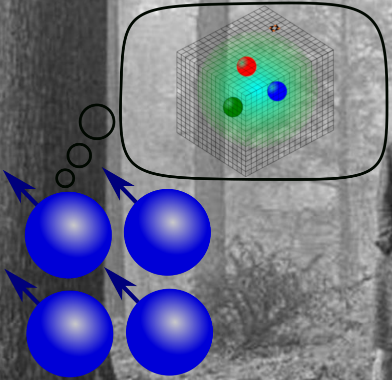
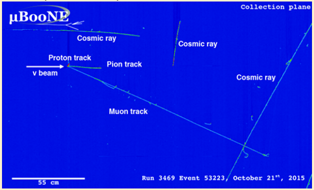
All Too Well:
Approximating SU(3) by only 1080 group elements

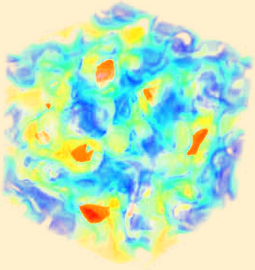
Hank Lamm



Formulating the problem of real-time dynamics

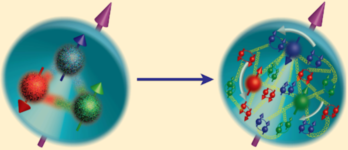
$\sigma(\nu N \to X) =$???





$\frac{\eta}{s}(T) =$???

$H^q(x, \xi, t) =$??? $E^q(x, \xi, t) =$???



Examples: $\nu - N$ scattering, QGP Transport, Hadron Tomography[1]

$$\langle \prod_i \mathcal{O}_i(t_i) \rangle = \int_{\psi(0)}^{\psi(T)} \mathcal{D}\psi \prod_i \mathcal{O}_i(t_i) e^{-iS} = \langle \psi(T) | \prod_i \mathcal{O}_i(t_i) | \psi(0) \rangle$$

We are concerned with **nonperturbative** results

[1] Carena, M. et al. In: *Snowmass 2021 LOI* TF10-077 (2020).

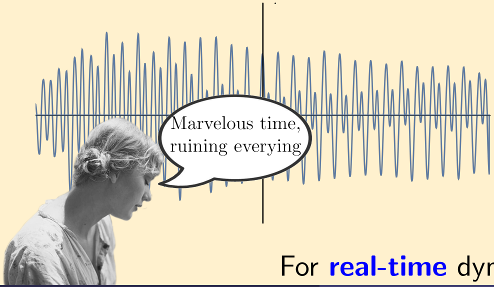
Monte Carlo methods struggle with sign problems

$S_I = 0$, $\langle \sigma \rangle_{S_R} = 1$

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\phi \, e^{-iS_I} \mathcal{O} e^{-S_R}}{\int \mathcal{D}\phi \, e^{-S_R}} \frac{\int \mathcal{D}\phi \, e^{-S_R}}{\int \mathcal{D}\phi \, e^{-S_R} e^{-iS_I}}$$

$$= \frac{\left\langle \mathcal{O} e^{-iS_I} \right\rangle_{S_R}}{\langle \sigma \rangle_{S_R}}$$

$S_I \neq 0$, $\langle \sigma \rangle_{S_R} \leq 1$



For **real-time** dynamics, $S_R = 0$!

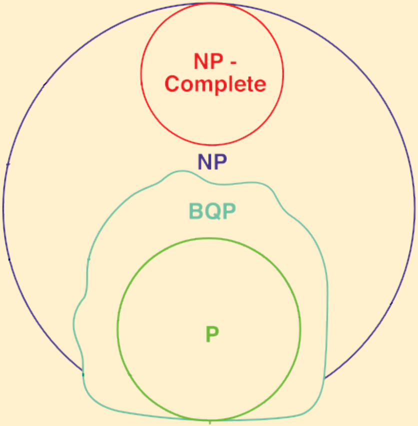
Stated succinctly...

$|\psi\rangle$ is a **complex-valued** probability amplitude

Fundamentally, physics needs quantum computers.

$$\langle\psi_f|U(t)|\psi_i\rangle = \langle\psi_f|e^{-iHt}|\psi_i\rangle = \int \mathcal{D}\phi e^{-S[\phi]}$$

QC can **efficently represent** superpositions and entanglement

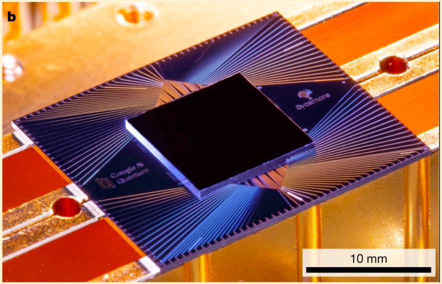


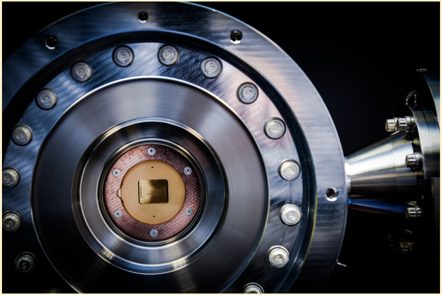
Credit: Scott Aaronson

What is the state of QC? Nasty, brutish and short

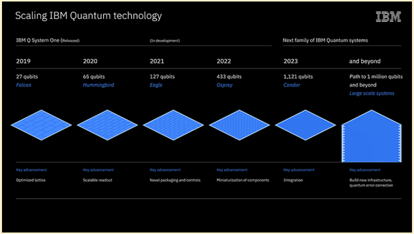
$\mathcal{O}(10^{1-2})$ qubits with entangling gate fidelities of $\sim 90 - 99\%$

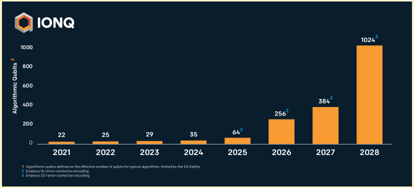
$\Longrightarrow \mathcal{O}(10^{1-2})$ clock cycles with $\mathcal{O}(10^3)$ CLOPs

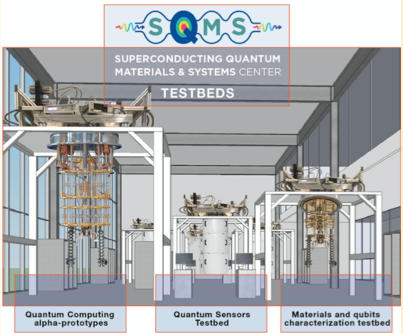




Where might we be in ten years?







Roadmaps: $\mathcal{O}(10^3)$ qubits in $\lesssim$**10 years**

Varying levels of QEC & circuit depth

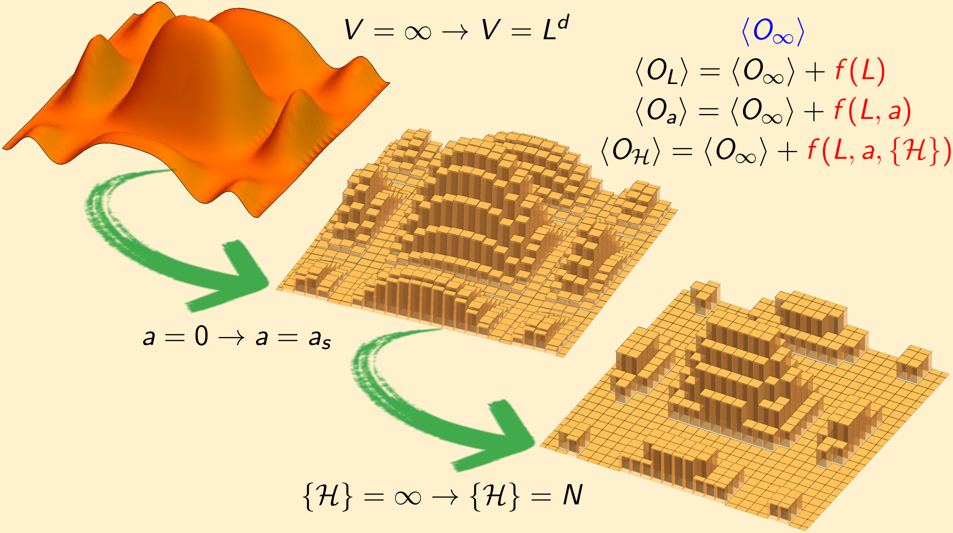
Similar to early LFT: $8^3 \times 20$ $\mathbb{Z}_2$[2]

Potential for small-scale **nonabelian** sims

[2] Creutz, M., L. Jacobs, and C. Rebbi. In: *Phys. Rev. D* 20 (1979). Ed. by Julve, J. and M. Ramón-Medrano.

How do we reckon with finite computers?

QFT is about infinities and how to regulate them



$$V = \infty \to V = L^d$$

$$\langle O_\infty \rangle$$

$$\langle O_L \rangle = \langle O_\infty \rangle + f(L)$$

$$\langle O_a \rangle = \langle O_\infty \rangle + f(L, a)$$

$$\langle O_{\mathcal{H}} \rangle = \langle O_\infty \rangle + f(L, a, \{\mathcal{H}\})$$

$$a = 0 \to a = a_s$$

$$\{\mathcal{H}\} = \infty \to \{\mathcal{H}\} = N$$

Example of digitization:

Start from **Kogut-Susskind** Hamiltonian (a **lattice-reg'd** version of H):

$$H_{KS} = \frac{c}{a_s}\left[\frac{g_H^2}{2}\sum_l E_l^2 + \frac{1}{g_H^2}\sum_p \text{Tr}\, U_p\right]$$

Notice there are two **natural** basis: E_l-basis & $U-$basis

Truncate the basis, e.g. $E_l \leq E_{\rm max}$ but now you aren't using H_{KS}

$$H_{\rm trunc} = \frac{c}{a_s}\left[\frac{g_H^2}{2}\sum_l E_l^2 + \frac{1}{g_H^2}\sum_p \text{Tr}\, U_p\right] + \mathcal{O}_{\rm trunc}$$

$\mathcal{O}_{\rm trunc}$ may break symmetries, unitarity – and could be **relevant** operator – and will be affected by **noise**

I'm going to talk about lattice actions

$$\langle x|e^{-iHt}|y\rangle = \int \mathcal{D}\phi e^{iS}$$

The **anisotropic Wilson** action is

$$S_{\mathrm{W}} = \frac{1}{g_t^2}\xi \sum_t \mathrm{Tr}\, U_t + \frac{1}{g_s^2}\frac{1}{\xi}\sum_s \mathrm{Tr}\, U_s$$

thru **transfer matrix,** $\langle i|e^{-a_0 H}|j\rangle$ derives the H_{KS}

$$H_{KS} = \frac{c}{a_s}\left[\frac{g_H^2}{2}\sum_l E_l^2 + \frac{1}{g_H^2}\sum_p \mathrm{Tr}\, U_p\right]$$

- H_{KS} **isn't** the Hamiltonian, but a choice with $O(a_s^2)$ errors[3]
- Osterwalder-Schrader reflection positivity allows relations via A.C.[4]

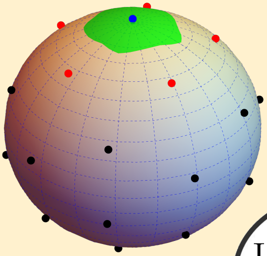
[3] Carena, M., **H. Lamm**, Y.-Y. Li, and W. Liu. In: (Mar. 2022). arXiv: 2203.02823 [hep-lat].

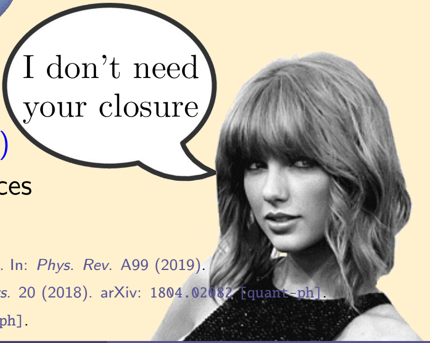
[4] Luscher, M. In: *Commun. Math. Phys.* 54 (1977).

So what is a good digitization scheme?

Discrete subgroups allow plug-and-play[5][6][7]

Replace $G \to H$ in $e^{-S}, e^{-i\mathcal{H}}$





- $SU(3) \to \mathbb{V}$ reduces qubits by $O(10^2)$
- I **believe** endgame will be **3x3** matrices

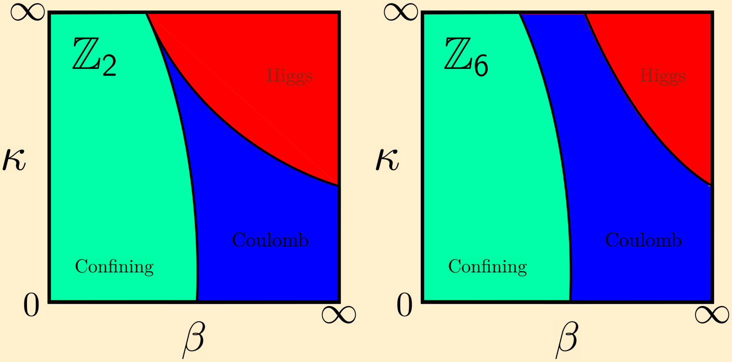
[5] Bhanot, G. In: *Phys. Lett.* 108B (1982), Hackett, D. C. et al. In: *Phys. Rev.* A99 (2019).

[6] Bender, J., E. Zohar, A. Farace, and J. I. Cirac. In: *New J. Phys.* 20 (2018). arXiv: 1804.02082 [quant-ph].

[7] Haase, J. F. et al. In: (June 2020). arXiv: 2006.14160 [quant-ph].

Discrete groups can't reach continuum[8][9][10]

$$S = \beta \sum \mathrm{Re}\,\mathrm{Tr}\, U_P + \kappa \sum \phi \cdot D\{U_P\} \cdot \phi^\dagger + c.c.$$



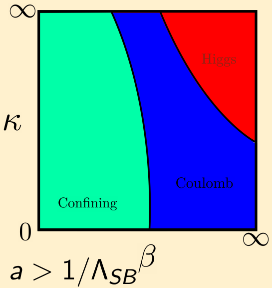
Integrating over ϕ leads to S_{eff} with new irreps of G

[8] Fradkin, E. H. and S. H. Shenker. In: *Phys. Rev. D* 19 (1979).

[9] Horn, D., M. Weinstein, and S. Yankielowicz. In: *Phys. Rev. D* 19 (1979).

[10] Labastida, J. M. F., E. Sanchez-Velasco, R. E. Shrock, and P. Wills. In: *Phys. Rev. D* 34 (1986).

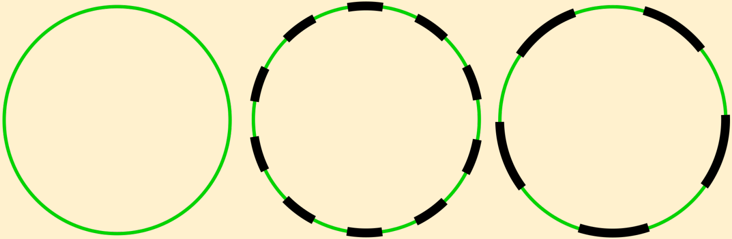
So, discrete groups are continuous groups+Higgs



- Starting from G **coupled to ϕ**
- The rep of ϕ **determines** the breaking $G \to H$
- **Higher** rep (larger H) $\to$ **larger** Λ_{SB} $\to$ **smaller** $a > 1/\Lambda_{SB}$
- **Dislike this?** note that $SO(4)$ is **never** recovered for $O(1/a)$ states
- On-going work to understand how Higgs couples to **Nonabelian** G[11]

[11] Das, S. and A. Hook. In: *JHEP* 10 (2020). arXiv: 2006.10767 [hep-ph].

So how can we predict a_f?[12]



$$\beta_{f,U(1)} = \frac{\log(1+\sqrt{2})}{1-\cos\left(\frac{2\pi}{N}\right)} \approx \kappa_2 N^2, \text{ which extends to } \beta_{f,SU(N)} \approx \kappa N^{\frac{N_c^2-1}{2}}$$

But whereas $\mathbb{Z}_N$ can be **taken to ∞**, **limited** number for $SU(N)$

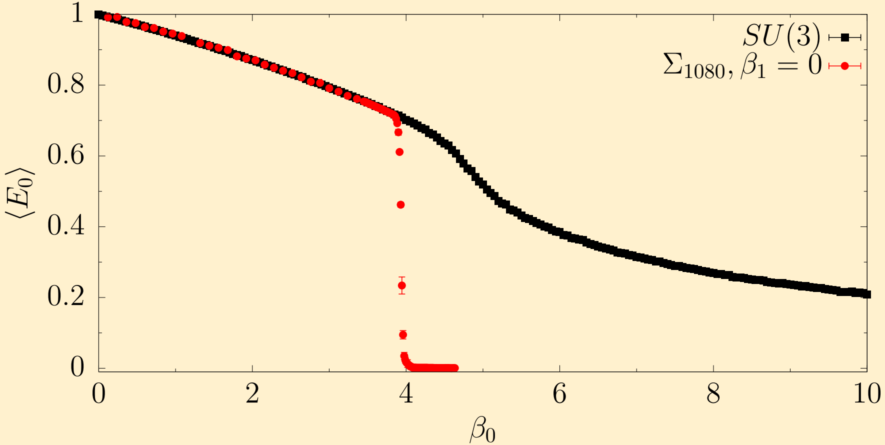
$$\beta \propto \frac{1}{\log(a)} \implies a_f \propto e^{-\beta_f}$$

So the important question is $a_s > a_f$?

[12] Petcher, D. and D. H. Weingarten. In: *Phys. Rev.* D22 (1980), Hartung, T., T. Jakobs, K. Jansen, J. Ostmeyer, and C. Urbach. In: (Jan. 2022). arXiv: 2201.09625 [hep-lat].

What do we know from Wilson Action?

- $U(1) \to \mathbb{Z}_N$, $N > 4$
- $SU(2) \to \mathbb{BO}, \mathbb{BI}$
- $SU(3) \to \mathbb{V}$ has $\beta_f = 3.935(5) < \beta_s \approx 6$
- One **1152** qubit $SU(3)$ link vs $\sim 4^3$ lattice of **11** qubits for $\mathbb{V}$ link



But why use the Wilson action?

The Wilson action is inadequate for many issues

$$S_W = \beta\,\mathrm{Re}\,\mathrm{Tr}[1 - U_p] \approx -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{12}a^2 D_\mu F_{\mu\nu} D_\mu F_{\mu\nu}$$

...which can be treated with **Symanzik improvement**[13]

$$\begin{aligned} S_{LW} =& \beta\,\mathrm{Re}\,\mathrm{Tr}[1 - U_p] + \beta_2\,\mathrm{Re}\,\mathrm{Tr}[1 - U_{rt}] + \beta_3\,\mathrm{Re}\,\mathrm{Tr}[1 - U_{par}] \\ &\approx -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + O(a^4) \end{aligned}$$

but you could also local terms proportional to other irreps...e.g.[14]

$$S_M = \beta\,\mathrm{Re}\,\mathrm{Tr}[1 - U_p] + \beta_a\,\mathrm{Re}\,\mathrm{Tr}[U_p]\,\mathrm{Tr}[U_p^\dagger] \tag{1}$$

[13] Symanzik, K. In: *Communications in Mathematical Physics* 18 (1970).

[14] Bhanot, G. In: *Phys. Lett.* 108B (1982), Fukugita, M., T. Kaneko, and M. Kobayashi. In: *Nucl. Phys. B* 215 (1983), Hasenbusch, M. and S. Necco. In: *JHEP* 08 (2004). arXiv: hep-lat/0405012 [hep-lat].

'Same' physics at $\beta_W \equiv f(\beta_f, \beta_a)$ have diff. errors[15]

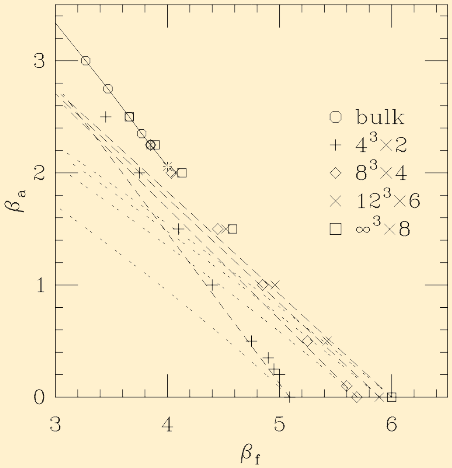
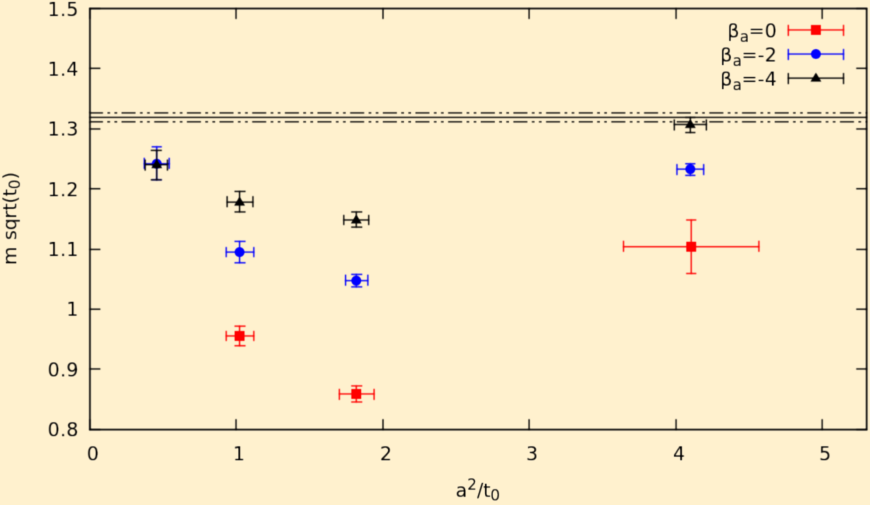


Figure 6: Lines of constant physics as predicted by perturbation theory (dotted lines) and tadpole improved perturbation theory (dashed lines) together with the deconfinement transitions for $N_t = 2$, 4, 6, and 8.

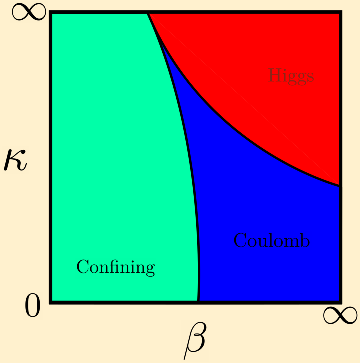
[15] Blum, T. et al. In: *Nucl. Phys.* B442 (1995). arXiv: hep-lat/9412038 [hep-lat].

S_M reduces lattice errors by avoiding FOPT[16]

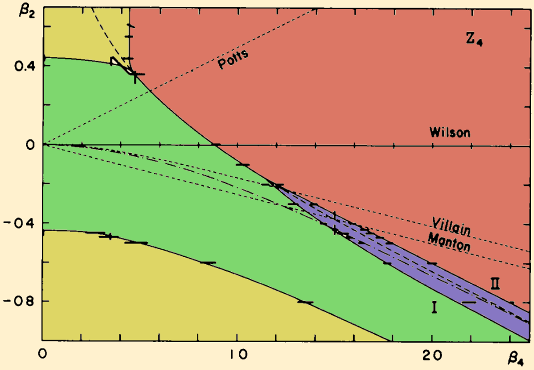


[16] Hasenbusch, M. and S. Necco. In: *JHEP* 08 (2004). arXiv: hep-lat/0405012 [hep-lat].

Modifed actions can lower truncation needed[17]



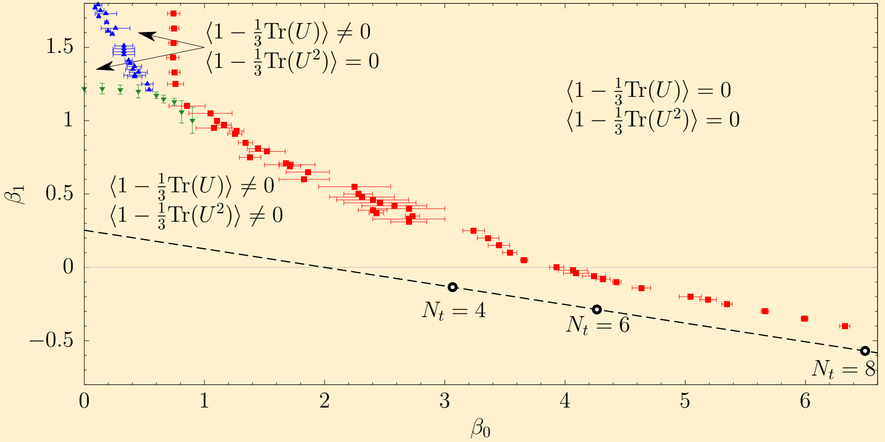
$$f(z) = \beta_0 + \tfrac{1}{2}\beta_4(z + z^{-1}) + \beta_2 z^2 .$$



[17] Fukugita, M., T. Kaneko, and M. Kobayashi. In: *Nucl. Phys. B* 215 (1983).

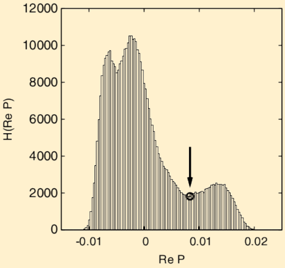
Can modified actions help $S(1080)$?

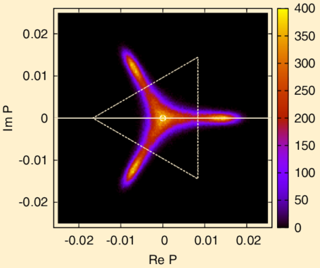
Define a trajectory to study continuum limit

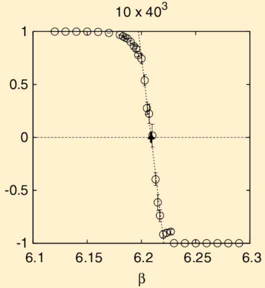


Find β_c for $N_t = 4, 6, 8$ and $N_s = 3N_t$ via separatix[18]

$T_c = \frac{1}{N_t a(\beta_c)}$ defines the transition from $\langle P \rangle \approx 0$ and $\langle P \rangle \approx 1$





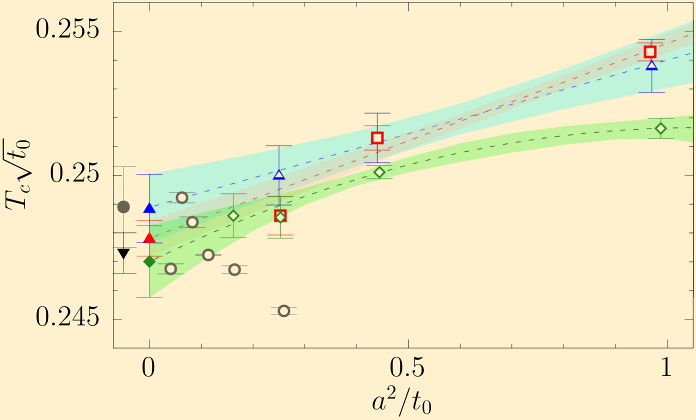


[18] Francis, A., O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno. In: *Phys. Rev.* D91 (2015).

$T_c\sqrt{t_0}$ suggests $a \approx 0.06$ fm ≈ 2 GeV^{-1} possible[20]

$$S = \sum \frac{\beta_0}{3} \,\mathrm{Re\,Tr}\, U + \beta_1 f(U) \text{ with } f(U) = \{\mathrm{Tr}^2\, U + \mathrm{Tr}\, U^2, |TrU|^2\}$$

Agrees **below 1%** with SU(3)[19]

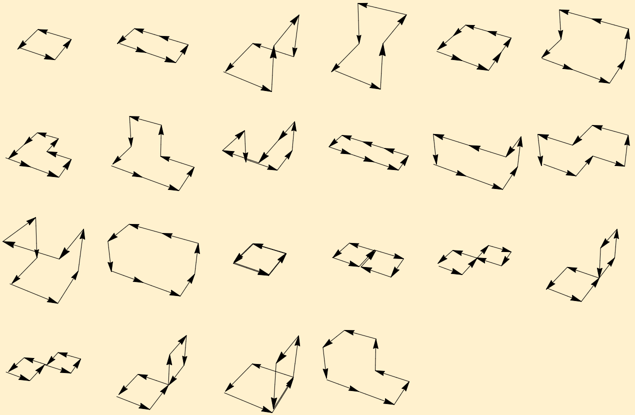


[19] Kitazawa, M., T. Iritani, M. Asakawa, T. Hatsuda, and H. Suzuki. In: *Phys. Rev.* D94 (2016).

[20] Alexandru, A. et al. In: *Phys.Rev.D* 100 (2019). arXiv: 1906.11213 [hep-lat].

But what about spectroscopy and higher energies?

Operator basis for glueballs



10,016 independent operators from $p = 0$ operators across 20 symmetry sectors with $n_{\text{smear}} = 2, 4, 6, 8$ levels of *stout-smearing*[21].

[21] Morningstar, C. and M. J. Peardon. In: *Phys. Rev.* D69 (2004). arXiv: hep-lat/0311018 [hep-lat].

Extracting glueball masses from correlators

Finite-volume m_g are best extracted from **matrices** of temporal correlators,

$$C_{ij}(\tau) = \sum_{\tau_0} \langle 0|\mathcal{O}_i(\tau+\tau_0)\mathcal{O}_j(\tau_0)^\dagger|0\rangle,$$

for $\mathcal{O}(\tau) = O(\tau) - \langle 0|O(\tau)|0\rangle$. We construct the matrix

$$\widetilde{C}(\tau) = U^\dagger C(\tau_0)^{-1/2} C(\tau) C(\tau_0)^{-1/2} U,$$

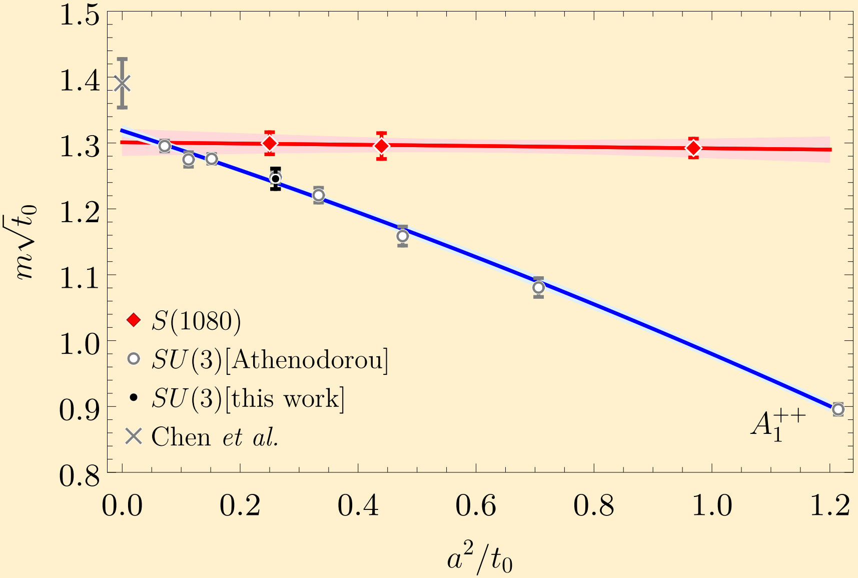
where U is built of eigenvectors of $G(\tau_d) = C(\tau_0)^{-1/2} C(\tau_d) C(\tau_0)^{-1/2}$.

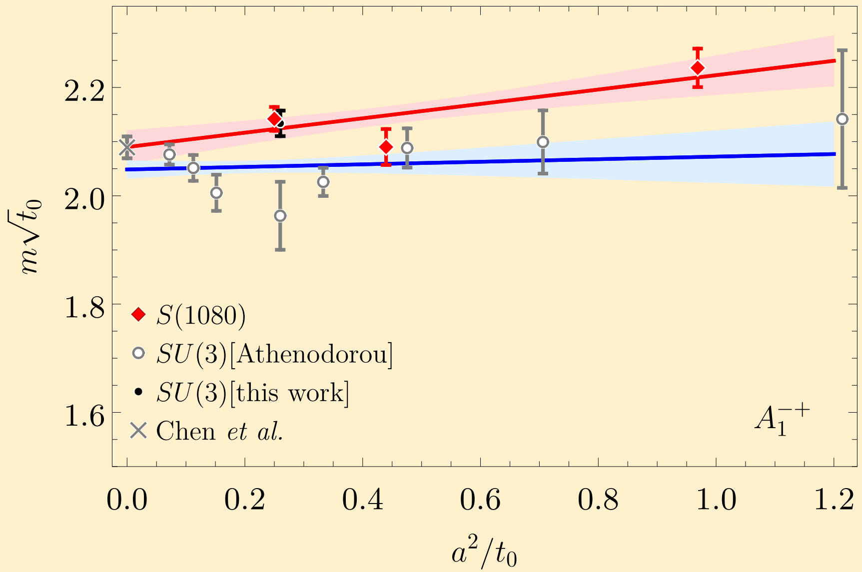
Input parameters

The top three lines are for $S(1080)$ and the forth is the $SU(3)$ calibration run. The parameters are: $n_{\rm therm} = 200$, $n_{\rm decorr}$ the number of updates between measurements, n_ρ and n_b the number of smearing and blocking levels respectively. For $SU(3)$ the value of $\sqrt{t_0}/a$ is from[22].

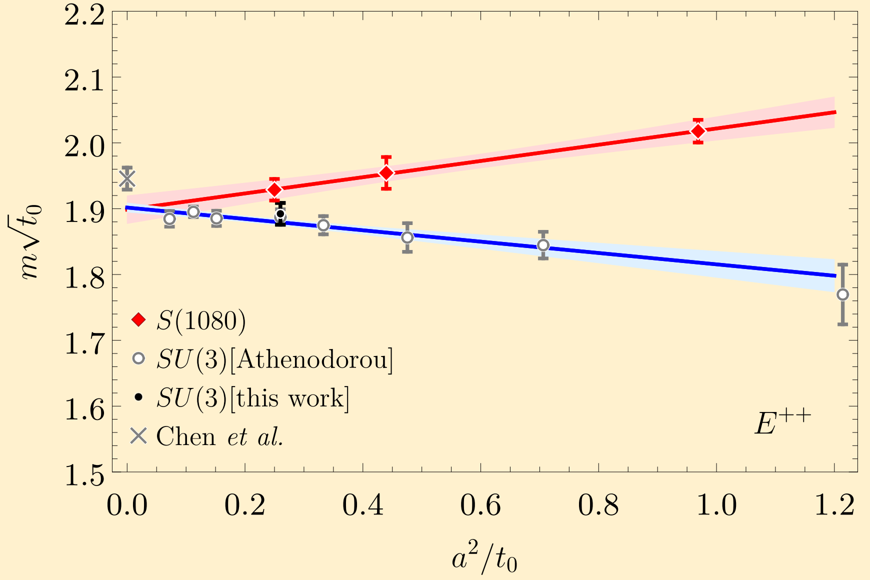
β_0	β_1	n^4	$n_{\rm decorr}$	$n_{\rm meas}$	$n_{\rm bins}$	$\sqrt{t_0}/a$
9.154	-0.9065	16^4	40	652500	1305	1.016(3)
12.795	-1.3677	16^4	40	650000	1300	1.508(3)
19.61	-2.2309	16^4	40	647500	1295	2.000(4)
6.0625	—	16^4	5	567500	1135	1.962(1)

[22] Francis, A., O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno. In: *Phys. Rev.* D91 (2015).


1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
$m\sqrt{t_0}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
a^2/t_0
$S(1080)$
$SU(3)$[Athenodorou]
$SU(3)$[this work]
Chen *et al.*
A_1^{++}



$m\sqrt{t_0}$
2.2
2.0
1.8
1.6
0.0
0.2
0.4
0.6
0.8
1.0
1.2
a^2/t_0
$S(1080)$
$SU(3)$[Athenodorou]
$SU(3)$[this work]
Chen *et al.*
A_1^{-+}


2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
$m\sqrt{t_0}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
a^2/t_0
$S(1080)$
$SU(3)$[Athenodorou]
$SU(3)$[this work]
Chen *et al.*
E^{++}

Low-lying glueball masses are consistent with $SU(3)$

irrep	$S(1080)$	$SU(3)$[23]	$SU(3)$[24]
A_1^{++}	1.301(20)	1.319(8)	1.391(37)
A_1^{-+}	2.090(31)	2.049(17)	2.089(20)
E^{++}	1.899(21)	1.902(7)	1.946(17)

$S(1080)$ reproduces $SU(3)$ at **10**$\times$ higher energy than $T_c\sqrt{t_0} \approx 0.25$

$S(1080)$ good until **at least** $\mathcal{O}(10^5)$ qubit devices

[23] Athenodorou, A. and M. Teper. In: *JHEP* 11 (2020). arXiv: 2007.06422 [hep-lat].

[24] Chen, Y. et al. In: *Phys. Rev.* D73 (2006). arXiv: hep-lat/0510074 [hep-lat].

It's time to go

So many things to do!...and lots can be done before the machine exists

Strong confidence that $S(1080)$ approximates $SU(3)$ for $a \gtrsim 0.07$ fm

- Digitizing SU(3)
  - **Spectroscopy** for $\mathbb{V}$ with dynamical fermions
  - $\mathbb{V}$ **circuits**
- Reducing the errors
  - e.g. Finite volume, finite a, a_t, decimation errors to make **realisitic** resource estimates
- Algorithms for **state prep, smearing**
- Investigate desirable properties
  - **PDF?, Viscosity?, Cosmology?**