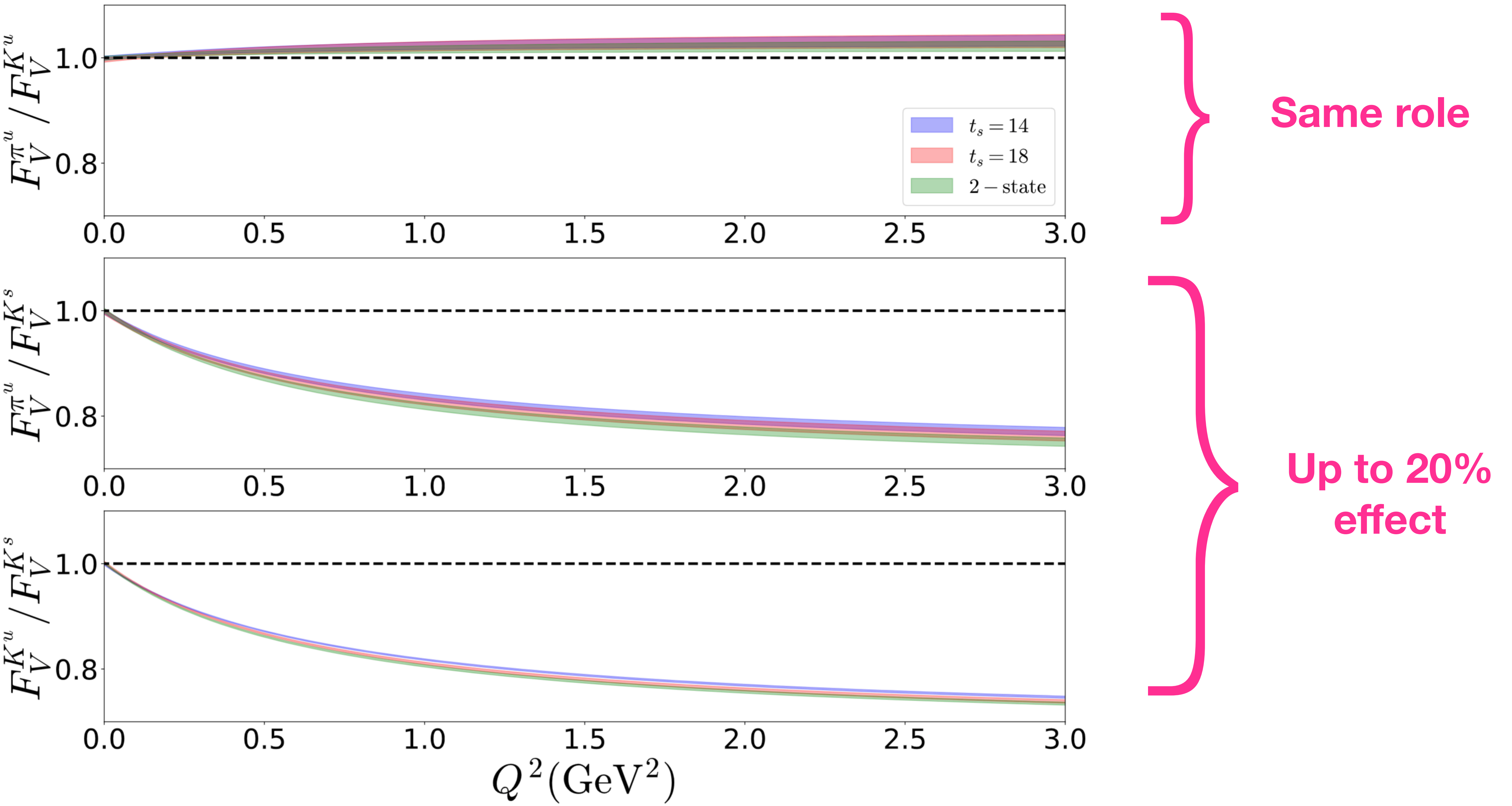


Same role

Up to 20% effect

SU(3) flavor symmetry breaking



★ **Suppressed excited-states effects compared to individual FFs**

★ **Similar picture for scalar and tensor FFs**

Transverse spin structure

Quark probability density in impact parameter space

$$\rho(b_\perp, s_\perp) = \frac{1}{2}\Big[F_V(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m}\,\frac{\partial F_T(b_\perp^2)}{\partial b_\perp^2}\Big]$$

$$F_\Gamma(Q^2) = \frac{F_\Gamma(0)}{1+\frac{Q^2}{\mathcal{M}_\Gamma^2}} \qquad F_{\mathcal{O}}(b_\perp^2) = \frac{1}{2\pi}\int d^2\Delta_\perp^2\, e^{-ib_\perp\cdot\Delta_\perp}\, F_{\mathcal{O}}(t=-\Delta_\perp^2)$$

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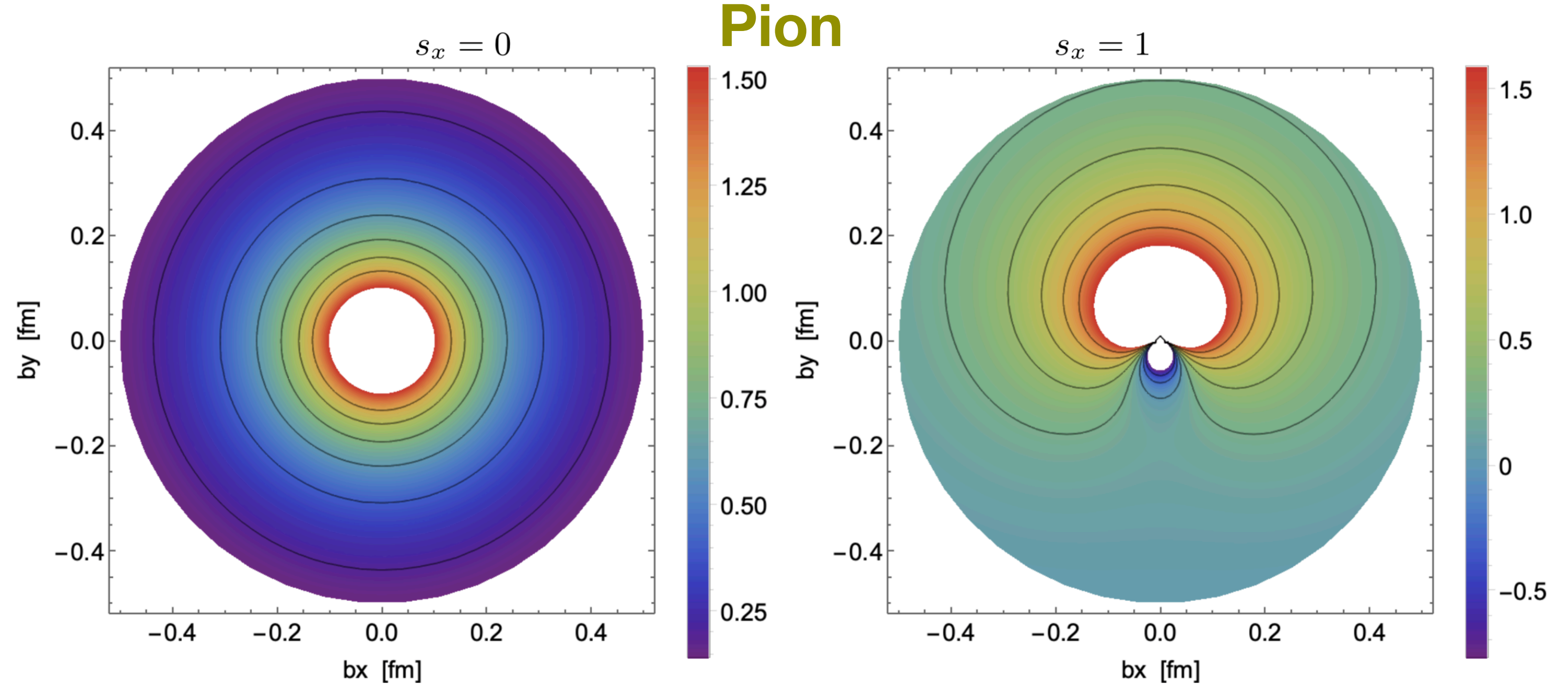
Pion



★ **Distortion for polarized quarks**

★ **Similar picture for kaon**

Concluding Remarks

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- ★ **Accessing Mellin moments up to $\langle x^3 \rangle$ without mixing is feasible**
- ★ **SU(3) flavor symmetry breaking non-negligible**
- ★ **Reconstruction of PDFs using up to $\langle x^3 \rangle$ is possible**
- ★ **Our lattice data propose a $(1-x)^2$ decay for both pion and kaon PDFs**

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★ Immediate next steps: generalized FFs and more ensembles

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Thank you

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