Small-x Helicity Phenomenology

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Proton Spin Puzzle

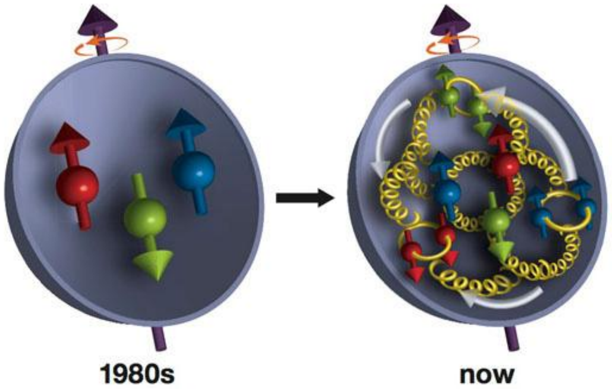
Jaffe-Manohar Spin Sum Rule:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$S_{q,g}$ = Helicity of quarks and gluons

$L_{q,g}$ = Orbital angular momentum

S_q ~ 30% of proton spin!

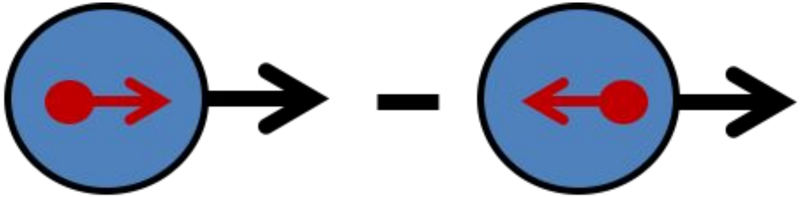


Quark Helicity Parton Distribution Functions

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q (\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2))$$

Helicity PDFs:

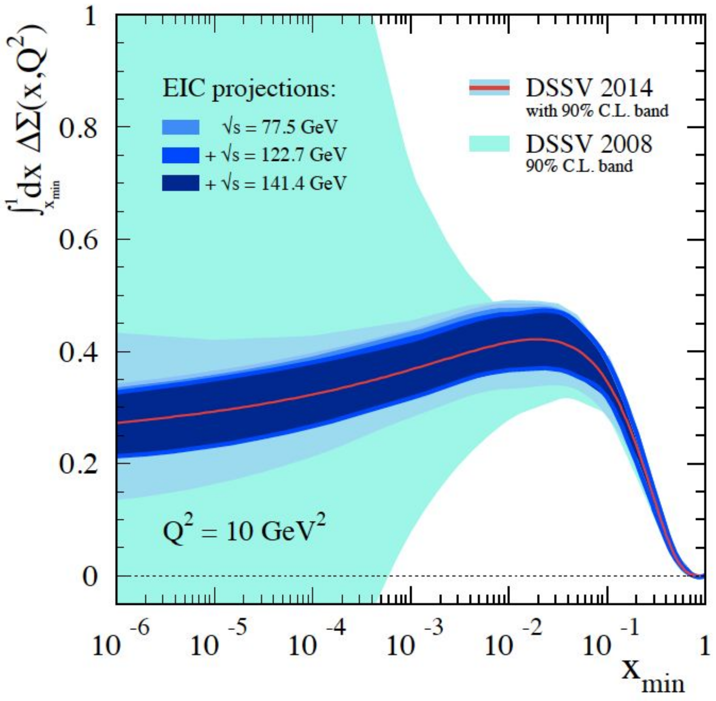
$$\Delta q =$$



- Q^2 = resolution at which we probe the proton
- Bjorken $x \sim \frac{1}{s}$. We need theory to extrapolate to *x*=0

Quark hPDF - DGLAP extraction

2 x (quark spin)

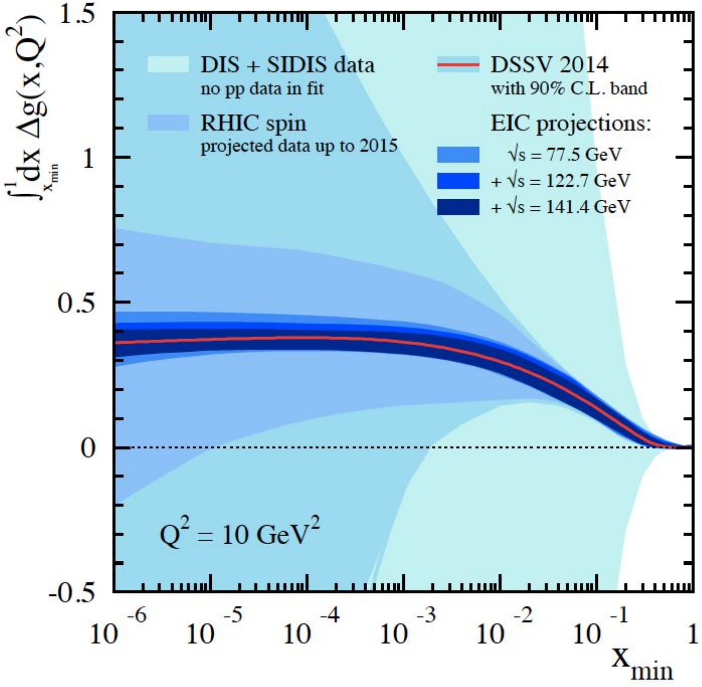


$$\Delta\Sigma = \sum_q (\Delta q + \Delta\bar{q})$$

- E. Aschenauer et al, **arXiv:1509.06489 [hep-ph]**, (DSSV = de Florian, Sassot, Stratmann, Vogelsang, DGLAP-based helicity PDF extraction from data)
- Large uncertainty at small-x!

Gluon Helicity Parton Distributions Function

gluon
spin



$$S_g(Q^2) = \int_0^1 dx \Delta G(x, Q^2)$$

ΔG = Gluon Helicity PDF

- Uncertainty consistently blows up when extrapolating beyond data

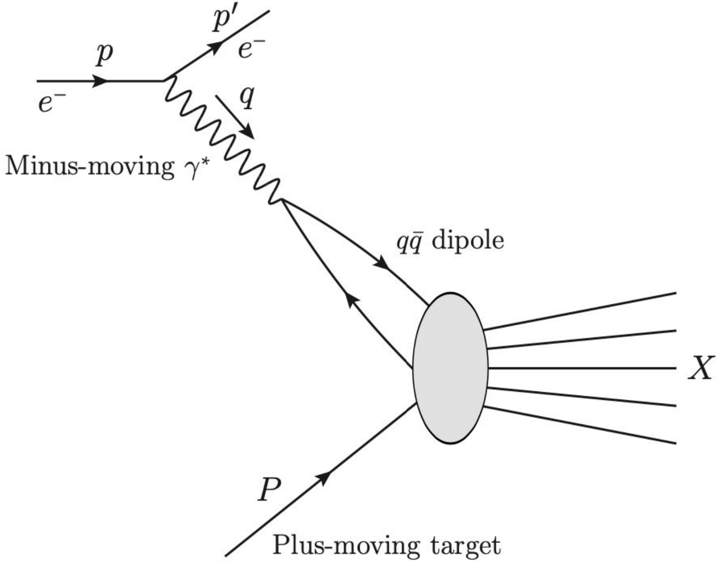
The Plan

Any complete description of quark and gluon helicity needs to

- Describe existing data $(5 \times 10^{-3} < x < 0.7)$
- Predict future, e.g EIC, data $(4 \times 10^{-3} < x < 5 \times 10^{-3})$
- Compare with said data
- Extrapolate down to $x = 0$
- While maintaining good control over theoretical uncertainty

Deep-Inelastic Scattering (DIS)

Probing the proton at small *x*



- Electron of momentum *p* scatters off proton of momentum *P*
- Transverse size given by virtuality of photon:

$$\frac{1}{x_\perp^2} \propto Q^2 = -q^2$$

- Bjorken-*x*: $x = \dfrac{Q^2}{2P \cdot q} \approx \dfrac{Q^2}{s}$

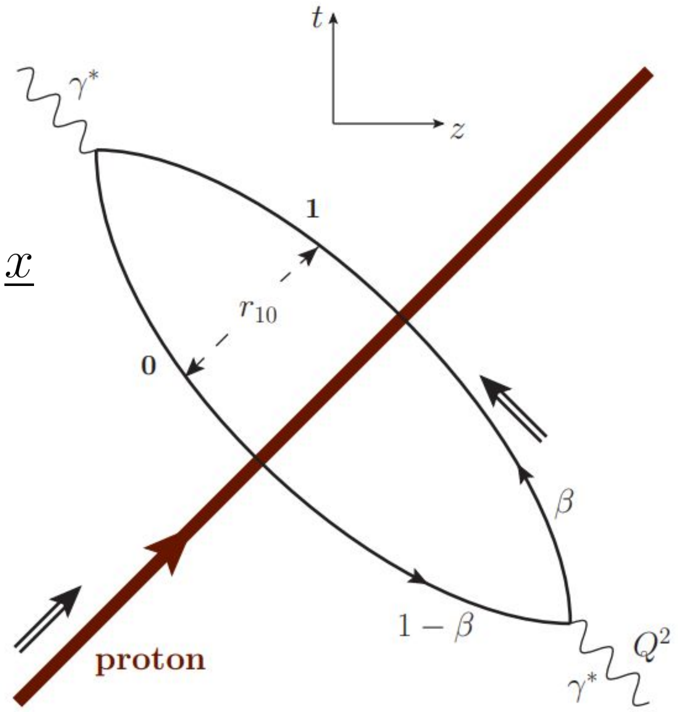
DIS in the Dipole Picture

$$W^{\mu\nu} = \frac{1}{4\pi M_p} \int d^4x \, e^{iq\cdot x} \, \langle P | j^\mu(x) \, j^\nu(0) | P \rangle$$

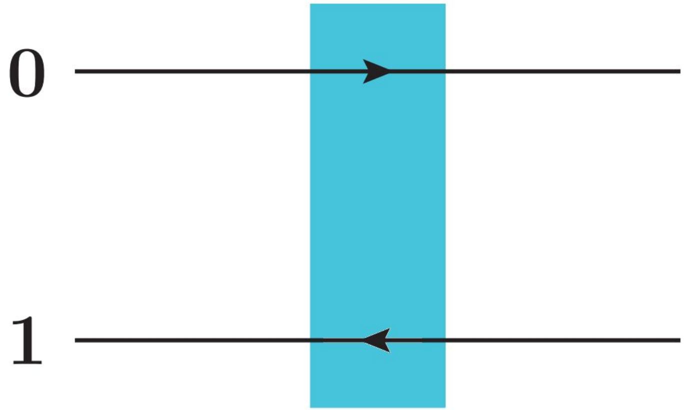
$$q^\mu = \left(\frac{-Q^2}{2q^-}, q^-, \underline{q} \right) \quad , \quad q \cdot x = q^- x^+ - \frac{Q^2}{q^-} x^- - \underline{q} \cdot \underline{x}$$

Large q^- □ large x^- separation

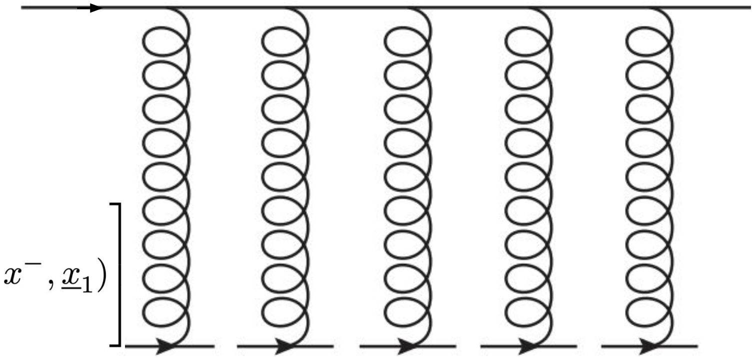
$$\Rightarrow F_2 \propto |\psi|^2 \otimes N$$



DIS in the Dipole Picture



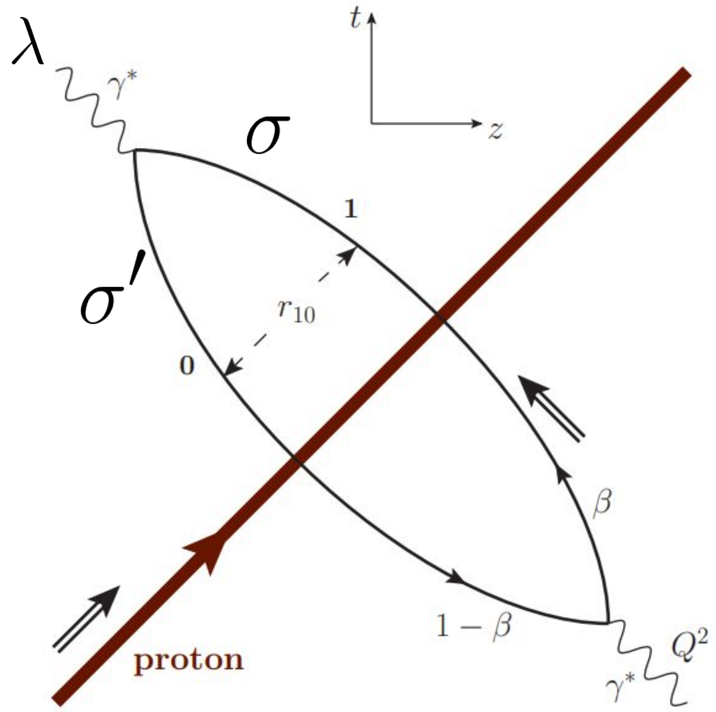
- The proton is a shockwave
- Working in light-cone gauge $A^- = 0$
- Unpolarized structure functions F1, F2 proportional to Dipole Amplitude
- $N(s) = 1 - \frac{1}{N_c}\langle \mathrm{tr}[V_{\underline{1}} V_{\underline{0}}^\dagger] \rangle(s)$



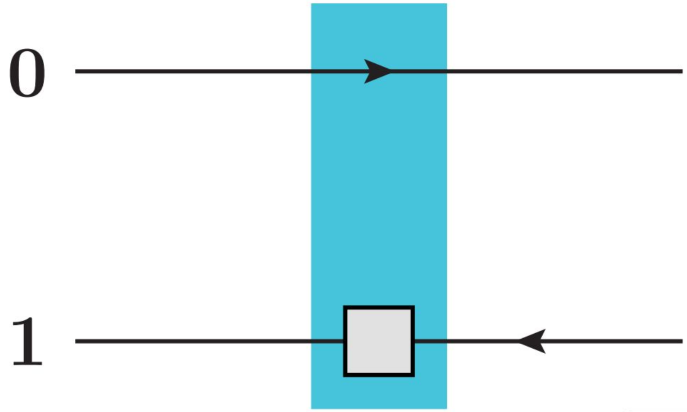
- Quarks undergo multiple eikonal re-scatterings
- Quark lines replaced with Wilson Lines
- $V_{\underline{1}}[x_f^-, x_i^-] \equiv V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp \left[ig \int\limits_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1) \right]$

(Polarized) DIS in the (Polarized) Dipole Picture

$$g_1 \propto |\psi|^2 \otimes (Q + 2G_2)$$



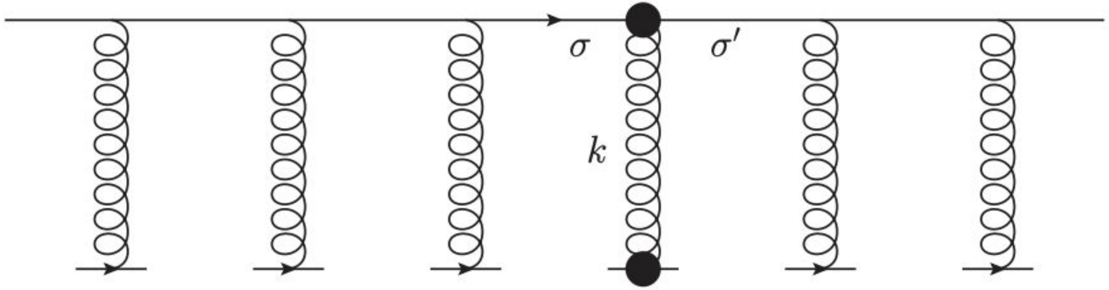
(Polarized) DIS in the (Polarized) Dipole Picture



- Quark line undergoes one extra helicity exchange, which is **sub-eikonal**

- In pDIS, the electron and proton have their helicity specified
- Cross-section now dependent on **Polarized Dipole Amplitudes**:

$$Q_q, G_2, \tilde{G}$$



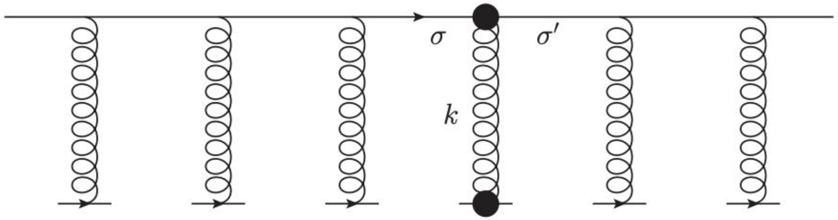
Sub-eikonal Expansion

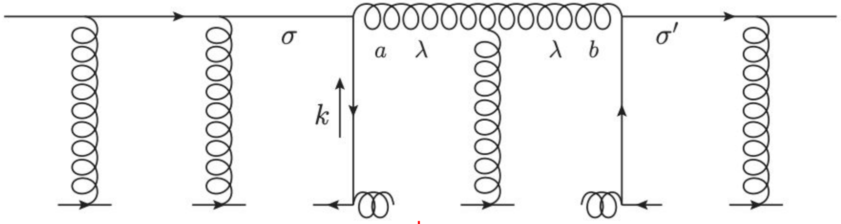
- Expansion in energy or in *x*

$1/x,$	$x^0,$	x^1
Eikonal	Sub-Eikonal	Sub-Sub-Eikonal
F_1, F_2	$g_1^{p,n}, \Delta q, \Delta \bar{q}$	Transversity

- No eikonal terms contain any helicity information - Wilson lines are helicity independent
- Must calculate sub-eikonal terms to access helicity

Polarized Wilson Lines





$\vec{\mu} \cdot \vec{B}$

- Chromo-magnetic field

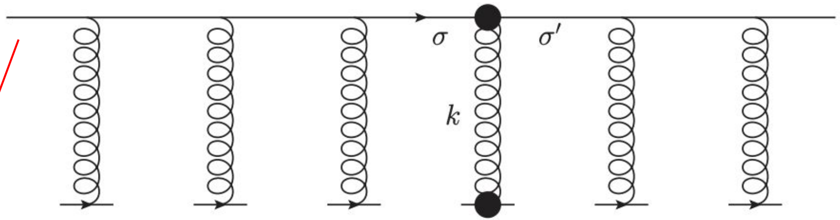
$\mu B_z \sim F_{12}$

$\bar{\psi}\gamma^+\gamma^5\psi$

- Axial Current

Polarized Dipole Amplitudes: "Q_q" , "$\tilde{G}$"

Polarized Wilson Lines



- Quark propagator

$$\int \frac{dk^+}{2\pi} e^{ikx} \frac{\not{k}}{2k^+k^- - k_\perp^2}$$

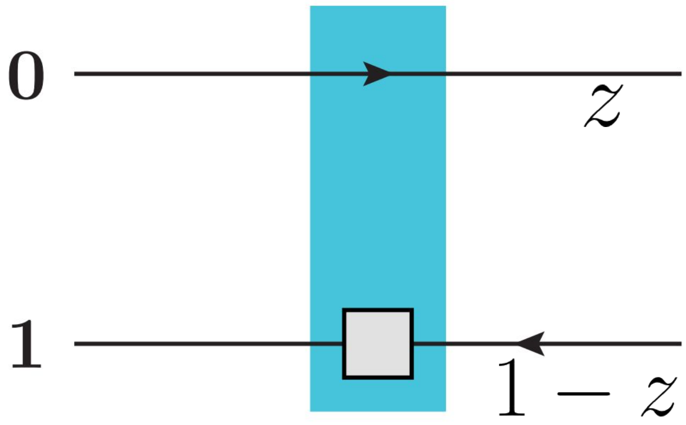
- Sub-eikonal phase expansion

$$e^{-ix^- \frac{k_\perp^2}{2k^-}} \approx 1 - ix^- \frac{k_\perp^2}{2k^-} \Rightarrow \partial_\perp^2 \rightarrow D_\perp^2 \rightarrow \text{“}G_2\text{”}$$

$$A_\perp$$

- Polarized gluon vertex

Polarized Dipole Amplitude - Degrees of Freedom



$$Q_q(s_{10}, \eta)$$

Polarized Dipole Amplitudes are functions of

- Transverse separation:

$$x_{10}^2 = (\underline{x}_1 - \underline{x}_0)^2$$

- Momentum Fraction times center of mass energy: zs
- Rescaled variables:

$$\eta = \sqrt{\frac{N_c}{2\pi}} \ln \frac{zs}{\Lambda^2} \qquad s_{10} = \sqrt{\frac{N_c}{2\pi}} \ln \frac{1}{x_{10}^2 \Lambda^2}$$

Calculating Helicity Distributions

$$\Delta q + \Delta\bar{q} = \frac{1}{N_c} \int_0^{\eta_{max}} d\eta \int_{s_{10}^{min}}^{\eta} ds_{10} \frac{1}{\alpha_s(s_{10})} \left(Q_q(s_{10}, \eta) + 2G_2(s_{10}, \eta)\right)$$

- We incorporate running coupling that runs with size of the dipole

- $\eta_{max} = \sqrt{N_c/2\pi} \ln(Q^2/x\Lambda^2)$
- $s_{10}^{min} = max[0, \eta - \sqrt{N_c/2\pi} \ln(1/x)]$

Calculating Helicity Distributions

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s(Q^2)} G_2\left(\sqrt{\frac{N_c}{2\pi}}\ln\frac{Q^2}{\Lambda^2}, \sqrt{\frac{N_c}{2\pi}}\ln\frac{Q^2}{x\Lambda^2}\right)$$

- Jaffe-Manohar Gluon Helicity Distribution
- Λ^2 Infrared cutoff

Energy Evolution

Evolution equations are constructed out of Renormalization Group flow equations:

$$\langle O \rangle_{Y+dY} \approx \langle O \rangle_Y + dY \frac{d}{dY} \langle O \rangle_Y$$

$$\langle O \rangle = \int \mathcal{D}(\delta A) O e^{iS[b+\delta A]}$$

$$\frac{d}{dY}\langle O \rangle \approx \langle \underbrace{\delta A_x \delta A_y}_{\text{Gluon Propagator}} J_{xy} \rangle$$

Evolution looks like gluon emission and absorption

Helicity Evolution

Using Light-Cone Operator Treatment, we need to resum all gluon exchanges that exchange helicity information

$\Rightarrow$ Resumming all terms containing: $$\alpha_s \int_x^1 \frac{dz}{z} \int_{1/s}^{1/Q^2} \frac{d^2 x_{21}}{x_{21}^2}$$

Longitudinal part. Present in un-polarized evolution

Transverse part. UV exactly cancelled in un-polarized evolution

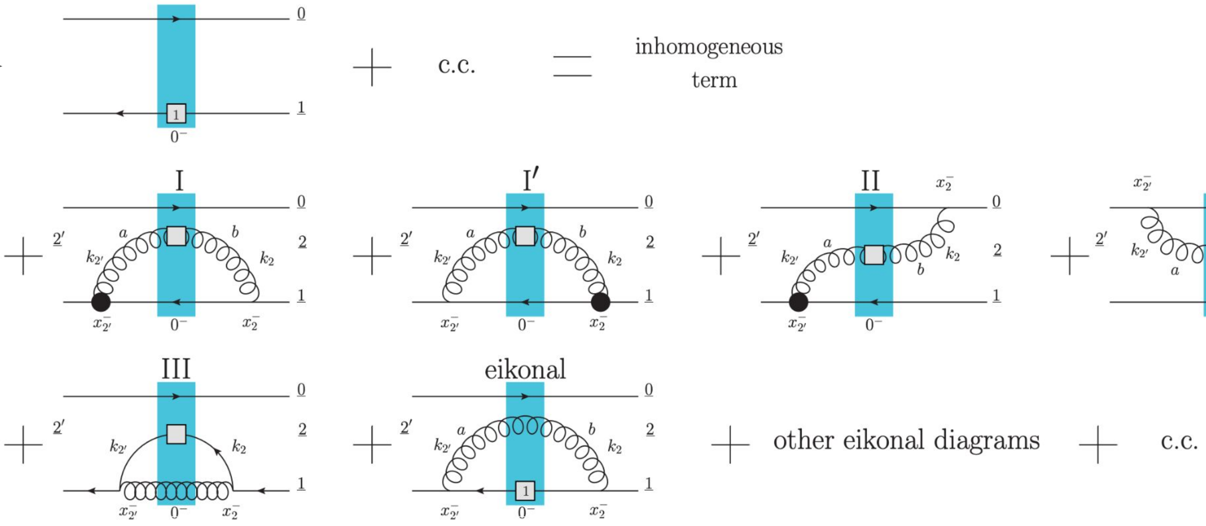
Resum double log (DLA) terms:

$$\alpha_s \ln^2(1/x)$$

Helicity Evolution

- Relate Polarized Dipole Amplitude to themselves at higher energies by resumming emission diagrams - resumming Double Log (DLA) contributions: $\alpha_s \ln^2(1/x)$

$Q_q(x_{10}^2, zs) \longrightarrow$



Large Nc&Nf Helicity Evolution

In the large Nc&Nf, Nc/Nf fixed limit, the evolution equations for the polarized dipole amplitudes close:

$$Q_q(s_{10},\eta) = Q_q^{(0)}(s_{10},\eta) + \int_{s_{10}+y_0}^{\eta} d\eta' \int_{s_{10}}^{\eta'-y_0} ds_{21} \Big[Q_q(s_{21},\eta') + 2\tilde{G}(s_{21},\eta') + 2\tilde{\Gamma} s_{10}, s_{21},\eta') - \bar{\Gamma}_f(s_{10},s_{21},\eta') + 2G_2(s_{21},\eta') + 2\Gamma_2(s_{10},s_{21},\eta') \Big]$$

$$+ \frac{1}{2} \int_{y_0}^{\eta} d\eta' \int_{\max\{0, s_{10}+\eta'-\eta\}}^{\eta'-y_0} ds_{21} \Big[Q_q(s_{21},\eta') + 2G_2(s_{21},\eta') \Big]$$

+ 9 more

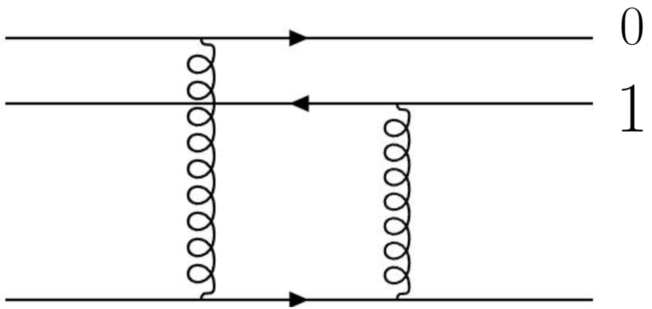
- 5 Polarized dipole amplitudes mix under evolution: $Q_{u,d,s}, \tilde{G}, G_2$
- With 5 auxiliary dipoles: $\Gamma_{u,d,s}, \tilde{\Gamma}, \Gamma_2$ - which impose lifetime ordering
- Small-*x* cutoff, $y_0 \propto \ln 1/x_0$

Large Nc&Nf Helicity Evolution

- 5 **Polarized dipole amplitudes** mix under evolution: $Q_{u,d,s}, \tilde{G}, G_2$
- With 5 auxiliary dipoles: $\Gamma_{u,d,s}, \tilde{\Gamma}, \Gamma_2$
- For a total of 10 equations that form a **closed system**
- Undetermined initial conditions: $Q^{(0)}_{u,d,s}, \tilde{G}^{(0)}, G^{(0)}_2$

Inhomogeneous term

The inhomogeneous term is given by a Born-inspired ansatz:



$$\propto \int_0^s \frac{dk_\perp^2}{k_\perp^2}\left(1 - e^{-\underline{k}\cdot\underline{x}_{10}}\right) = \pi \ln(s x_{10}^2)$$

$$\propto \eta - s_{10}$$

$$\Gamma_q^{(0)} = Q_q^{(0)} = a\eta + b s_{10} + c$$

- Same form of the other Dipole Amplitudes
- Parameters a,b,c need to be extracted from data

Recap:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q (\Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2))$$

$$S_g(Q^2) = \int_0^1 dx \Delta G(x,Q^2)$$

$$\Delta q + \Delta\bar{q} = \frac{1}{N_c}\int_0^{\eta_{max}} d\eta \int_{s_{10}^{min}}^{\eta} ds_{10} \frac{1}{\alpha_s(s_{10})}\left(Q_q(s_{10},\eta) + 2G_2(s_{10},\eta)\right)$$

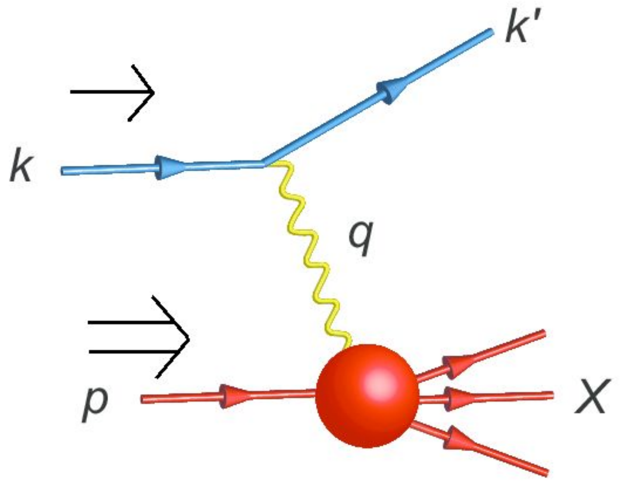
$$\Delta G(x,Q^2) = \frac{2N_c}{\alpha_s(Q^2)} G_2\left(\sqrt{\frac{N_c}{2\pi}}\ln\frac{Q^2}{\Lambda^2}, \sqrt{\frac{N_c}{2\pi}}\ln\frac{Q^2}{x\Lambda^2}\right)$$

Large $N_c \& N_f$ Helicity Evolution

$$Q_q^{(0)}, \tilde{G}^{(0)}, G_2^{(0)}$$

Observables - Double Spin Asymmetries in DIS

$$A_{||} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto A_1 \propto g_1^{p,n}$$



- $\uparrow (\downarrow)$ is positive (negative) helicity electron
- $\Uparrow (\Downarrow)$ is positive (negative) helicity proton
- A_1 is virtual photoproduction asymmetry

Describing Observables - pDIS

What enters into observables are linear combinations of hPDFs

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$
$$\Delta q^- = \Delta q - \Delta \bar{q}$$

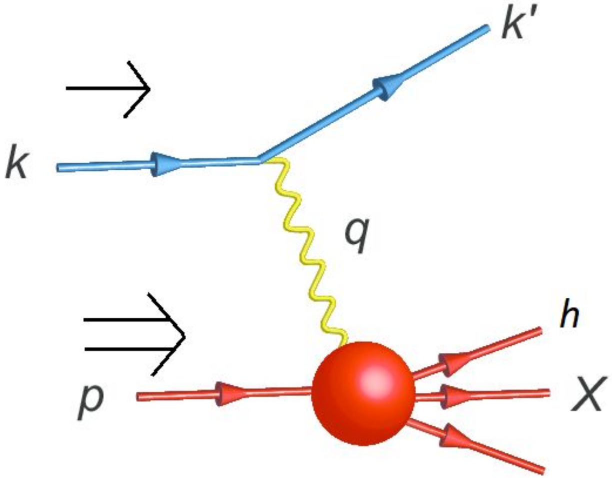
- Three relevant hPDFs in DIS: $\Delta u^+, \Delta d^+, \Delta s^+$
- Data exist for two observables that contain these hPDFs in linearly independent combinations: g_1^p and g_1^n

$$g_1^p(x, Q^2) = \frac{1}{2}\sum_q Z_q^2 \Delta q^+(x, Q^2)$$

- Z_q is the quark charge fraction

Observables - Double Spin Asymmetries in SIDIS

$$A_{||}(z) = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto g_1^h(z)$$



- *h* is the tagged hadron
- *z* is the momentum fraction of the virtual photon carried by the tagged hadron

Describing Observables - pSIDIS

- 2 observables are not enough to describe 3 hPDFs.
- Expand our horizons to Semi-Inclusive DIS - all hPDFs are relevant here, both singlet, Δq^+ and non-singlet, Δq^-
- **Non-singlet distributions obey their own small-*x* evolution that has been solved**

$$\Delta q^- = \frac{N_c}{2\pi^3} \int d\eta \int ds_{10} Q_q^{NS}(s_{10}, \eta)$$

- Q_q^{NS} is the non-singlet Polarized Dipole Amplitude - obeys its own evolution equation
- pSIDIS grants us access to the semi-inclusive, spin dependent structure functions g_1^h

g_1^h Structure Functions

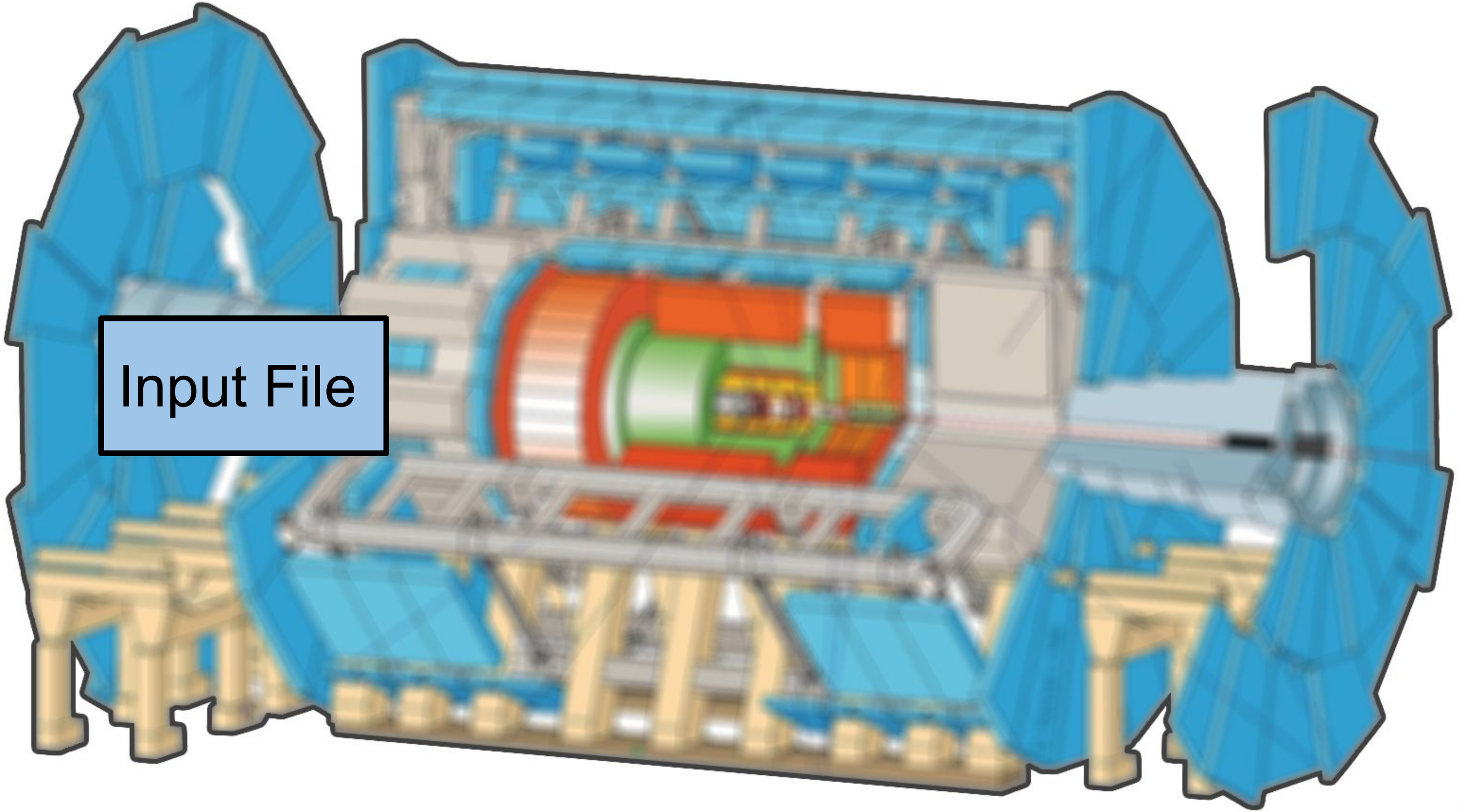
$$g_1^h(x,z,Q^2) = \frac{1}{2}\sum_q Z_q^2 \Delta q(x,z,Q^2) D_q^h(z,Q^2)$$

- D_q^h are fragmentation functions - giving the probability quark *q* fragments into hadron *h*
- z Is the fraction of the virtual photons momentum carried by the hadron
- The flavour hPDF is obtained via $\Delta q = \frac{1}{2}(\Delta q^+ + \Delta q^-)$
- In pSIDIS, we are able to scatter on 2 targets (proton, neutron), tag 2 outgoing hadrons (pion, kaon) that each have 2 charges - 2x2x2=8 new observables

Actually doing phenomenology - JAM framework

The Jefferson Angular Momentum (JAM) framework is a pipeline that enables the statistical comparison of theory to data. Thanks to Nobuo Sato and Wally Melnitchouk for granting us access. The JAM framework is

- Modular - We can insert our theoretical description of the hPDFs without disrupting the rest of the pipeline
- Robust - The parts that aren't our theory (E.g. *F1* structure function) have already been established
- Capable of extracting parameters - It can find the parameters of our initial condition that minimise the χ^2
- Returns statistical uncertainties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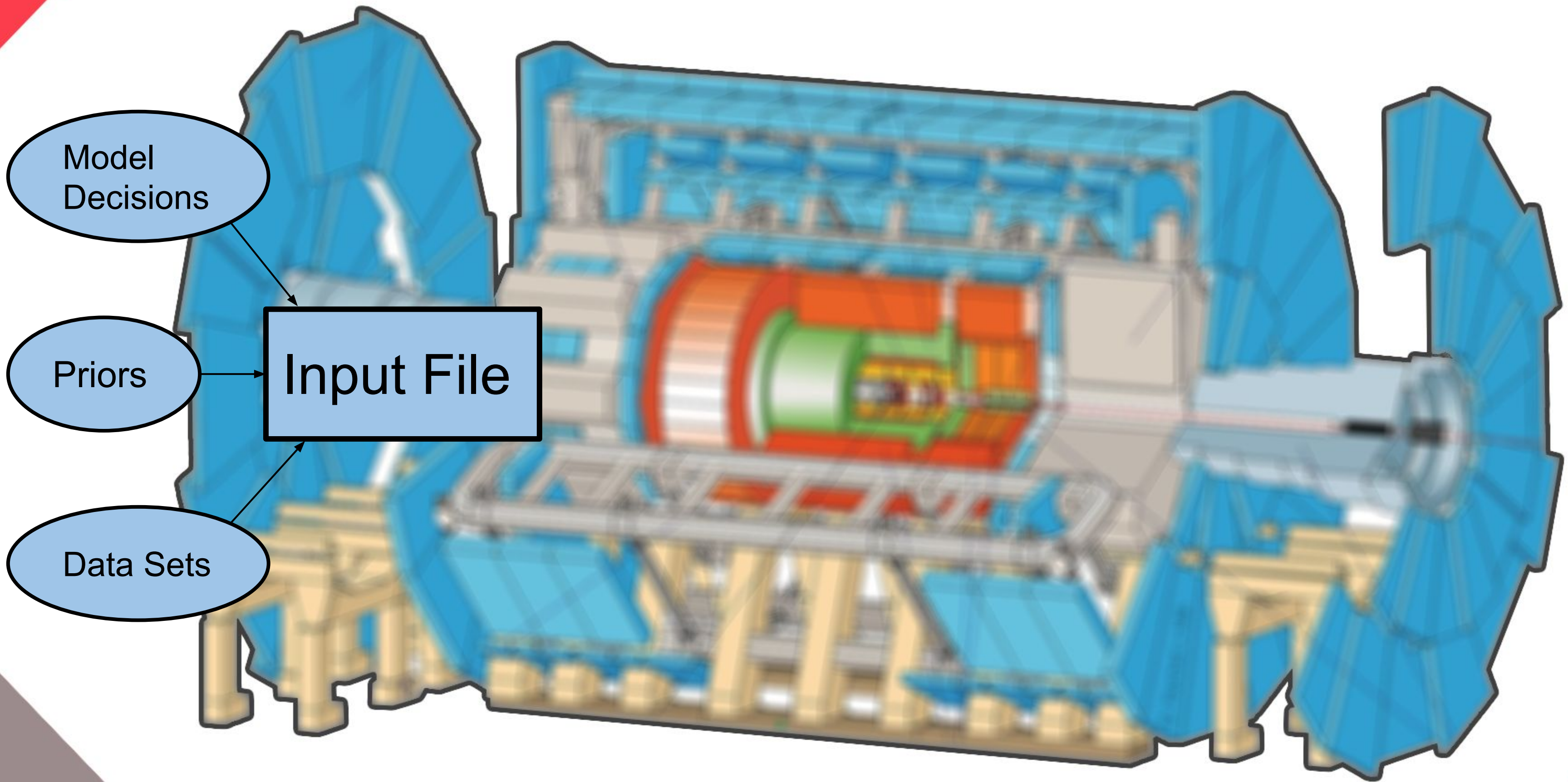
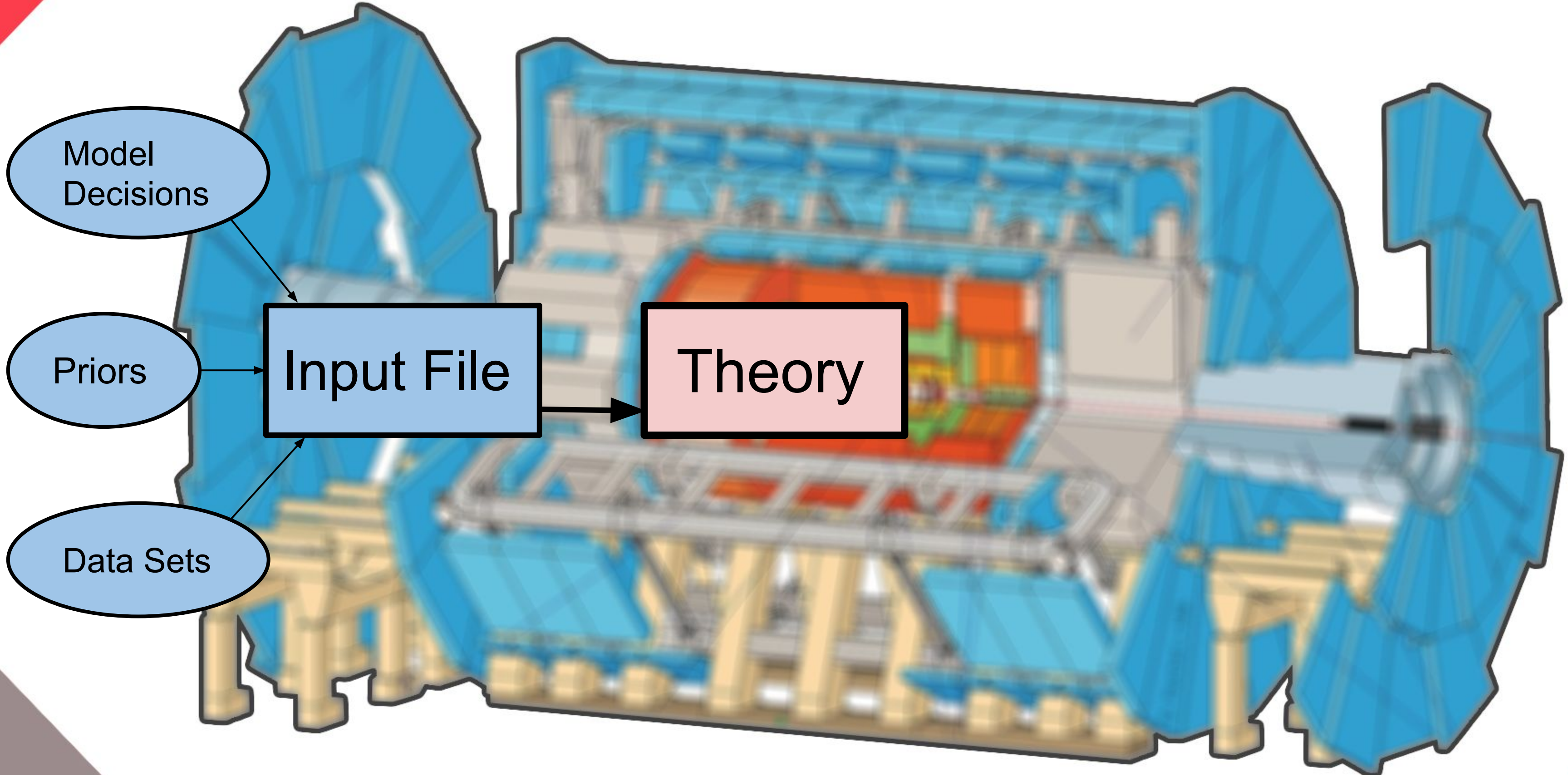

Model Decisions
Priors
Data Sets
Input File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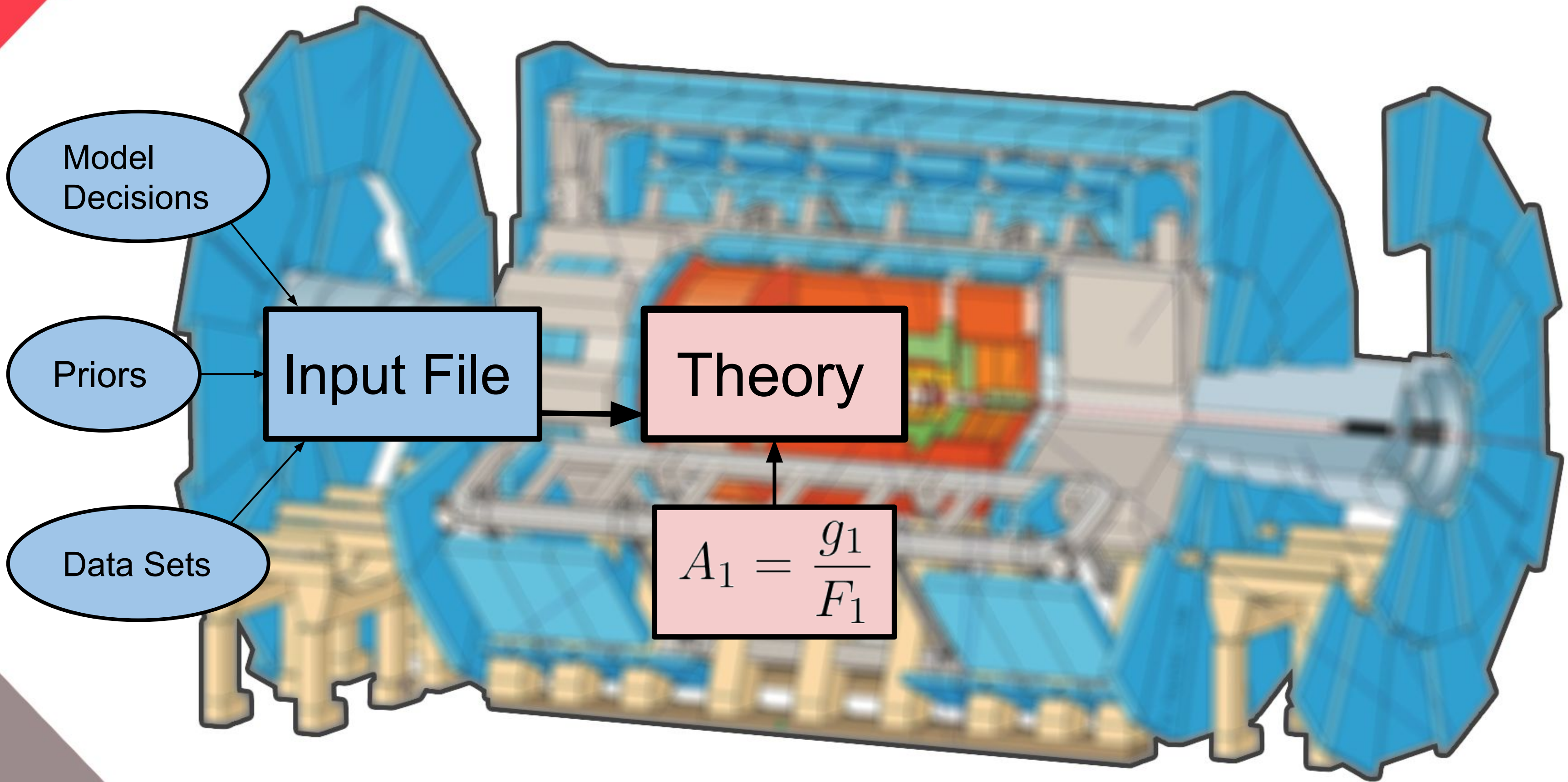

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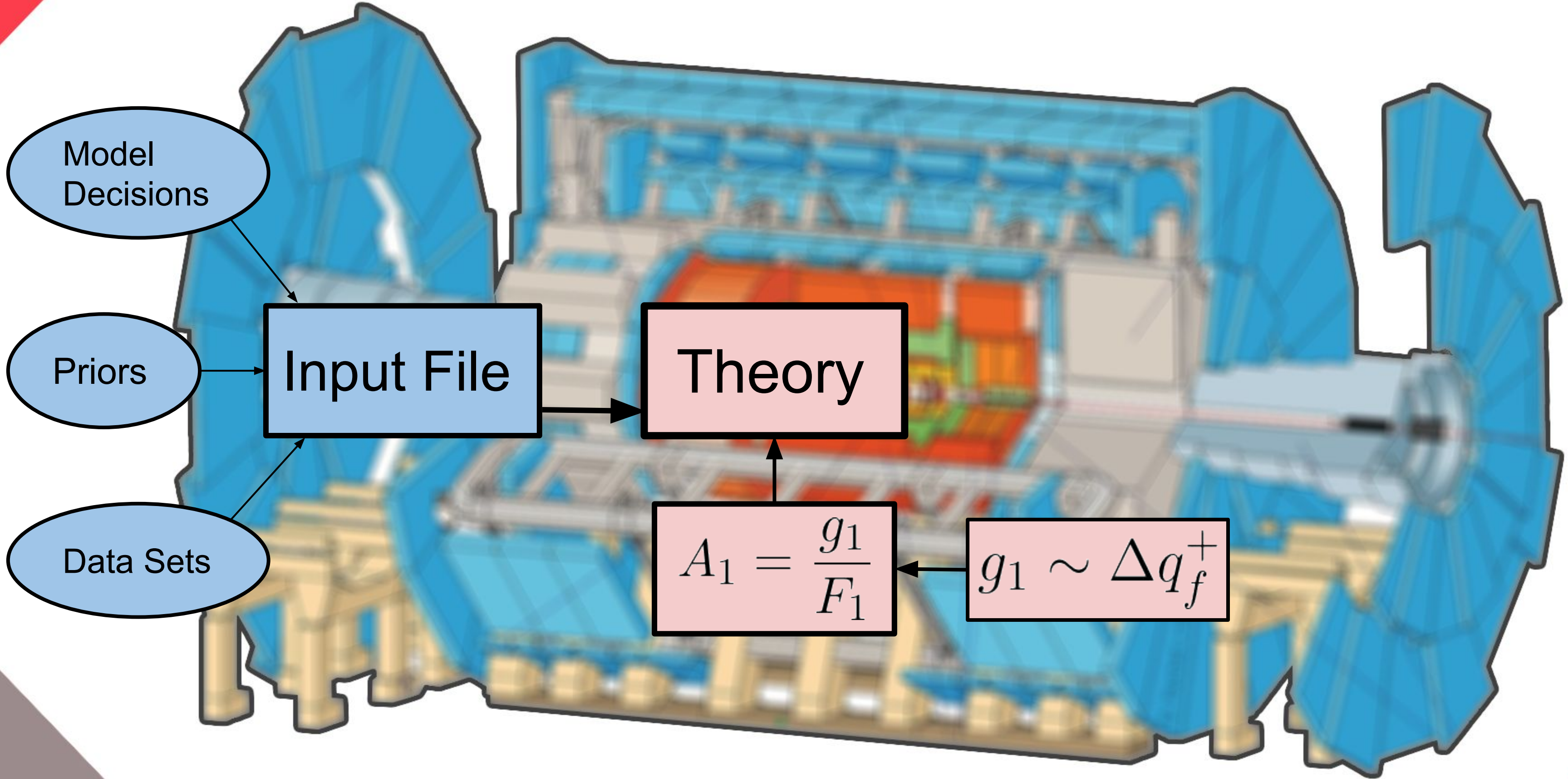
$A_1 = \frac{g_1}{F_1}$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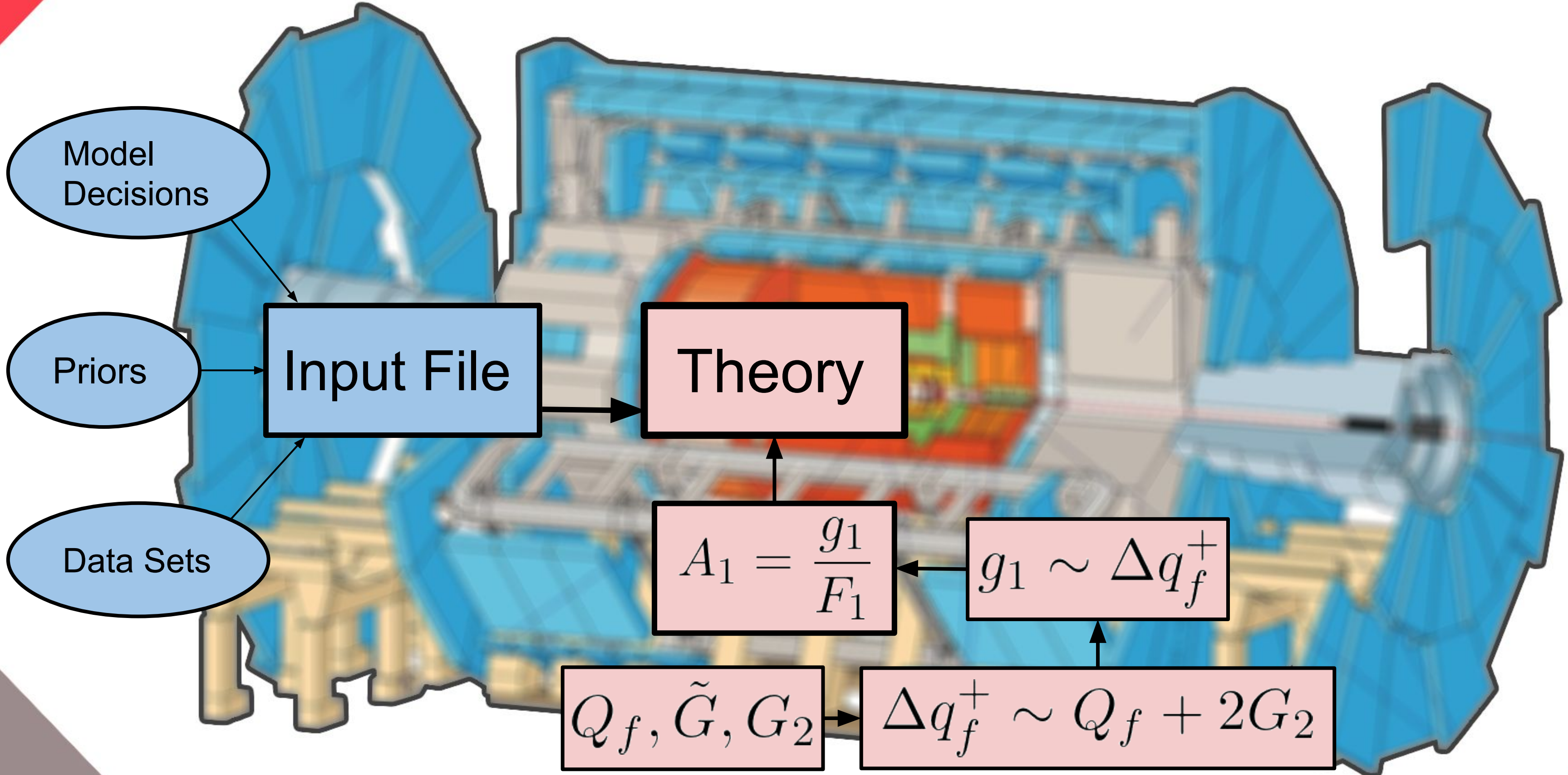
$g_1 \sim \Delta q_f^+$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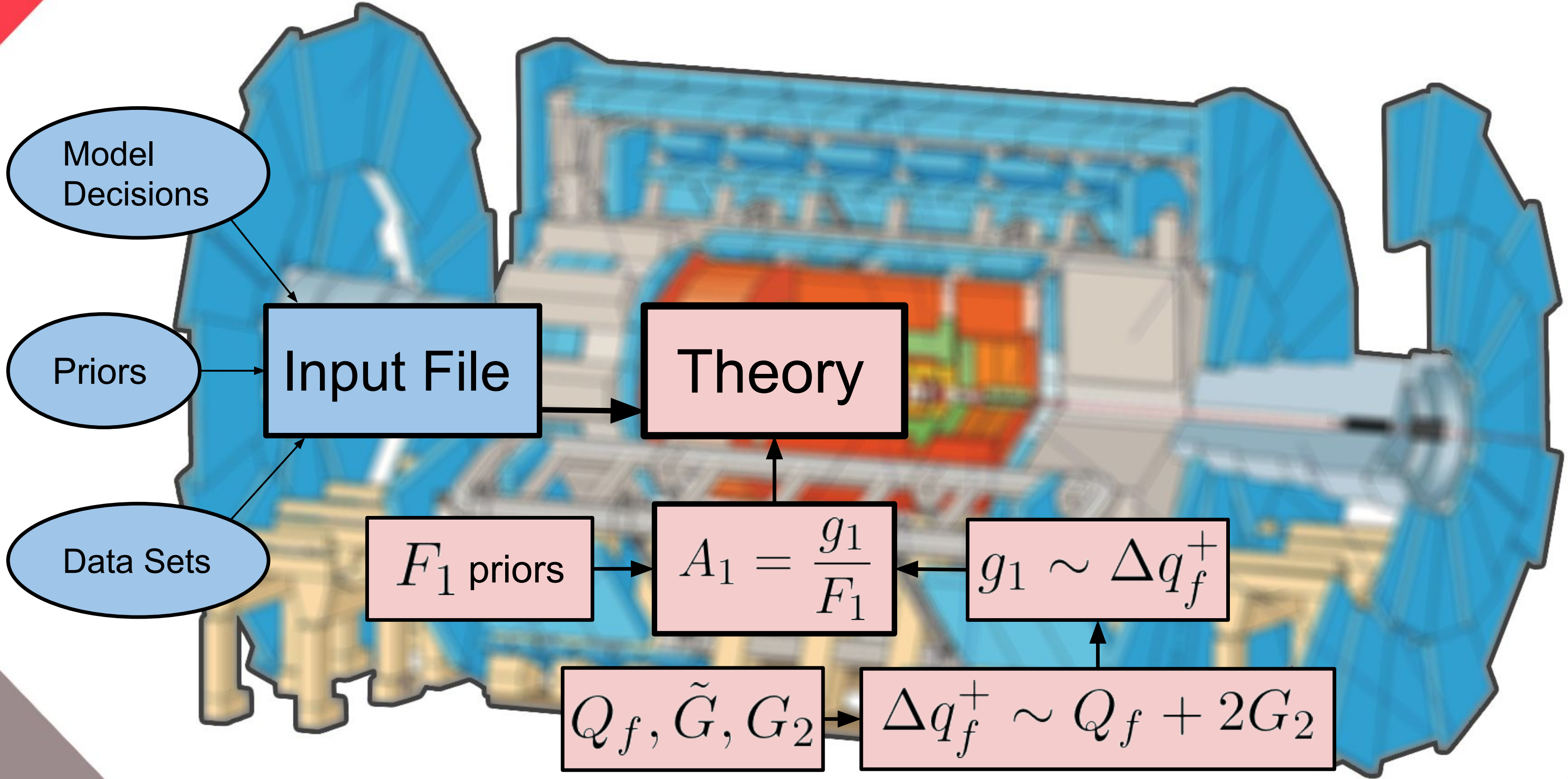

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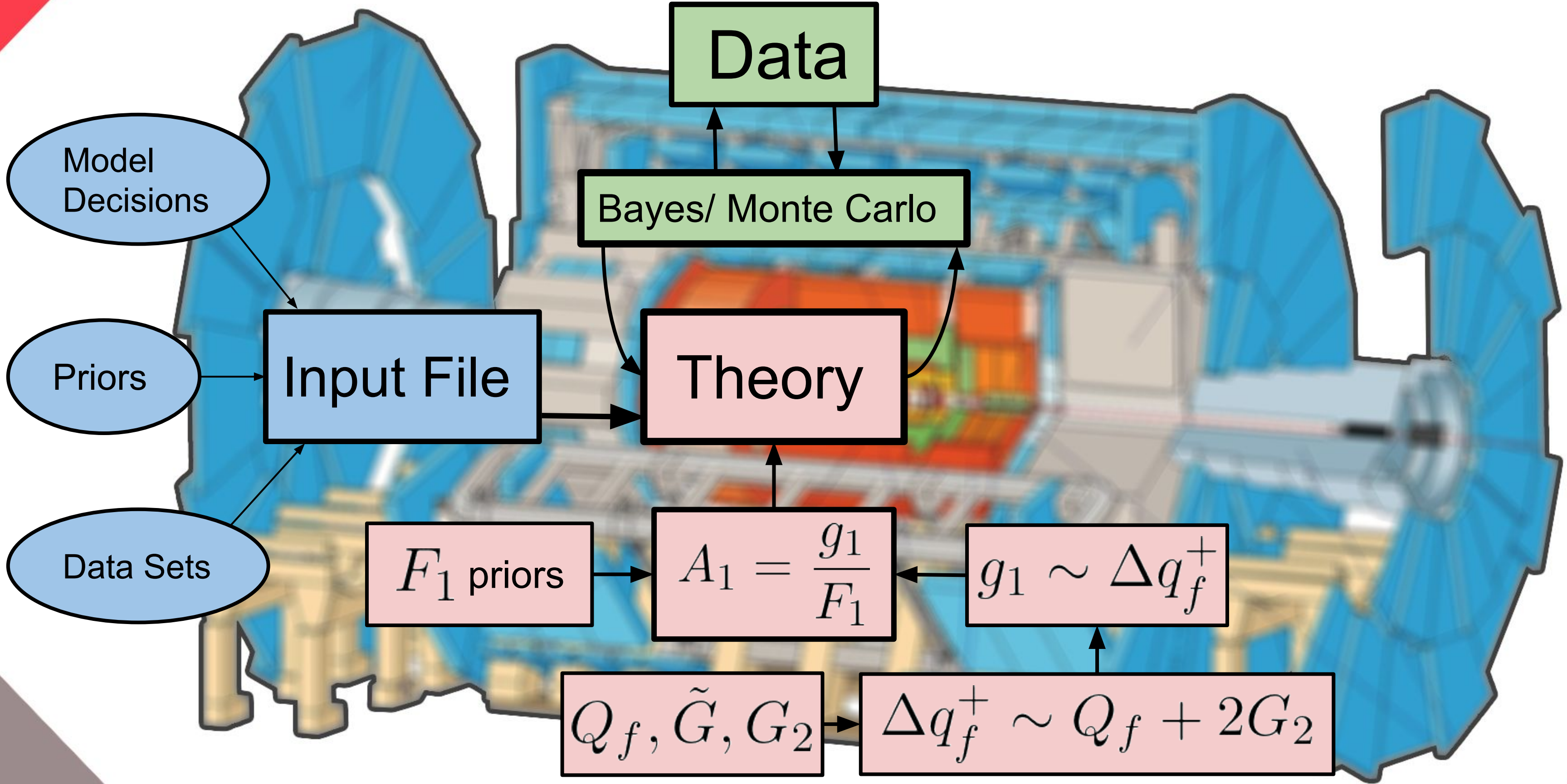

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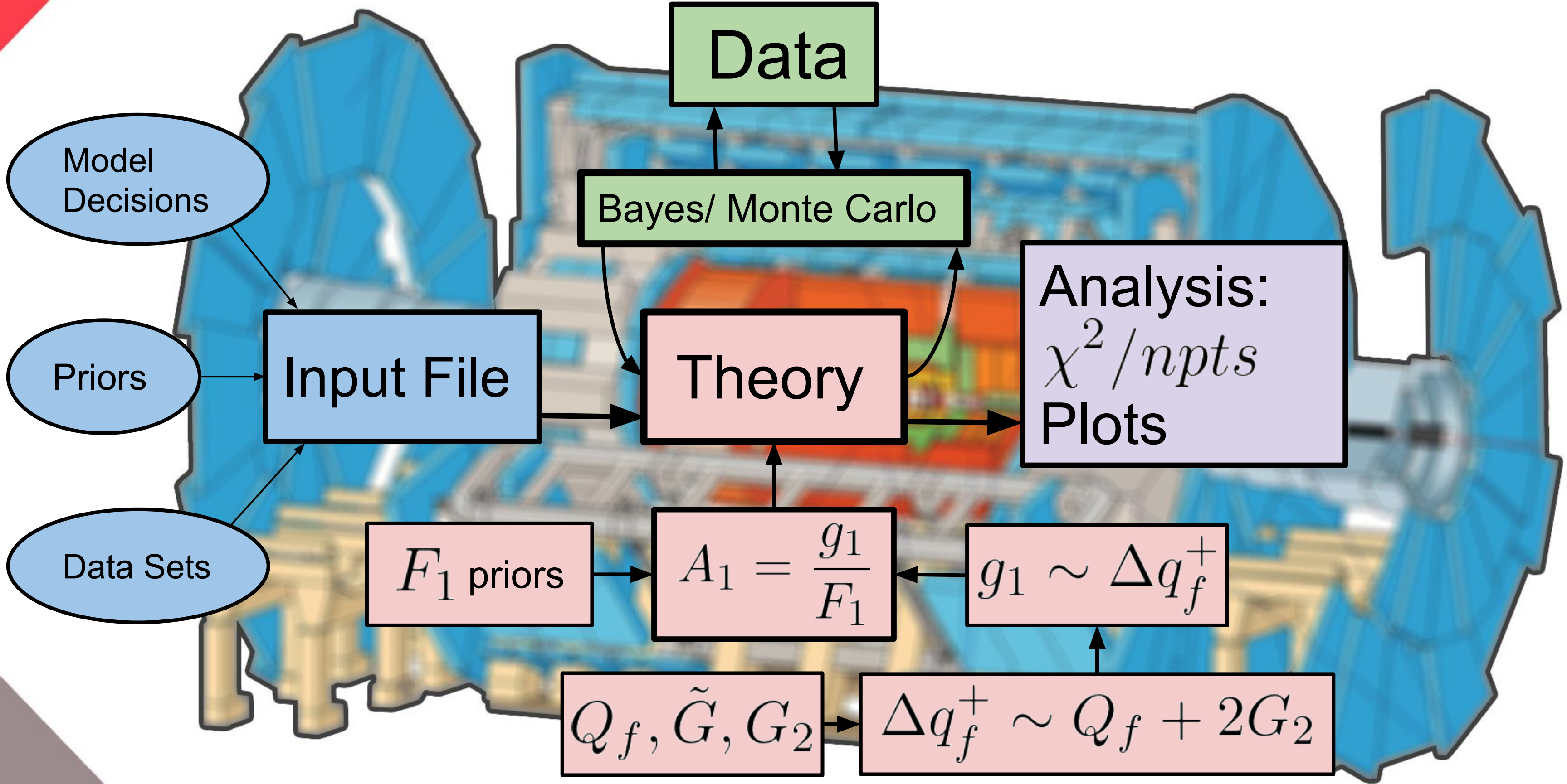
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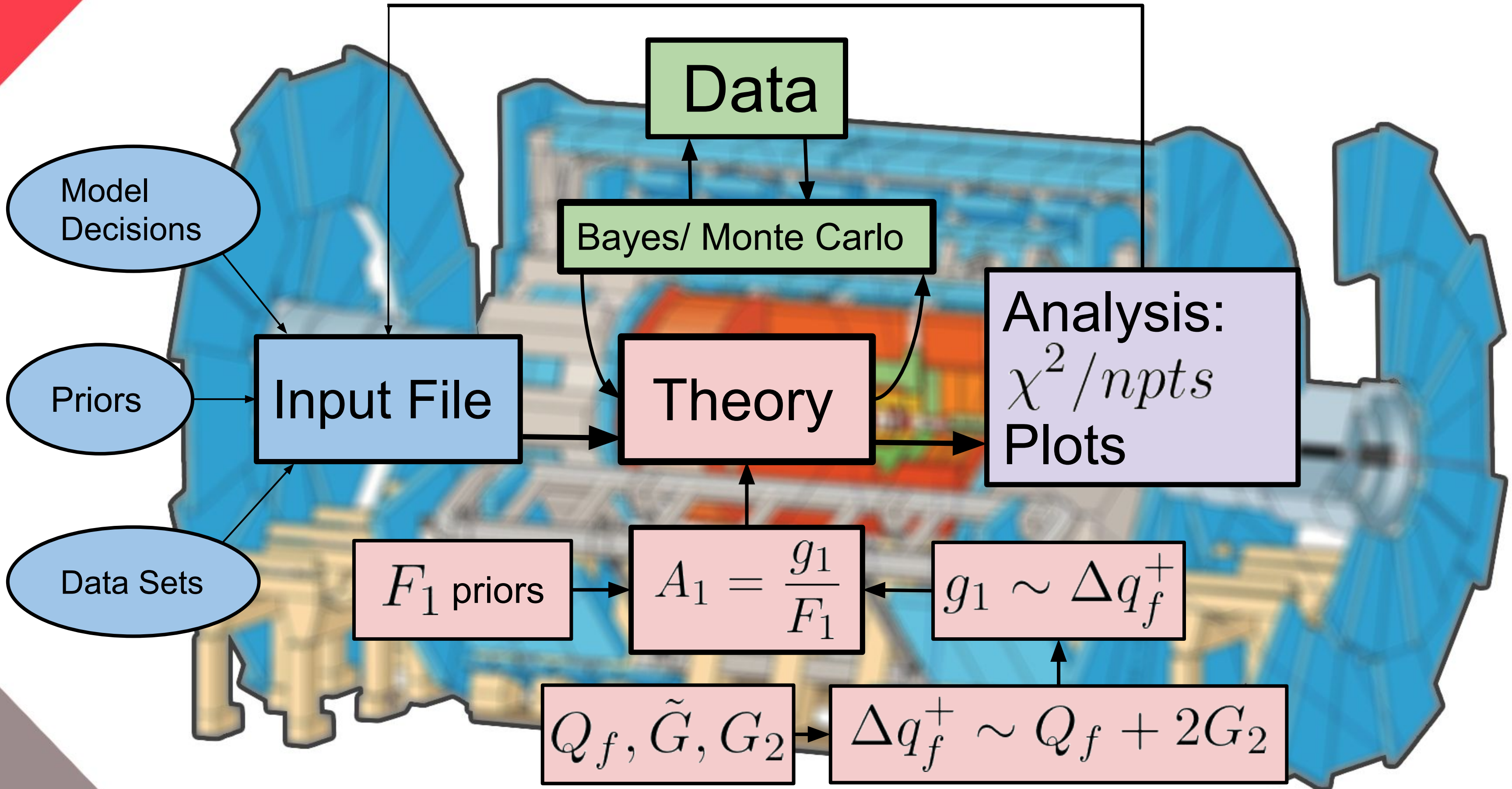
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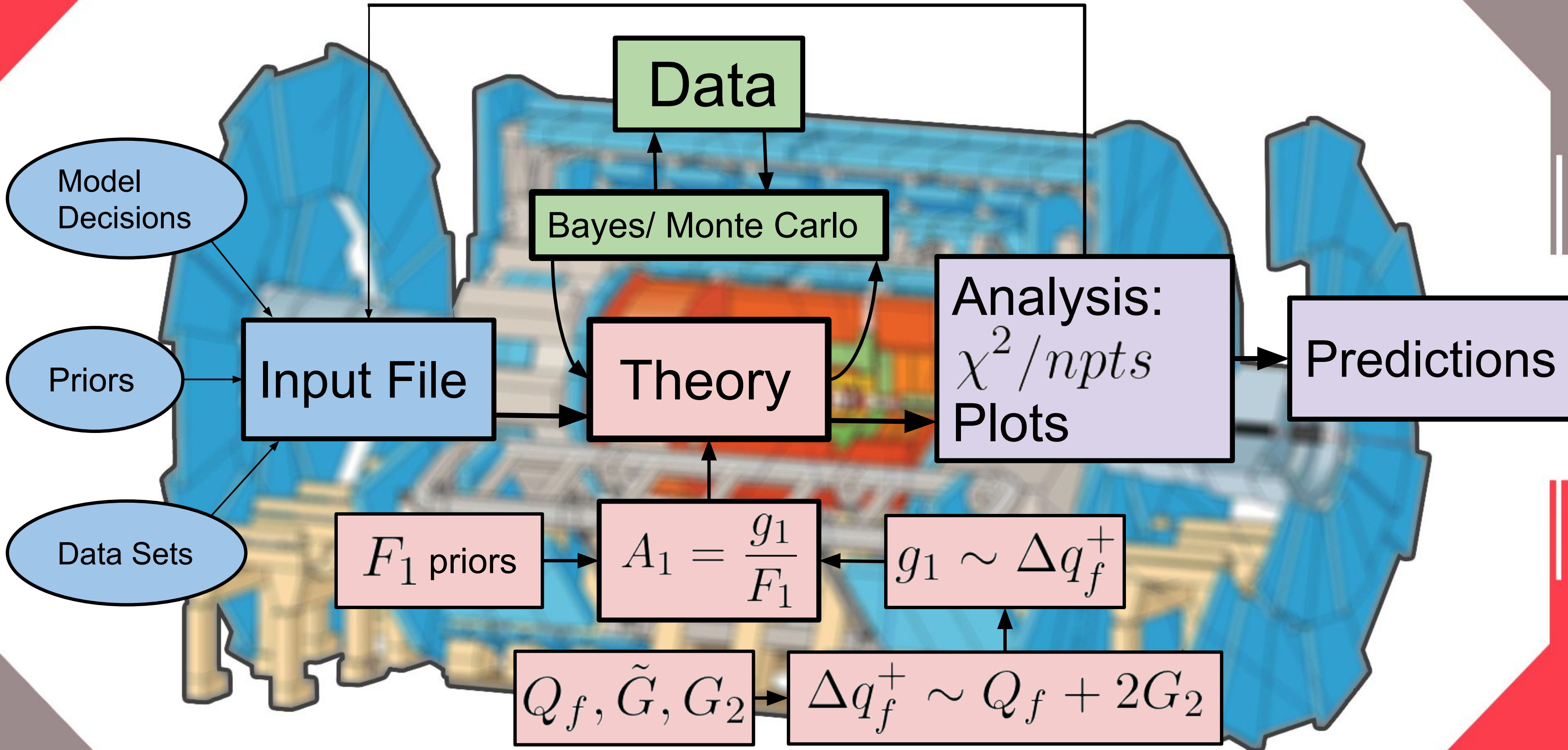
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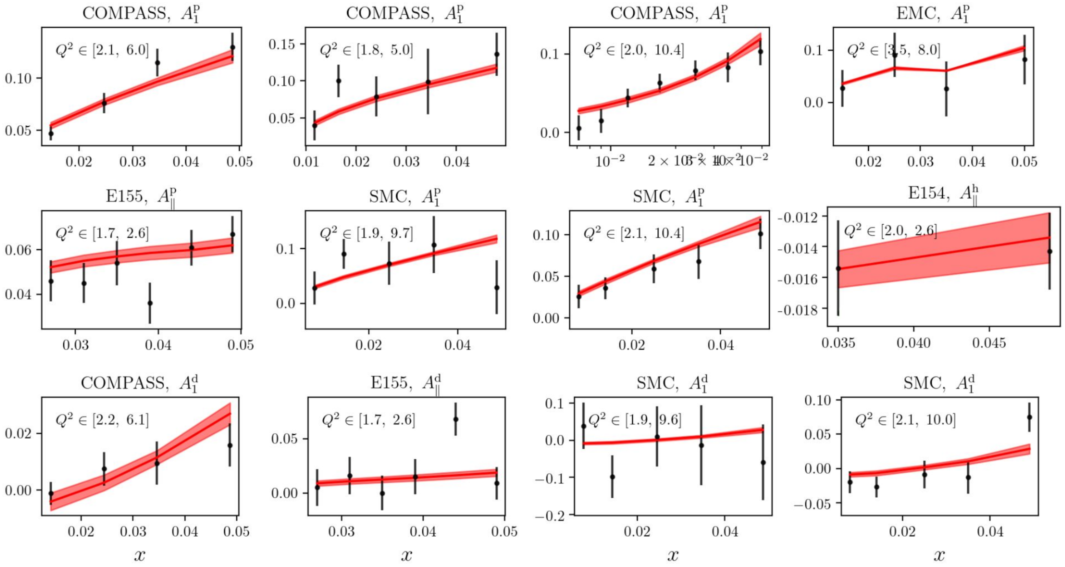
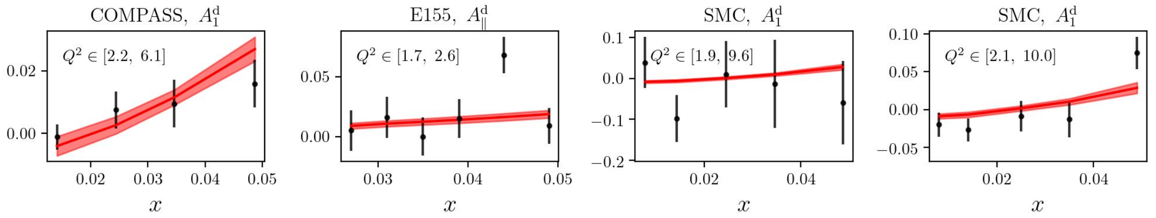
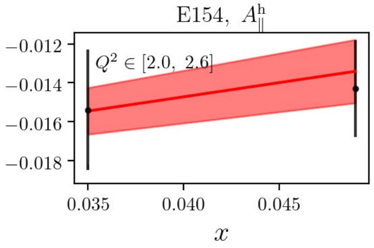
$\Delta q_f^+ \sim Q_f + 2G_2$

Global fit of DIS - Data vs Theory

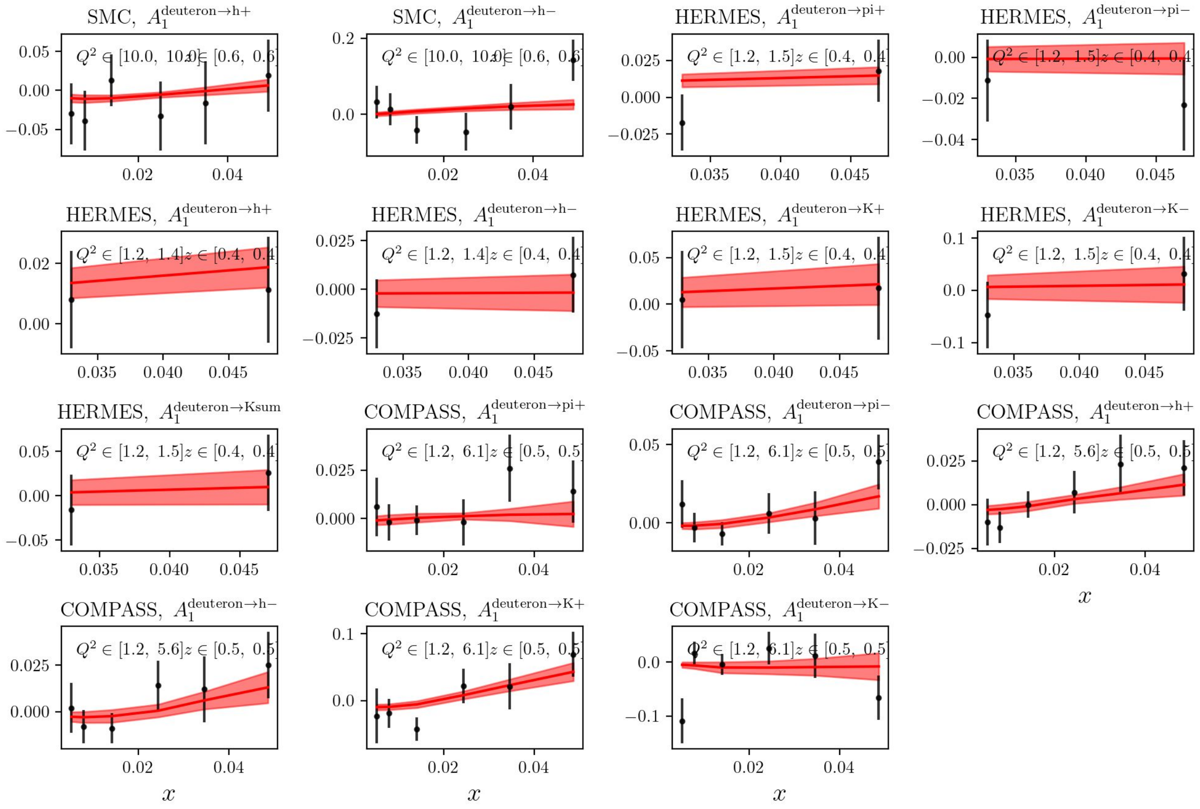


- Red curves - our theory
- Black dots - data
  - COMPASS
  - EMC
  - SMC
  - SLAC
  - HERMES
- Preliminary results

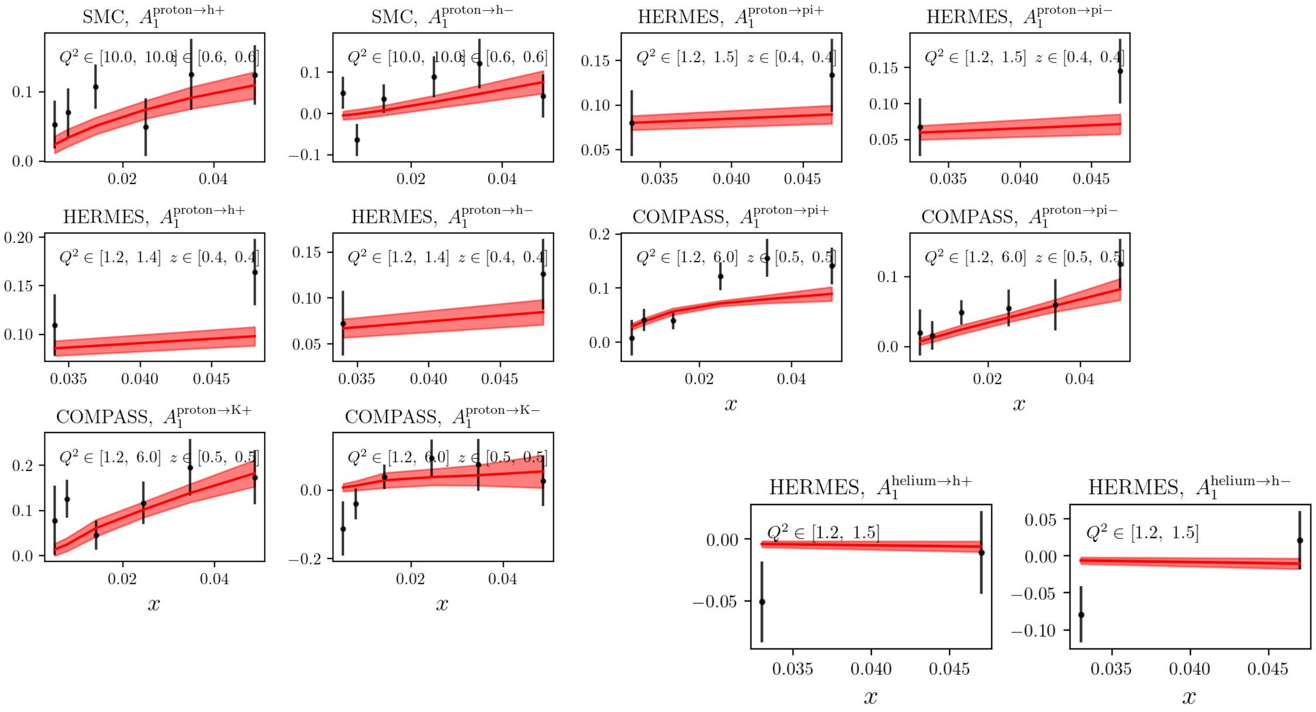




Global fit of SIDIS - Data vs Theory



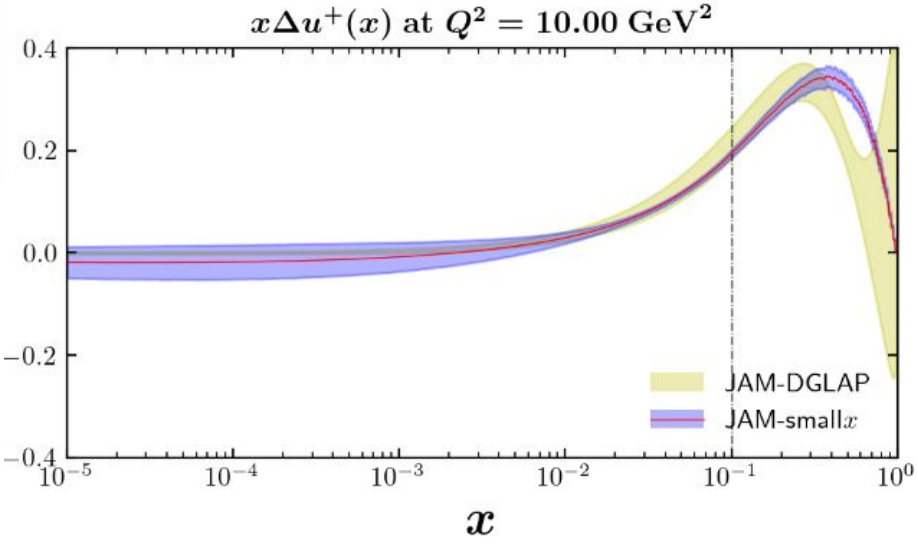
Fitting SIDIS - Data vs Theory

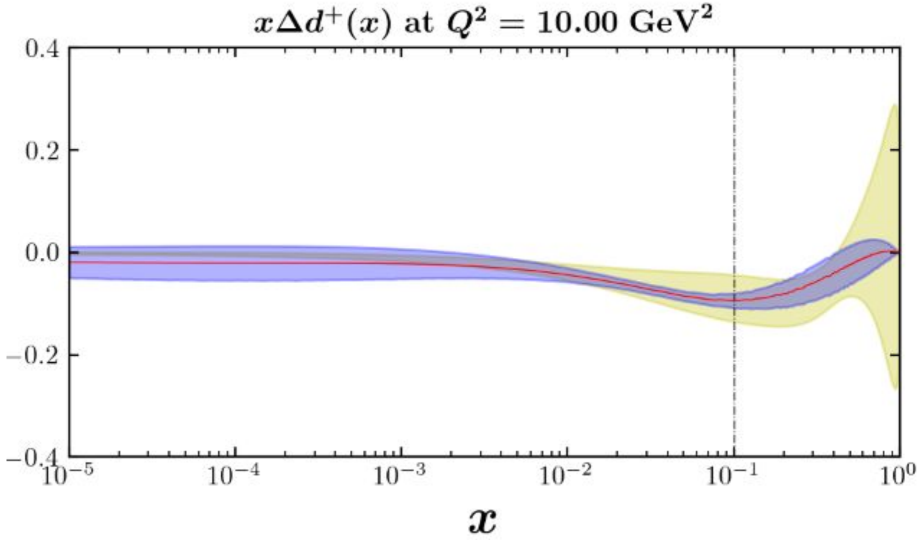


χ^2 and Data Cuts

- First simultaneous fit of small-x theory to polarized DIS & SIDIS data
- Cut of $0.005 < x < 0.1$
- Cut of $1.0 GeV^2 < Q^2 < 10.4 GeV^2$
- Cut of $0.2 < z < 1.0$
- Describing 234 data points
- With a $\chi^2/npts = 1.01$

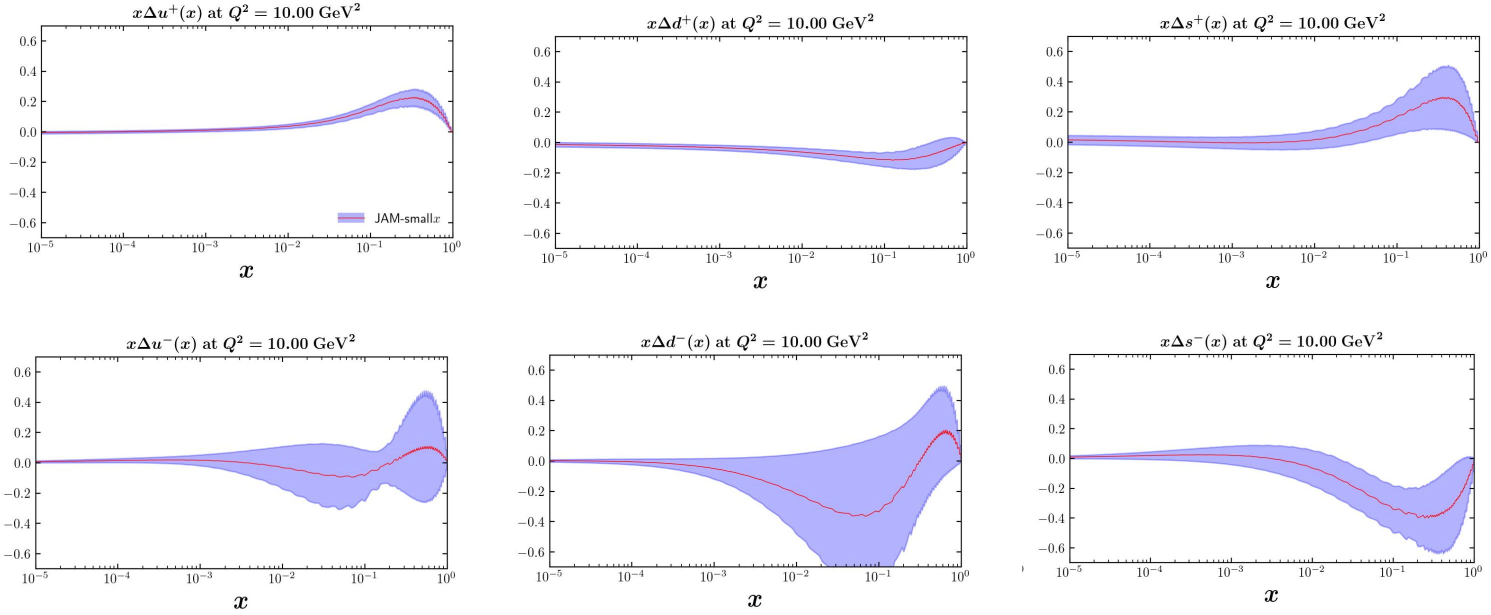
hPDFs - (Preliminary)





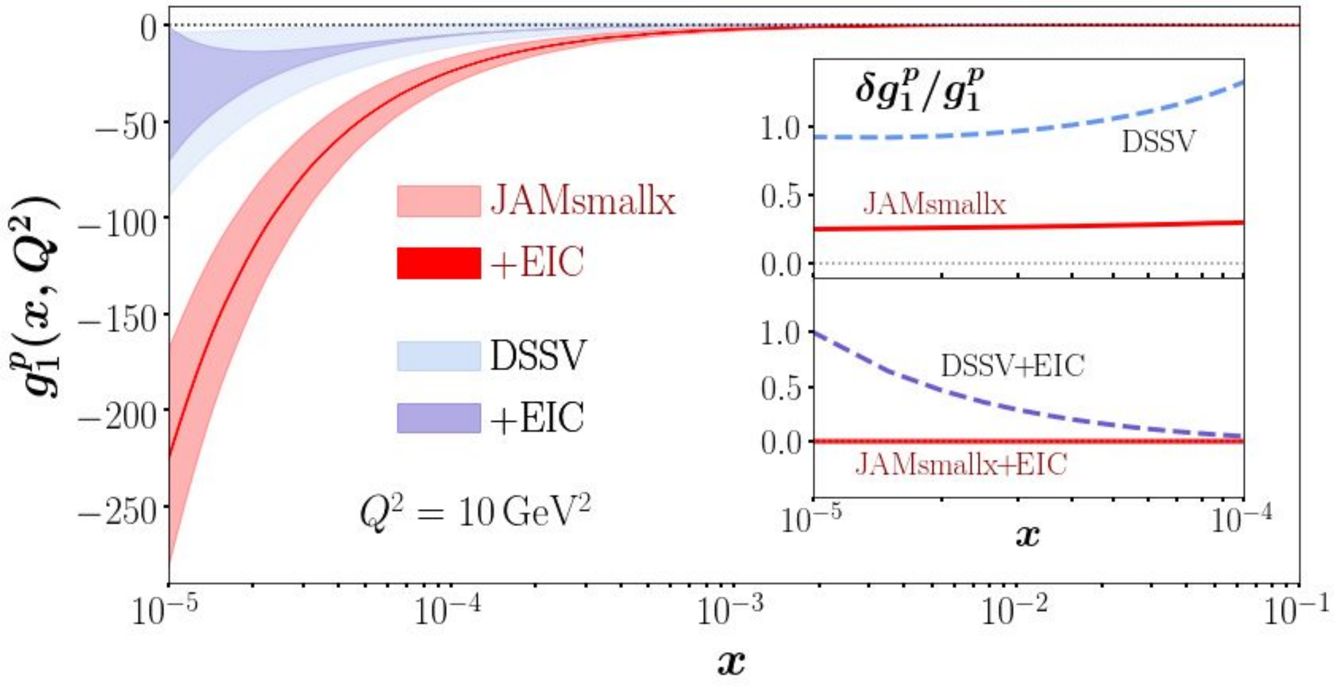
- DIS only: Strange distribution set to zero

hPDFs - Preliminary



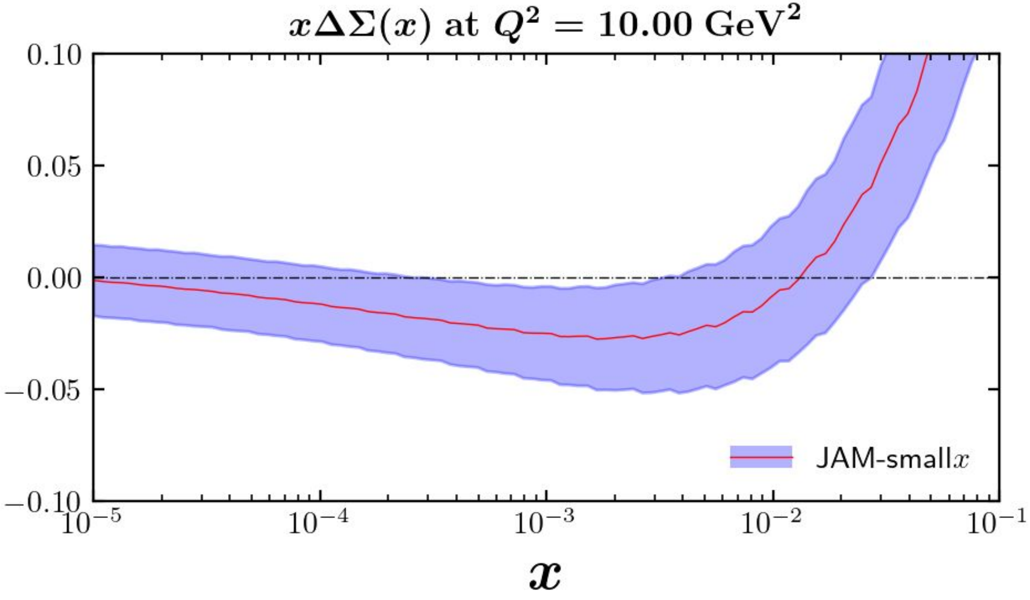
- Old version of evolution

(Preliminary) Extraction of g_1^p

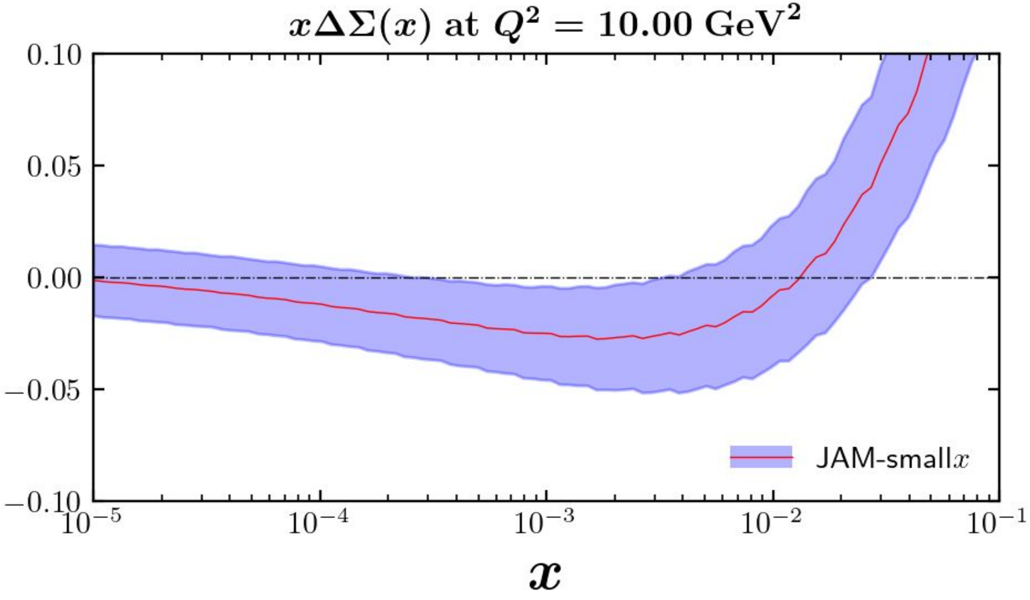


- DSSV uses DGLAP - rational function extrapolation of *x*
- We use small-*x* helicity evolution to predict the *x* behaviour
- Leads to control over uncertainty

Contribution from Quark Spin $\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$



Contribution from Quark Spin $\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$

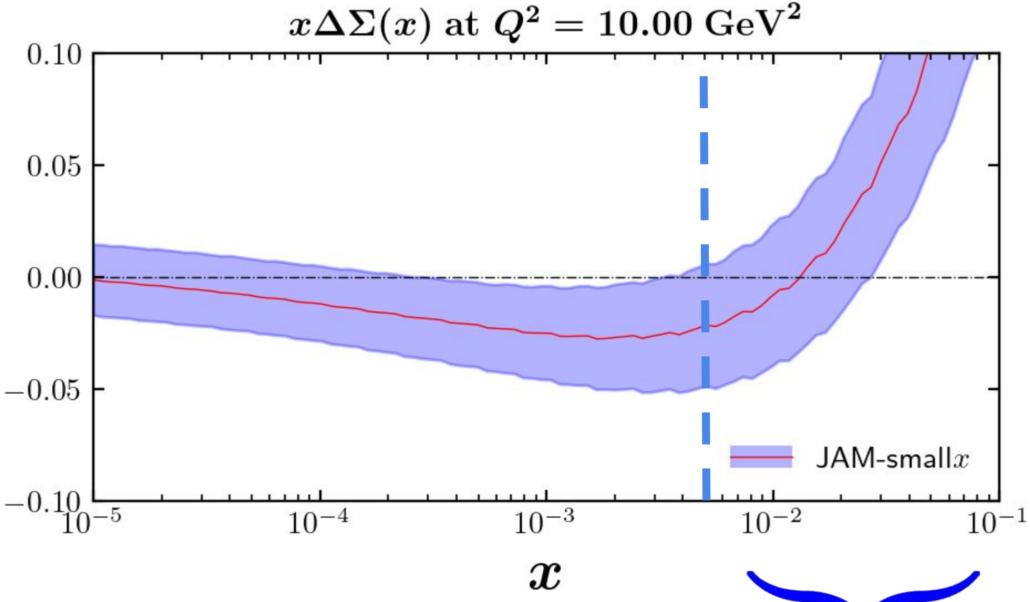


Large-x region

$$\int_{0.01}^{0.7} dx \Delta\Sigma(x) = \pm 0.36$$

Contribution from Quark Spin

$$\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$$



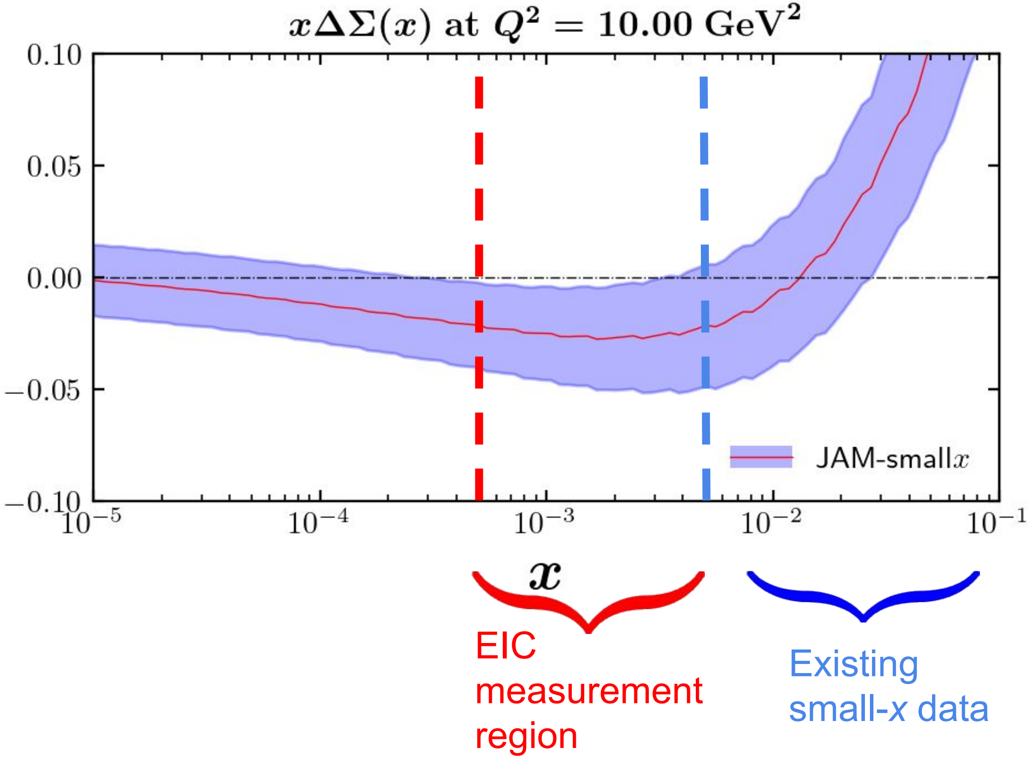
Existing small-*x* data

Large-*x* region

$$\int_{0.01}^{0.7} dx \Delta\Sigma(x) = \pm 0.36$$

Contribution from Quark Spin

$$\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$$

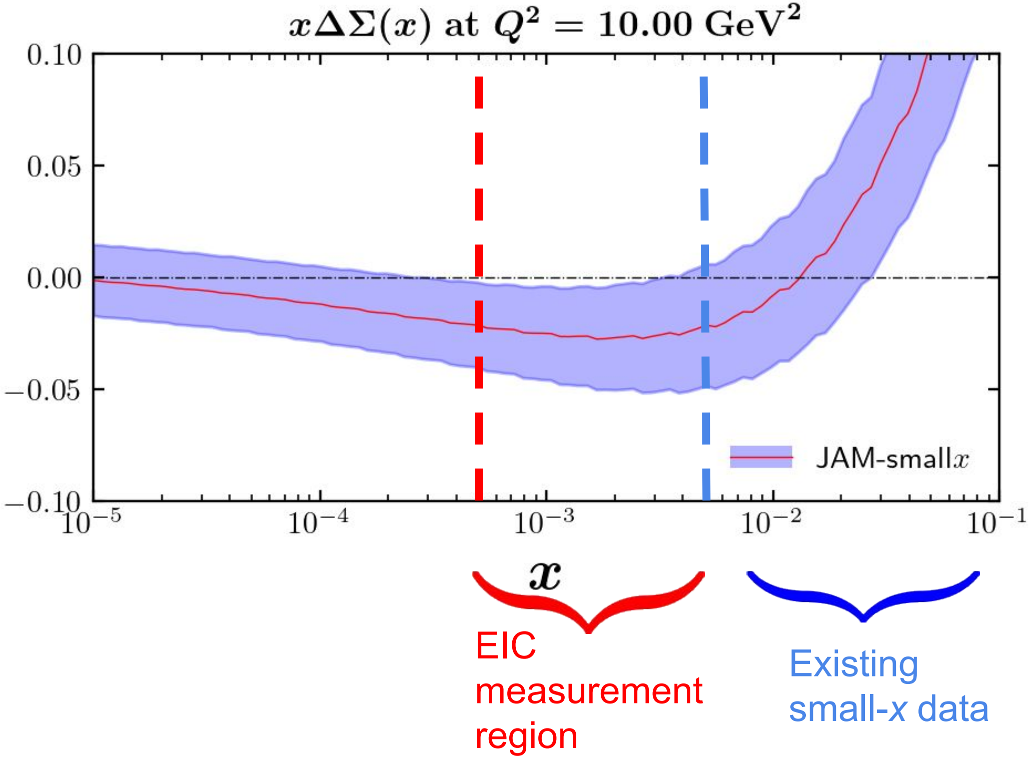


Large-x region

$$\int_{0.01}^{0.7} dx\Delta\Sigma(x) = \pm 0.36$$

Contribution from Quark Spin $\Delta\Sigma = \sum_q \Delta q^+(x, Q^2)$

Saturation region



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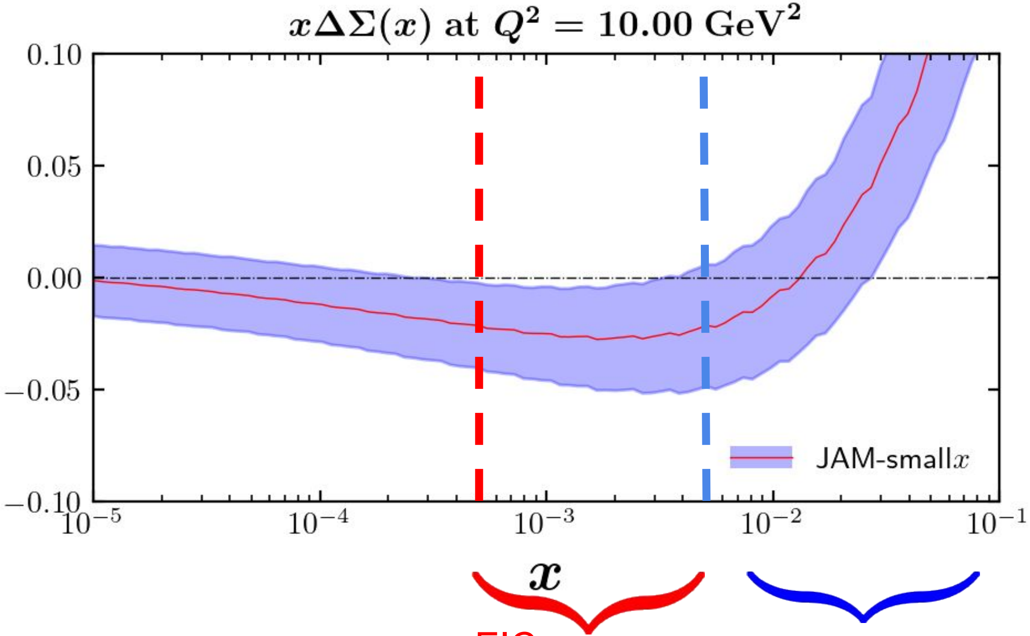

Large-*x* region

$$\int_{0.01}^{0.7} dx \Delta\Sigma(x) = \pm 0.36$$

Compare with:

$$\int_{10^{-5}}^{10^{-3}} dx \Delta\Sigma(x) = -0.1 \pm 0.1$$

Constraining the rest of the Polarized Dipole Amplitudes

$$g_1^{p,n} \sim Q_u, Q_d, Q_s, G_2$$

- 2 observables, 4 polarized dipole amplitudes. Under constrained system

$$g_1^h \sim Q_q, G_2, Q_q^{NS}$$

- 8 new observables, 3 new polarized dipole amplitudes. Exactly constrained - but $\tilde{G}$ does not enter directly into observables

$$pp \to jets \sim G_2, \tilde{G}$$

- Particle production might provide final constraints

Conclusions

- In order to resolve the spin puzzle, the small-*x* behaviour of the hPDFs need to be understood
- This is accomplished using small-*x* evolution
- Along with fitting to data
- Potentially a significant amount if spin is hiding in the small-*x* region
- More work needs to be done to constrain small-*x* behavior of the various polarized dipoles - especially G_2 and $\tilde{G}$
- Could be constrained by studying particle production in *pp* collisions