Trace anomaly form factors from lattice QCD

arXiv:2401.05496 [hep-lat]

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Jefferson Lab Theory Seminar
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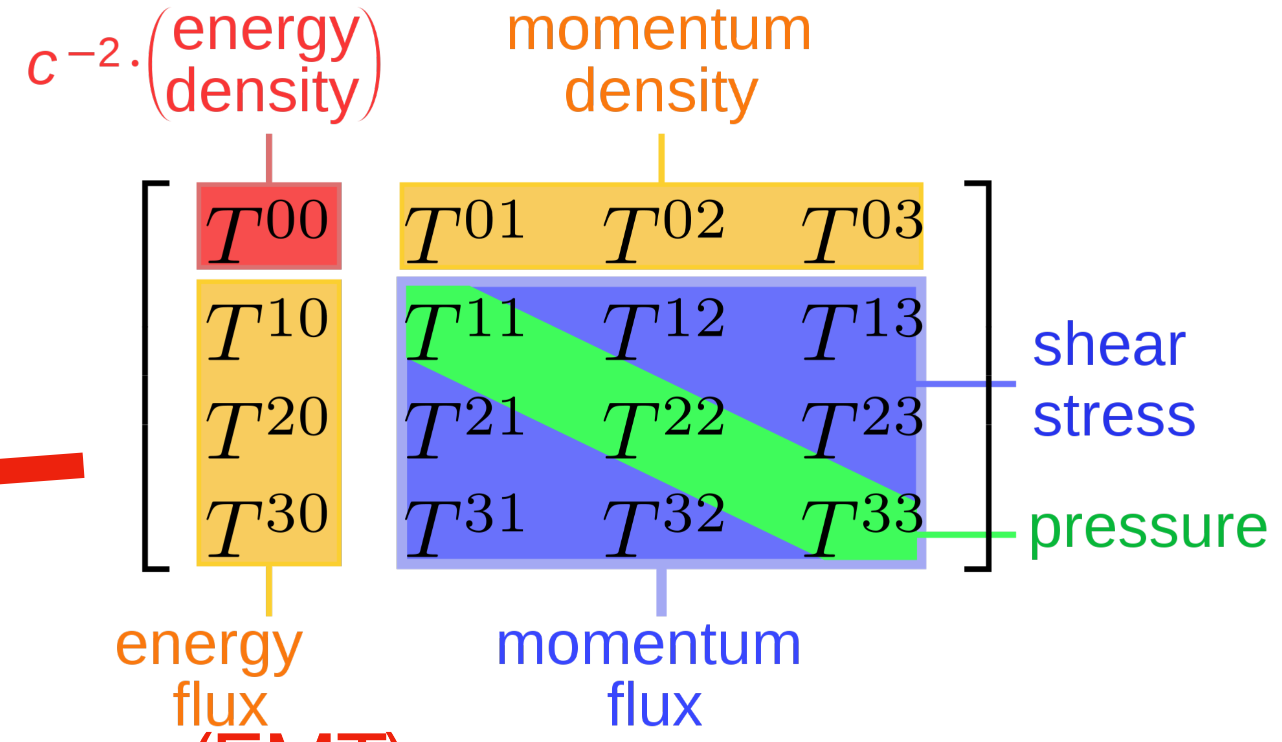
Outline

- Introduction
  - Scale invariance(?), the trace anomaly and hadron mass
  - The “pion mass puzzle” & the mass distribution in the pion
  - Radius and spatial distribution of the trace anomaly
- Calculation of the QCD trace anomaly from lattice QCD (glue part)
  - Numerical setup
  - Extract form factors from 2-point, 3-point correlation functions
- Results
  - Trace anomaly form factors, mass radii, and spatial distributions
- Conclusion and outlook

Scale invariance, trace anomaly and hadron mass

- The Standard Model $\mathscr{L}_{\mathrm{SM}}$ has symmetries (invariances under transformations):
  - Noether's theorem: symmetry → conserved current J^μ: $\partial_\mu J^\mu = 0 \rightarrow$ time independent classical charge $\dot{Q} = 0$
  - Non-symmetries: (classically) look like symmetries but actually not!↔ **anomaly**
    - Scale invariance at $m_q = 0$: $\psi(x) \rightarrow \lambda^{3/2}\psi(\lambda x), A^a_\mu(x) \rightarrow \lambda A^a_\mu(\lambda x)$, one can find $\mathscr{L} \rightarrow \mathscr{L}' = \lambda^4 \mathscr{L}(\lambda x), S \rightarrow S$
      - This is not true with the quantum effects (from QCD): $\partial_\mu J^\mu \neq 0 \leftrightarrow$ **anomaly**

Figure from Wikipedia



Energy-momentum tensor (EMT)

- Why do we call it **trace** anomaly?
  - Corresponding Noether current of the classical scale invariance is:

$$J^\mu_{\mathrm{scale}} = x_\nu T^{\mu\nu}.$$

  - The conserved current J^μ_{scale}:

$$\partial_\mu J^\mu_{\mathrm{scale}} = \partial_\mu (x_\nu T^{\mu\nu}) = (\partial_\mu x_\nu) T^{\mu\nu} + (\partial_\mu T^{\mu\nu}) x_\nu = T^\mu_\mu \neq 0$$

trace anomaly

- What are the **physics** related to it?

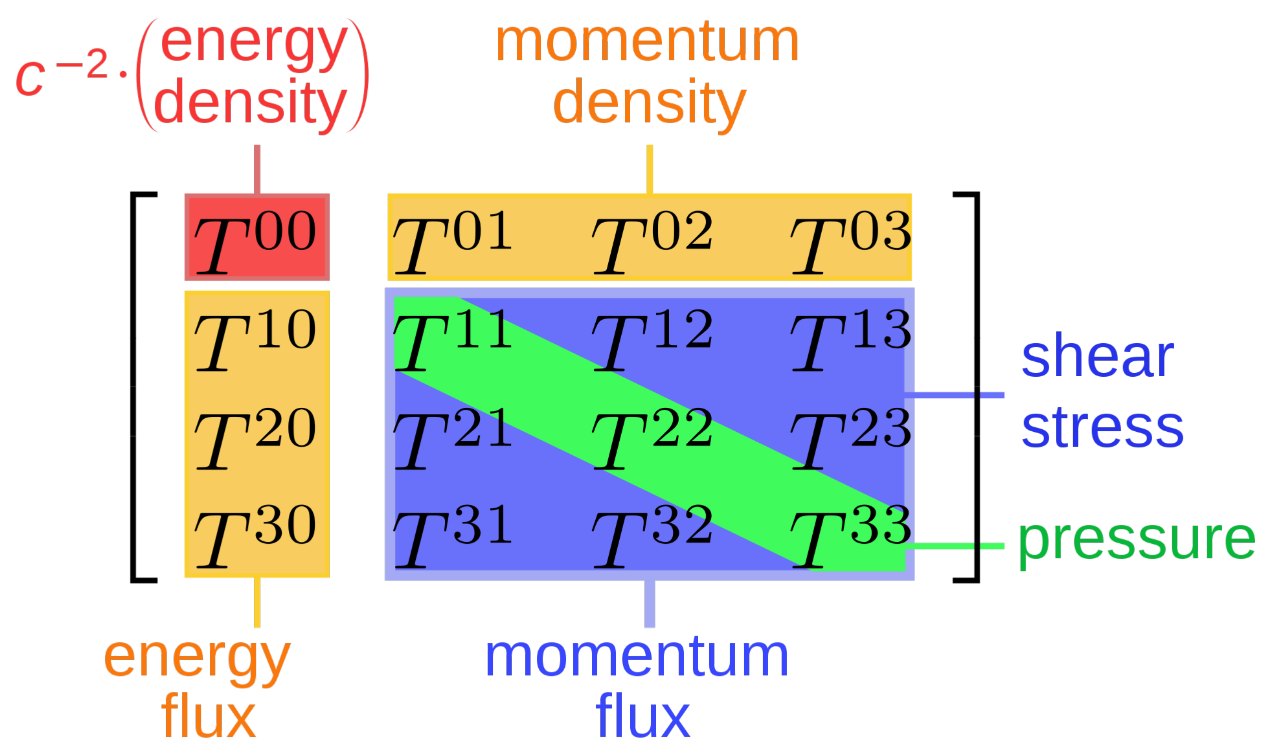
Scale invariance, trace anomaly and hadron mass

- What are the **physics** related to the **trace anomaly**? Hadron mass!
  - the matrix element of the EMT: $\langle \mathrm{H(k)} | T^{\mu\nu} | \mathrm{H(k)} \rangle = 2k^\mu k^\nu / 2m_\mathrm{H}$
  - the trace of EMT yields the mass of the hadron:

$$\langle \mathrm{H(k)} | T^\mu_\mu | \mathrm{H(k)} \rangle = m_\mathrm{H}$$

  - If $T^\mu_\mu = 0$, hadrons are massless
  - However, $T^\mu_\mu \neq 0$ and all hadrons are NOT massless:

Figure from Wikipedia



$$\langle \mathrm{H(k)} | T^\mu_\mu | \mathrm{H(k)} \rangle = m_\mathrm{H} = \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_\mathrm{H}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_\mathrm{H}}_{\langle (T^\mu_\mu)^\mathrm{a} \rangle \text{ trace anomaly}}$$

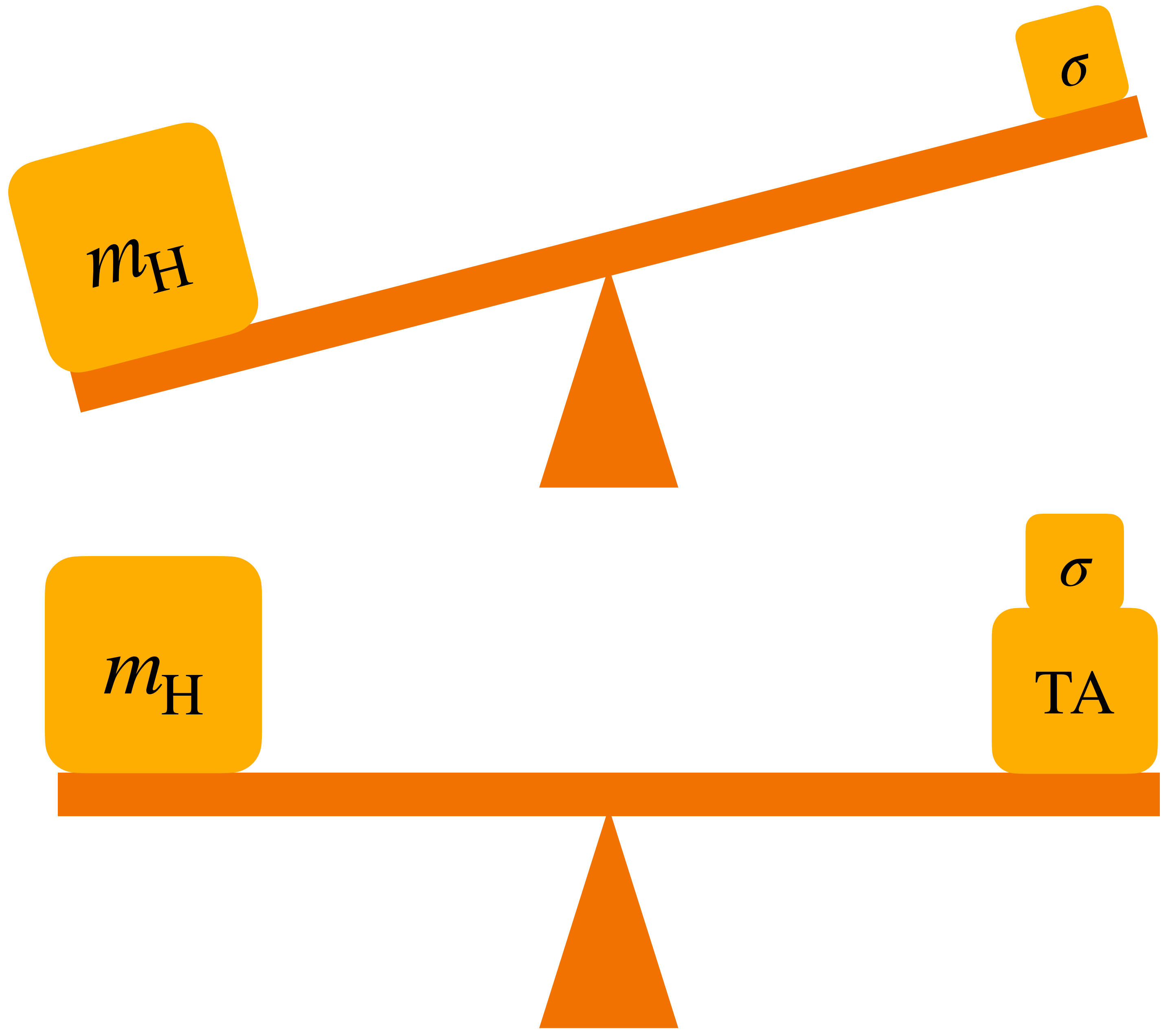
from dimensional regularization

R. Tarrach, Nucl. Phys. B 196, 45 (1982)

Scale invariance, trace anomaly and hadron mass

$$m_{\rm H} = \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\rm H}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\rm H}}_{\langle (T^\mu_\mu)^{\rm a} \rangle \text{ trace anomaly}}$$

RG Invariant

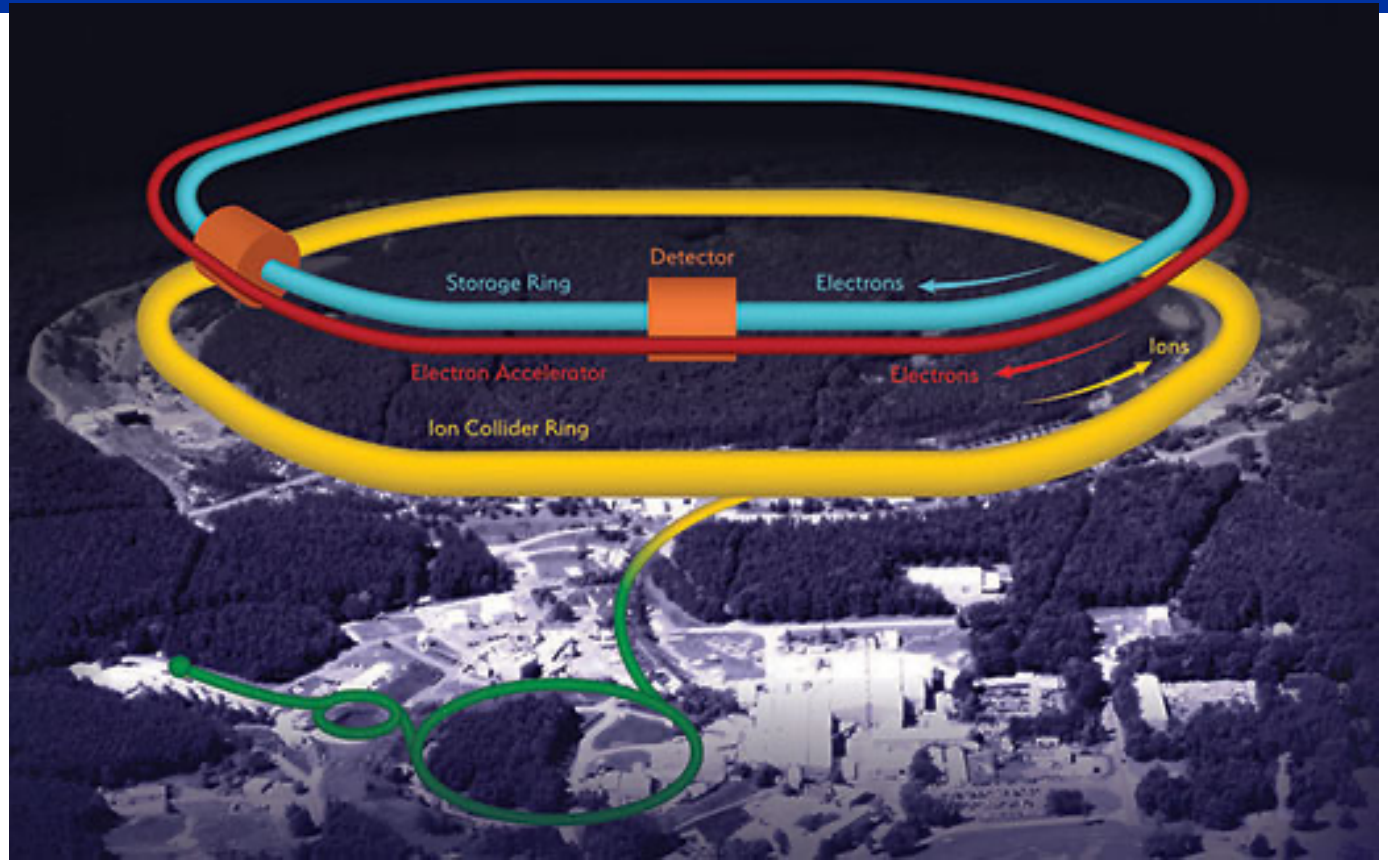


- **Trace anomaly:** a very important term in the hadron mass!
  - For the **nucleon**, the σ term is small (i.e. ~ 8.5% of the nucleon mass), **trace anomaly dominates**. [Liu, PhysRevD.104.076010] [YB Yang, et al. Phys. Rev. Lett. 121, 212001 (2018)]
  - For the **pion**, the σ term contributes **half of the pion mass.** (*Gellmann-Oakes-Renner relation* and *Feynman-Hellman theorem*)therefore the **trace anomaly** term contributes **another half.**

Scale invariance, trace anomaly and hadron mass

$$m_{\rm H} = \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\rm H}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\rm H}}_{\langle (T^\mu_\mu)^{\rm a} \rangle \text{ trace anomaly}}$$

RG Invariant



EIC at BNL(https://flic.kr/p/2ncjFe7)

A. e. a. Accardi, The European Physical Journal A 52,268 (2016)

A. Ali et al. (GlueX), Phys. Rev. Lett. 123, 072001 (2019), arXiv:1905.10811 [nucl-ex]

K. A. Mamo and I. Zahed, Phys. Rev. D 106, 086004 (2022), arXiv:2204.08857 [hep-ph]

D. E. Kharzeev, Phys. Rev. D 104, 054015 (2021),arXiv:2102.00110 [hep-ph]

Y. Hatta, D.-L. Yang, Phys. Rev. D 98, 074003 (2018), arXiv:1808.02163 [hep-ph]

- The origin of the nucleon mass:
  one of the major scientific goals of the Electron-Ion Collider (EIC)
- Calculations of the trace anomaly (highly non-perturbative):
  - With lattice QCD

    F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

Motivation: the "pion mass puzzle"

$$m_\pi = \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_\pi}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_\pi}_{\langle (T^\mu_\mu)^{\text{a}} \rangle \text{ trace anomaly}}$$

- Using $SU(2)$ **chiral** perturbation theory:
  - Left hand side: $m_\pi \propto \sqrt{m_f}, \quad for \quad m_f = m_u = m_d$
  - Right hand side: by using *Gellmann-Oakes-Renner relation* and *Feynman-Hellman theorem*
    - the σ term: $\frac{1}{2} m_\pi \propto \sqrt{m_f}$
- Therefore, the trace anomaly term must also $\propto \sqrt{m_f}$.
- Why does the trace anomaly(conformal symmetry breaking) have a chiral-symmetry-related behavior?

  K.-F. Liu, Phys. Rev. D 104, 076010 (2021)

- **What kind of structure** change can facilitate this attribute when approaching the chiral limit? Spatial distributions

  K.-F. Liu, arXiv:2302.11600 [hep-ph]

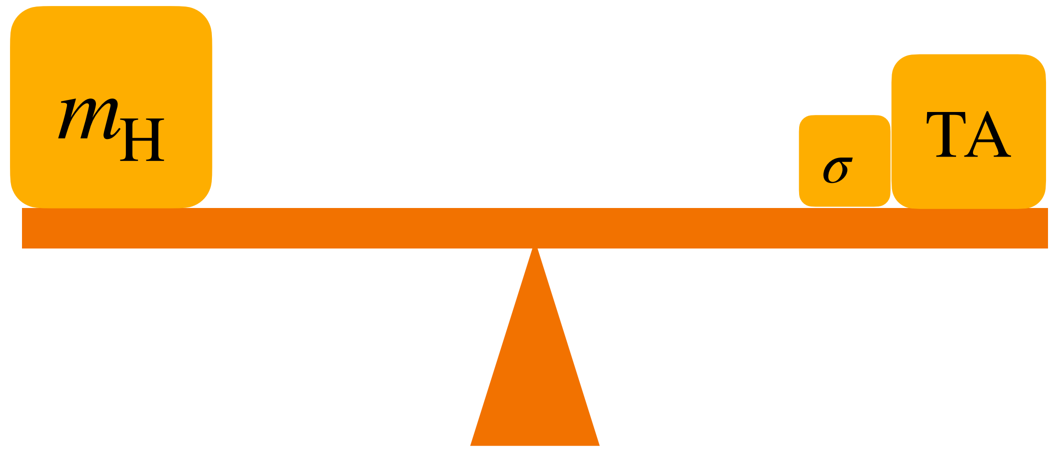
From mass to its spatial distribution

Hadron mass?

Matrix elements

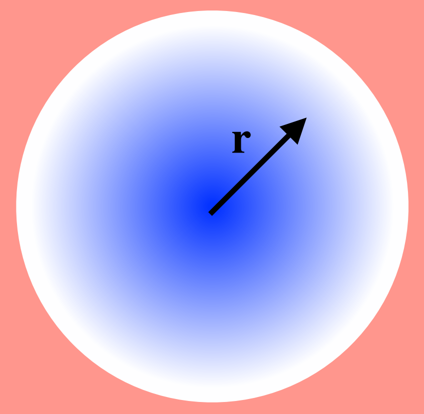
$$\langle p' | T^\mu_\mu | p \rangle$$

$$\underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\rm H}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\rm H}}_{\langle (T^\mu_\mu)^{\rm a} \rangle \text{ trace anomaly,RG invariant}}$$



?

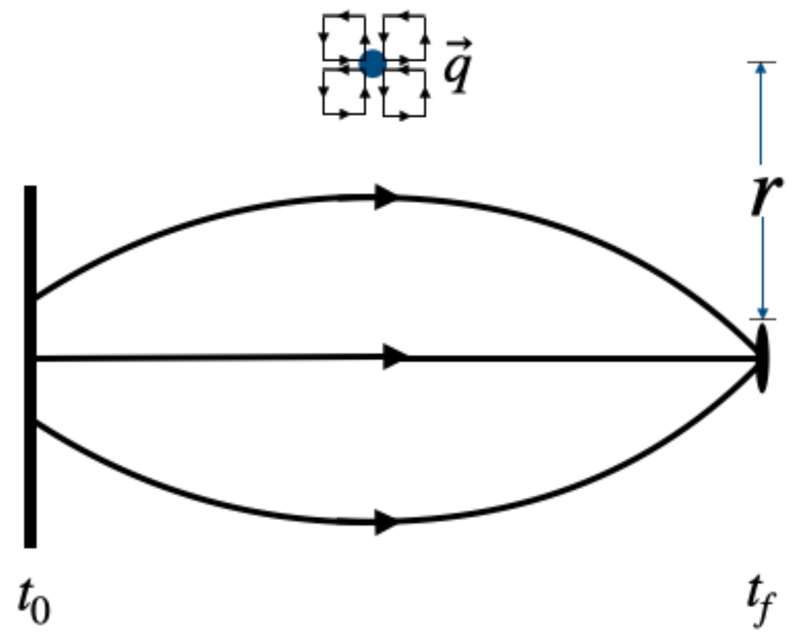
Spatial distributions of mass?

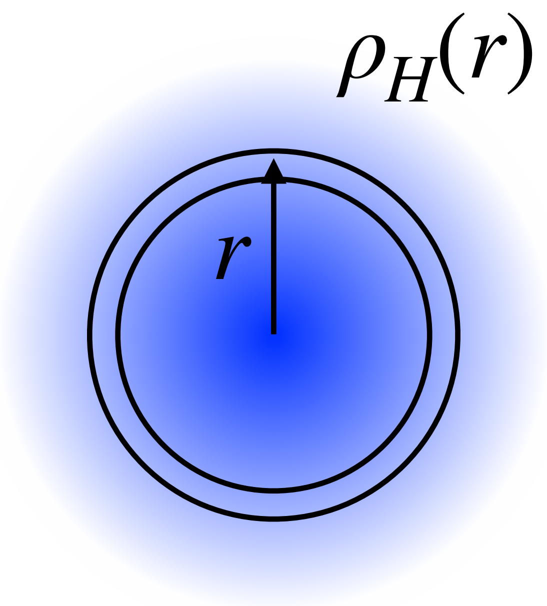


$\langle r^2 \rangle_{\rm m}({\rm H})$

$\rho(\mathbf{r})$

Motivation: the "pion mass puzzle"

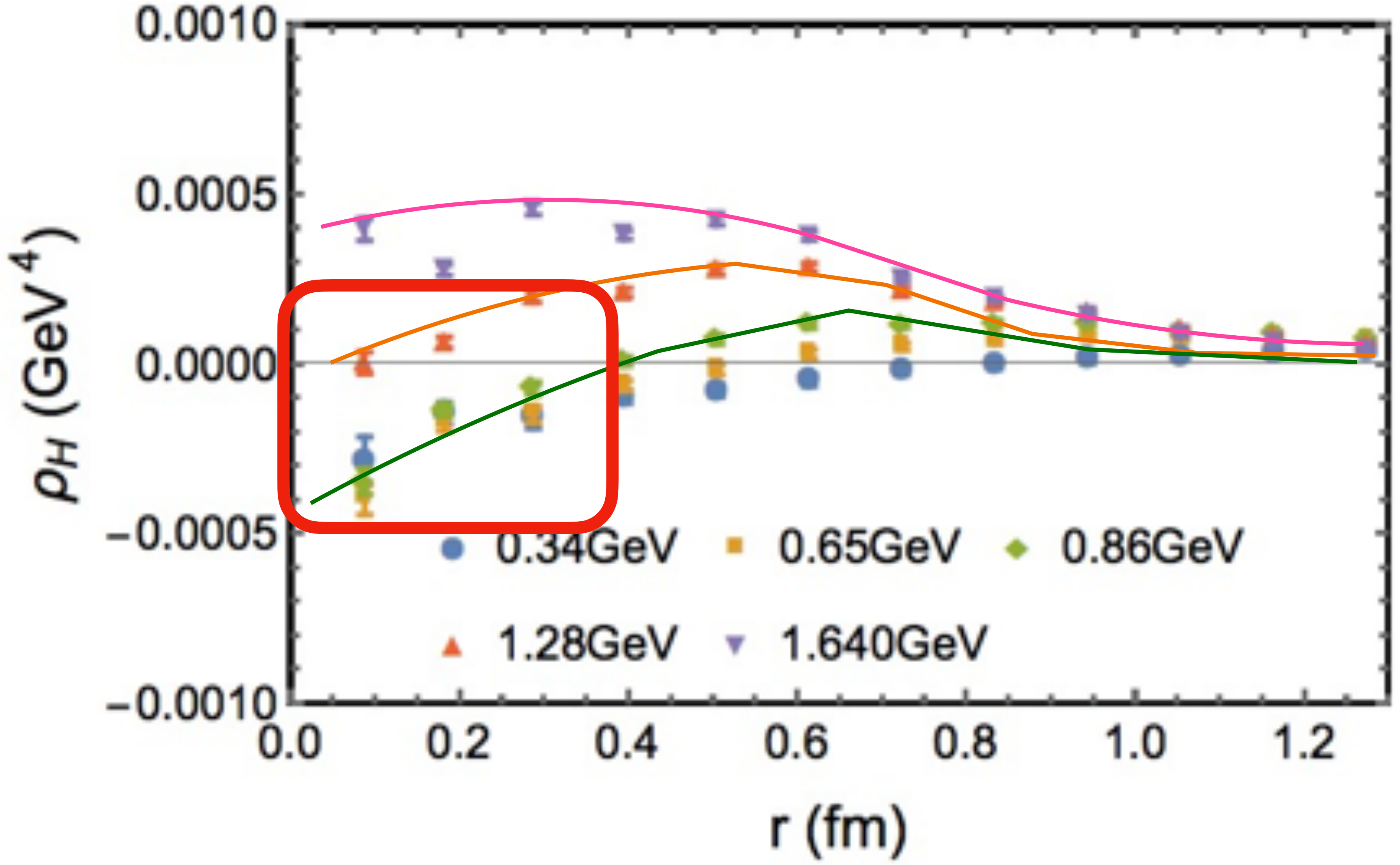




$$\underbrace{m_\pi}_{\propto \sqrt{m_f}} = \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_\pi}_{\text{the } \sigma \text{ term } \propto \sqrt{m_f}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_\pi}_{\langle (T^\mu_\mu)^a \rangle \text{ trace anomaly } \propto \sqrt{m_f}}$$

- What kind of structure change can facilitate this attribute when approaching the chiral limit? **F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)**

- As $m_f \to 0$, the density function $\rho_H(r)$ **changes sign**.



- **Why not calculate the form factors?**
  - Radius, spatial distribution of mass, etc.
  - Experimentally measurable(?)

From the form factor to spatial distribution

Hadron mass?

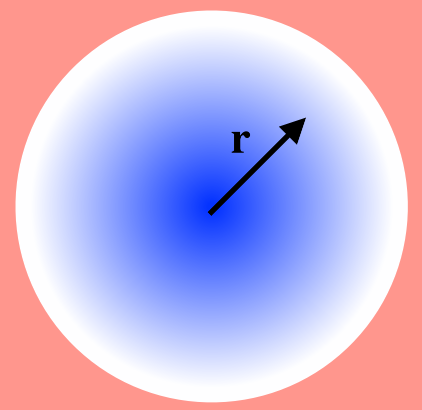
Matrix elements

$\langle p' | T^{\mu}_{\mu} | p \rangle$

Form factors

$\mathscr{F}_{\mathrm{m,H}}(Q^2)$

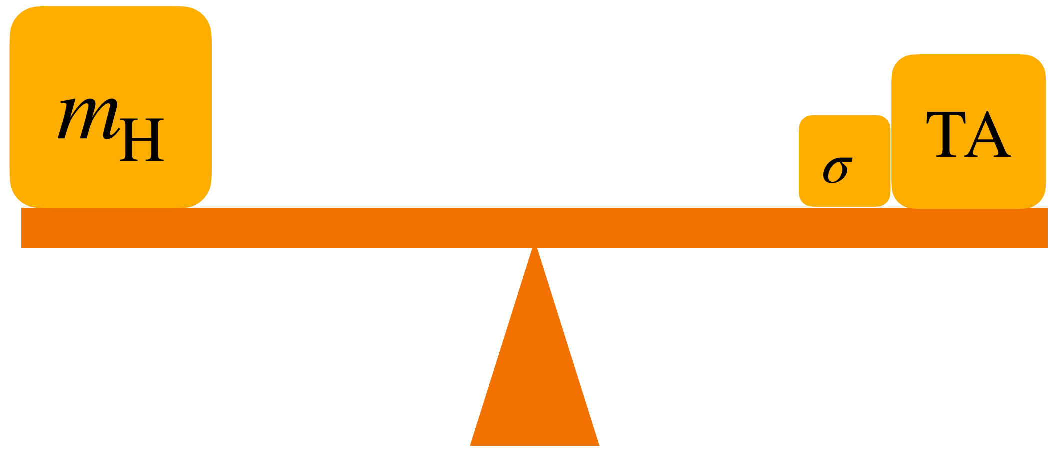
Spatial distributions of mass?



$\rho(\mathbf{r})$

$$\langle r^2 \rangle_{\mathrm{m}}(\mathrm{H}) = -6 \left. \frac{d\mathscr{F}_{\mathrm{m,H}}(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

$$\underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\langle (T^{\mu}_{\mu})^{\mathrm{a}} \rangle \text{ trace anomaly, RG invariant}}$$



- **Why do we calculate the form factors?**
  - Radius, spatial distribution, etc.

    Eg. $G_E(Q^2) \rightarrow$ charge radius
  - Experimentally measurable(?)

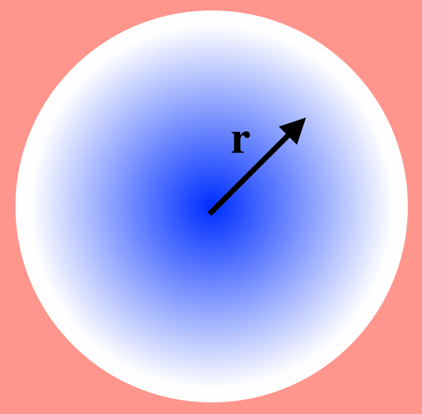
From the matrix element to form factor

Hadron mass?

Spatial distributions of mass?

Matrix elements → **Form factors** →

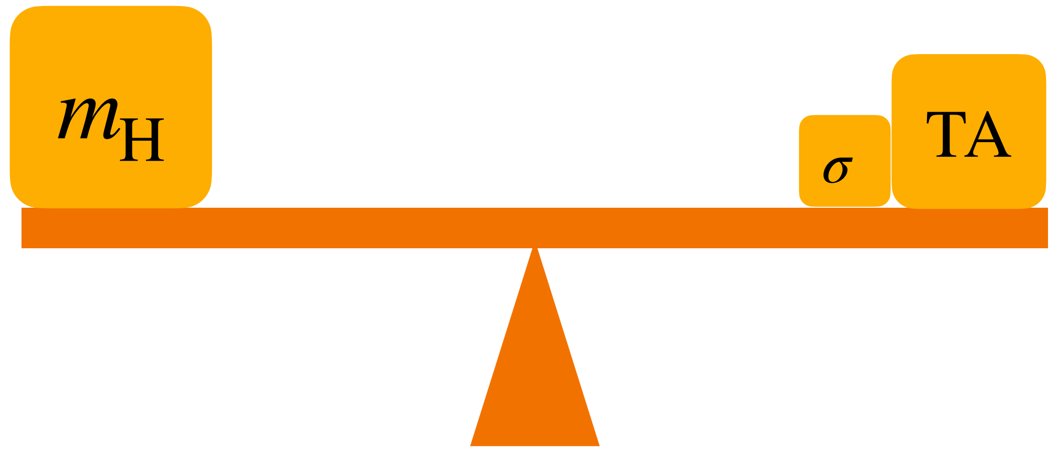
$\langle p' | T^{\mu}_{\mu} | p \rangle$ $\mathscr{F}_{\mathrm{m,H}}(Q^2)$



$\rho(\mathbf{r})$

$$\langle r^2 \rangle_{\mathrm{m}}(\mathrm{H}) = -6 \frac{d\mathscr{F}_{\mathrm{m,H}}(Q^2)}{dQ^2}\bigg|_{Q^2 \to 0}$$

$$\underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\langle (T^{\mu}_{\mu})^{\mathrm{a}} \rangle \text{ trace anomaly,RG invariant}}$$



From the matrix element to form factor

$$T^\mu_\mu = \underbrace{\sum_f m_f \overline{\psi}_f \psi_f}_{\text{the } \sigma \text{ term}} + \underbrace{\frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f}_{\langle (T^\mu_\mu)_{\rm a} \rangle \text{ trace anomaly,RG invariant}}$$

- Normalization convention: $1 = \int \frac{d^3p}{(2\pi)^3} |p\rangle \frac{m}{E_p} \langle p|, \quad |p\rangle = \sqrt{\frac{E_p}{m}} a_p^+ |\Omega\rangle$
- Define a **dimensionless scalar** form factor $\mathcal{F}_{\rm m,H}$, where $Q^2 = -(p'-p)^2$:
    - for spin-$\frac{1}{2}$particle like the nucleon:

$$\langle p', \mathsf{s}' | T^\mu_\mu | p, \mathsf{s} \rangle = m_{\rm N} \mathcal{F}_{\rm m,N}(Q^2) \bar{u}(p', \mathsf{s}') u(p, \mathsf{s}),$$

    - for spin-0 particle like the pion:

matrix element — form factor

$$\langle p' | T^\mu_\mu | p \rangle = m_\pi \mathcal{F}_{\rm m,\pi}(Q^2) \,.$$

- $\mathcal{F}_{\rm m,H}(Q^2 = 0) = 1$: the total mass of hadron $\rm H$ (in percentage).

From the form factor to spatial distribution

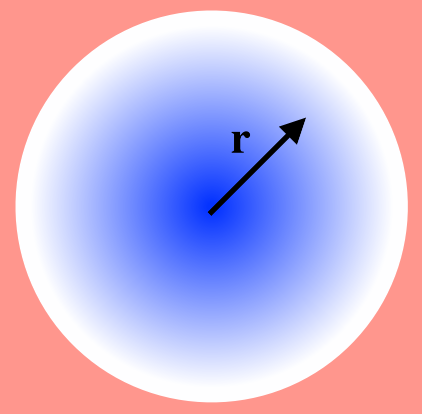
Hadron mass?

Spatial distributions of mass?

Matrix elements → **Form factors** → $\rho(\mathbf{r})$

$\langle p' | T^{\mu}_{\mu} | p \rangle$

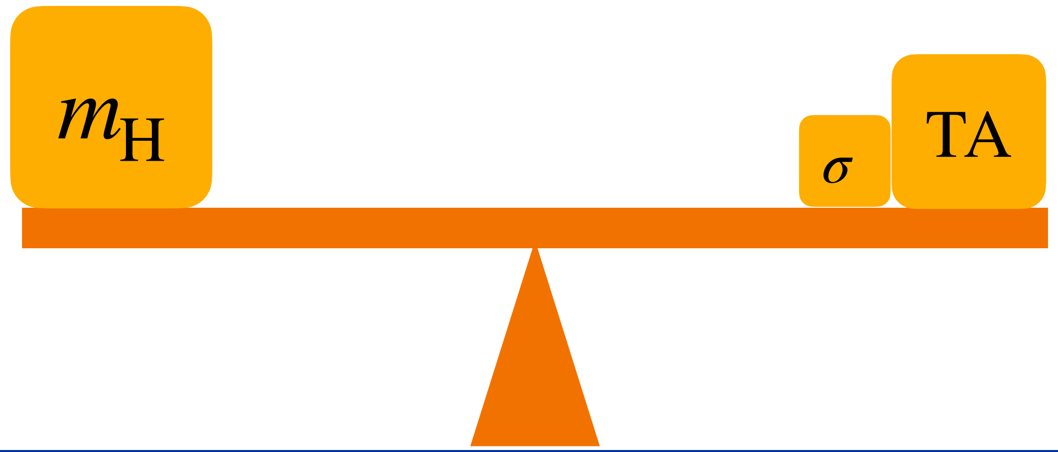
$\mathscr{F}_{\mathrm{m,H}}(Q^2)$



$\rho(\mathbf{r})$

$$\langle r^2 \rangle_{\mathrm{m}}(\mathrm{H}) = -6 \frac{d\mathscr{F}_{\mathrm{m,H}}(Q^2)}{dQ^2}\Bigg|_{Q^2 \to 0}$$

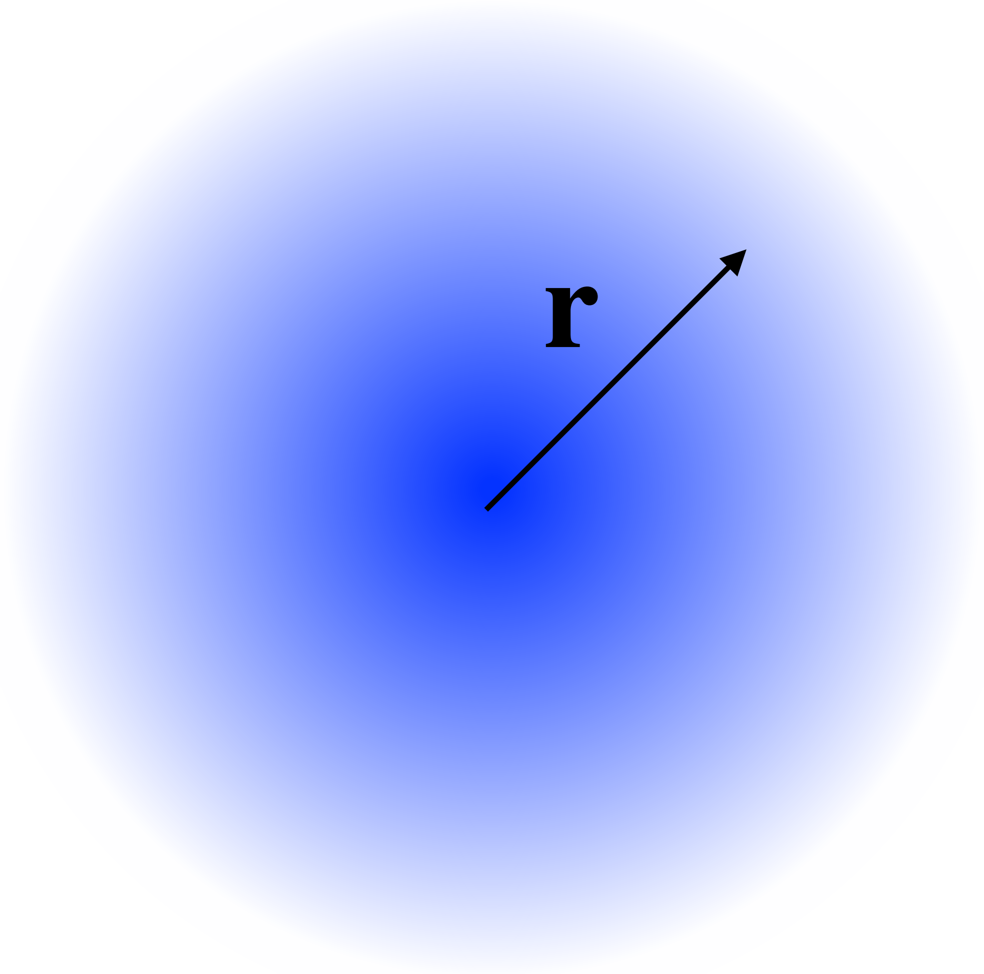
$$\underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\langle (T^{\mu}_{\mu})^{\mathrm{a}} \rangle \text{ trace anomaly, RG invariant}}$$



From the form factor to spatial distribution

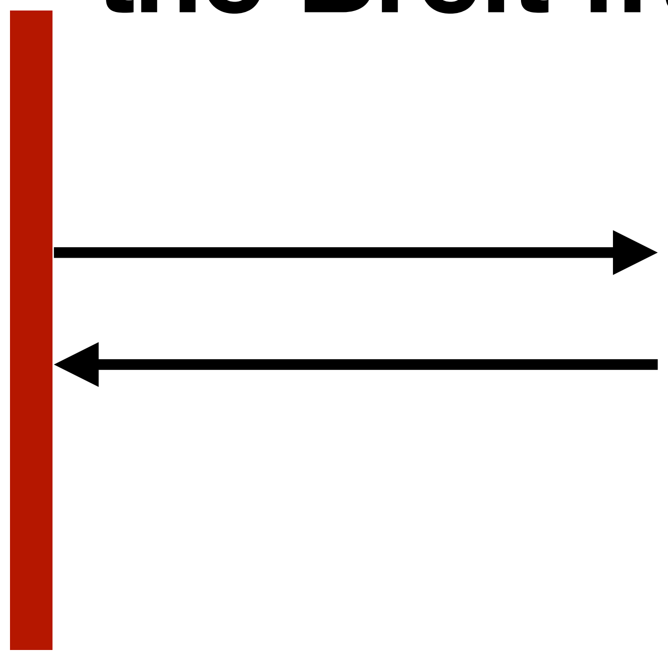
- Form factors $\mathscr{F}_{\mathrm{m,H}}(Q^2)$ R. G. Sachs, Phys. Rev. 126, 2256 (1962)

- Mass radius $\langle r^2\rangle_{\mathrm{m}}(\mathrm{H}) = -6 \left.\frac{d\mathscr{F}_{\mathrm{m,H}}(Q^2)}{dQ^2}\right|_{Q^2\to 0}$



- 3-dimensional spatial distribution in

the Breit frame



$$\Delta^0 = 0$$

$$\mathbf{P} = (\vec{p} + \vec{p}')/2 = \mathbf{0}$$

$$\Delta = p - p' \qquad \mathrm{P} = p + p'$$

$$\rho_{\mathrm{H}}(\mathbf{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{m_{\mathrm{H}}}{E_{\mathrm{H}}} e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}} \mathscr{F}_{\mathrm{m,H}}(Q^2)$$

only valid for the non-relativistic systems

$\Delta \ggg 1/m$

R. L. Jaffe, Phys. Rev. D 103, 016017 (2021)

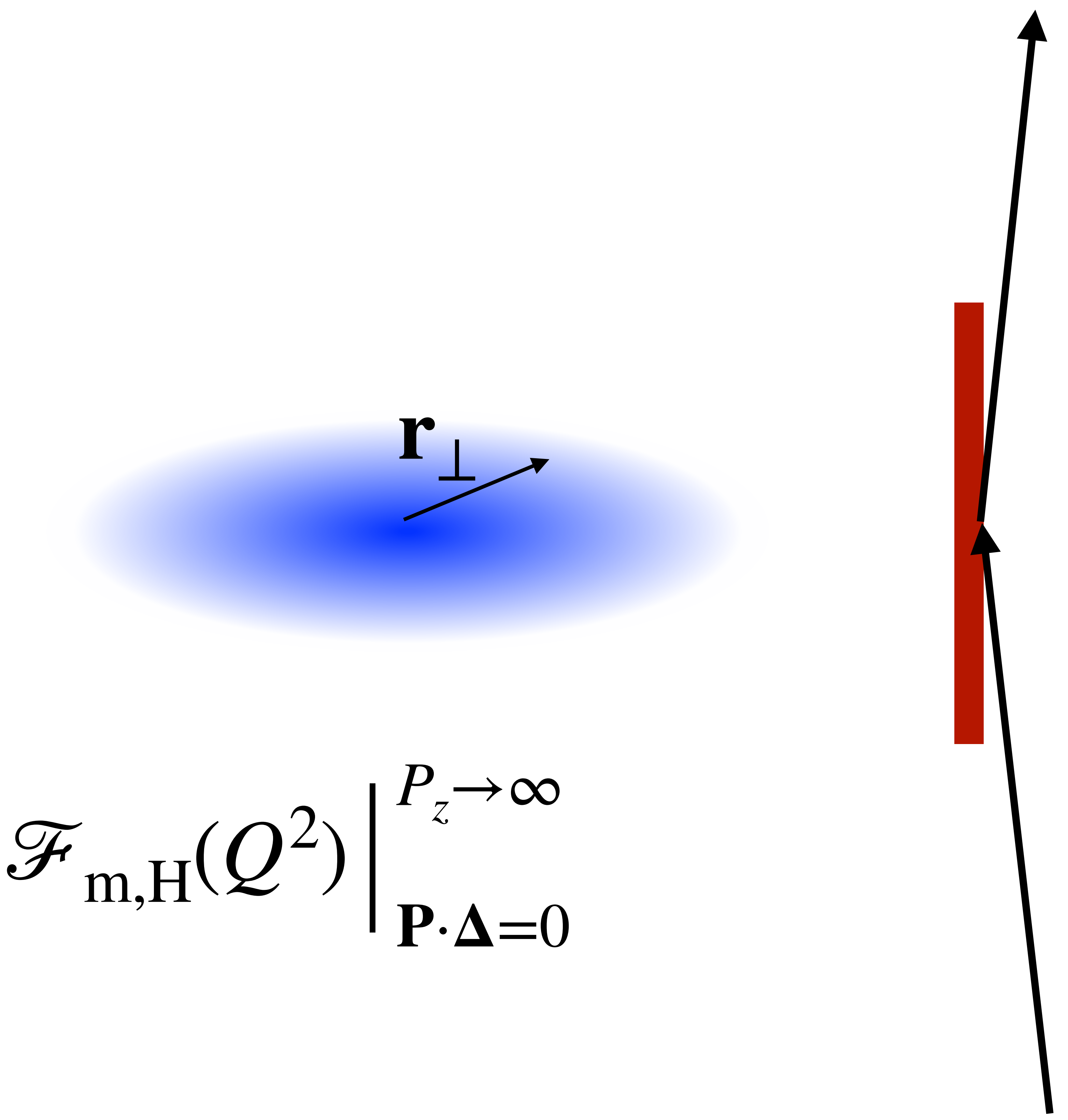
From the form factor to spatial distribution

- Form factors $\mathscr{F}_{\mathrm{m,H}}(Q^2)$
- Mass radius $\langle r^2 \rangle_{\mathrm{m}}(\mathrm{H}) = -6 \left. \frac{d\mathscr{F}_{\mathrm{m,H}}(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$
- 2-dimensional spatial distribution in

the infinite momentum frame (IMF)

$P_z \to \infty$ and $\mathbf{P} \cdot \mathbf{\Delta} = 0$

$$\rho_{\mathrm{H}}^{\mathrm{IMF}}(\mathbf{r}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{r}_\perp} \mathscr{F}_{\mathrm{m,H}}(Q^2) \Big|_{\mathbf{P}\cdot\mathbf{\Delta}=0}^{P_z\to\infty}$$



$\Delta = p - p'$ $\quad$ $\mathrm{P} = p + p'$

C. Lorcé, H. Moutarde, and A. P. Trawiński, Eur. Phys. J. C 79, 89 (2019), arXiv:1810.09837 [hep-ph]

From the matrix element to form factor

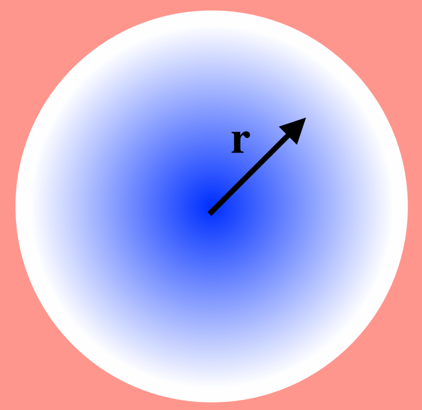
Hadron mass?

Spatial distributions of mass?

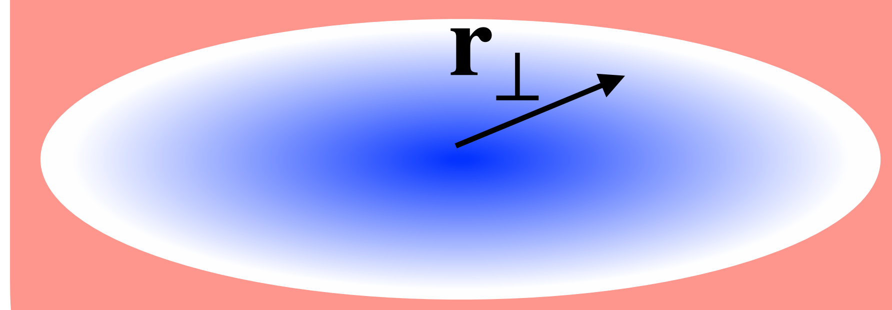
Matrix elements → Form factors →

$\langle p' | T^{\mu}_{\mu} | p \rangle$

$\mathcal{F}_{\mathrm{m},\pi}(Q^2)$

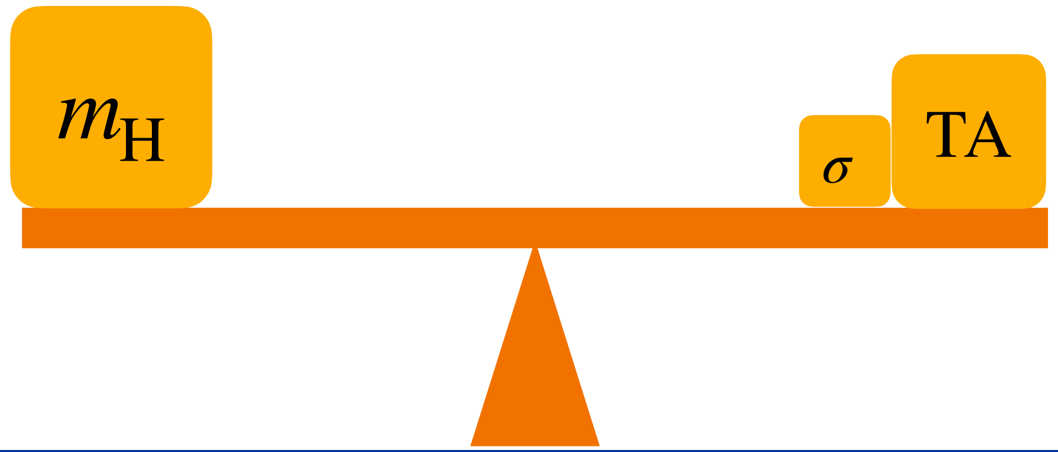


$$\langle r^2 \rangle_{\mathrm{m}}(\mathrm{H}) = -6 \frac{d\mathcal{F}_{\mathrm{m,H}}(Q^2)}{dQ^2}\bigg|_{Q^2\to 0}$$



$\rho^{\mathrm{IMF}}(\mathbf{r}_\perp)$

$$\underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\langle (T^{\mu}_{\mu})^{\mathrm{a}} \rangle \text{ trace anomaly, RG invariant}}$$



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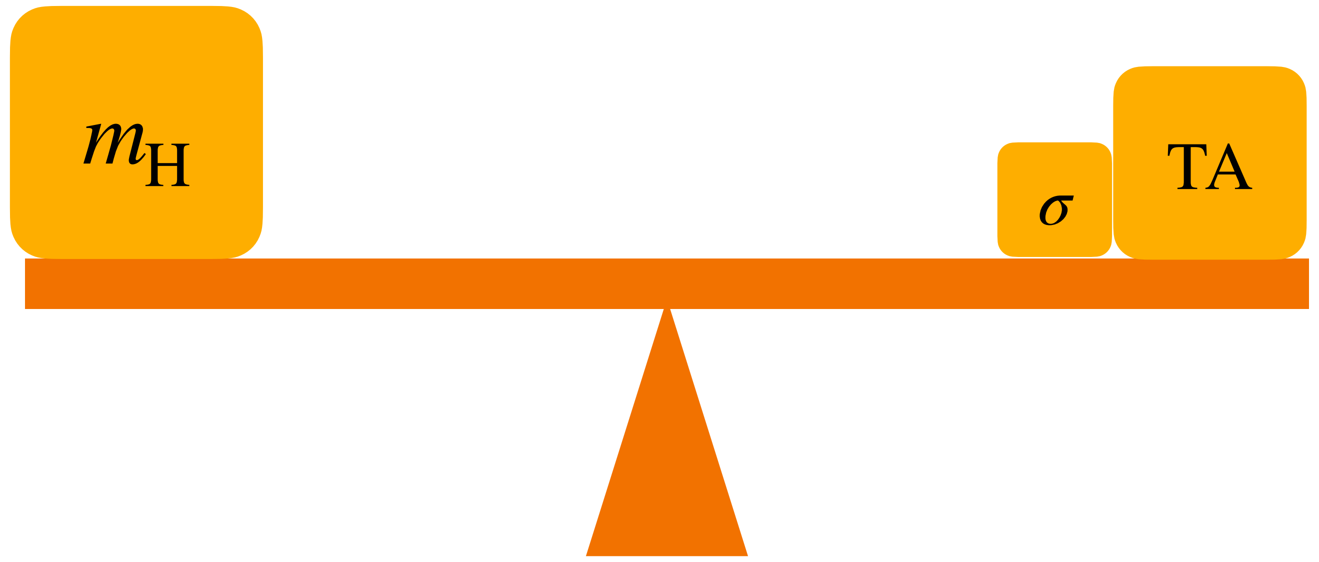
Trace anomaly form factors

$$\langle \mathrm{H}(P') \,|\, T^{\mu}_{\mu} \,|\, \mathrm{H}(P)\rangle = \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\langle (T^{\mu}_{\mu})^{\mathrm{a}} \rangle \text{ trace anomaly, RG invariant}}$$

$$\langle \frac{\beta}{2g} F^2 \rangle \rightarrow G_{\mathrm{H}}(Q^2)$$

$$\mathcal{F}_{\mathrm{m,H}}(Q^2) = \mathcal{F}_{\sigma,\mathrm{H}}(Q^2) + \mathcal{F}_{\mathrm{ta,H}}(Q^2)$$



- For the nucleon, the σ term is small (i.e. ~ 8.5% of the nucleon mass), the glue part of the trace anomaly dominates. **[Liu, PhysRevD.104.076010]** **[YB Yang, et al. Phys. Rev. Lett. 121, 212001 (2018)]**
- For the pion, the trace anomaly term $\sim \frac{1}{2} m_\pi$ (*Gellmann-Oakes-Renner relation* and *Feynman-Hellman theorem*) assuming γ_m is not large, the glue part dominates the trace anomaly term.
- In this work, we calculate the **glue** trace anomaly form factors $G_{\mathrm{H}}(Q^2)$

Trace anomaly form factors (glue)

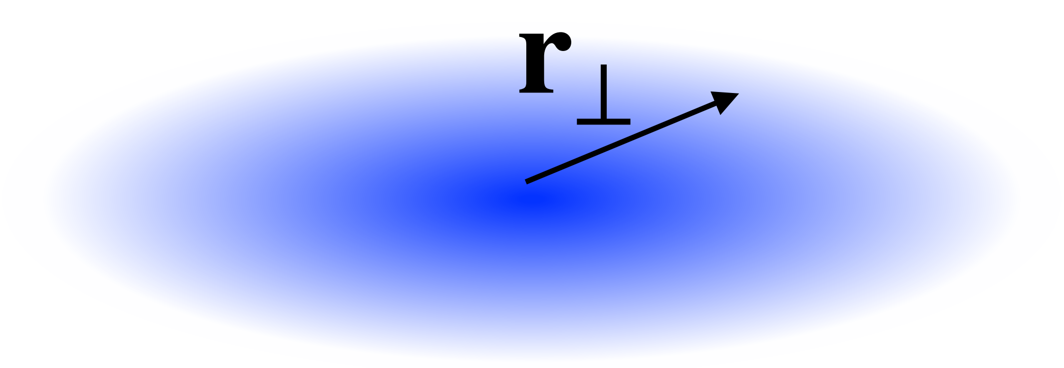
- Form factors $\mathscr{F}_{\mathrm{m,H}}(Q^2) = \mathscr{F}_{\sigma,\mathrm{H}}(Q^2) + \mathscr{F}_{\mathrm{ta,H}}(Q^2)$

$\mathscr{F}_{\mathrm{ta,H}}(Q^2) \rightarrow G_{\mathrm{H}}(Q^2)$

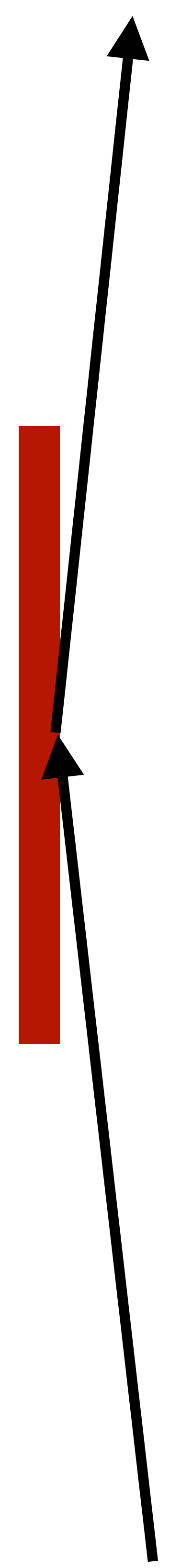
- Mass radius $\langle r^2 \rangle_{\mathrm{m}}(\mathrm{H}) = -6 \left. \dfrac{d\mathscr{F}_{\mathrm{m,H}}(Q^2)}{dQ^2} \right|_{Q^2 \to 0} \sim -6 \left. \dfrac{dG_{\mathrm{H}}(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$

- 2-dimensional spatial distribution in the infinite momentum frame (IMF)

$P_z \to \infty$ and $\mathbf{P} \cdot \mathbf{\Delta} = 0$



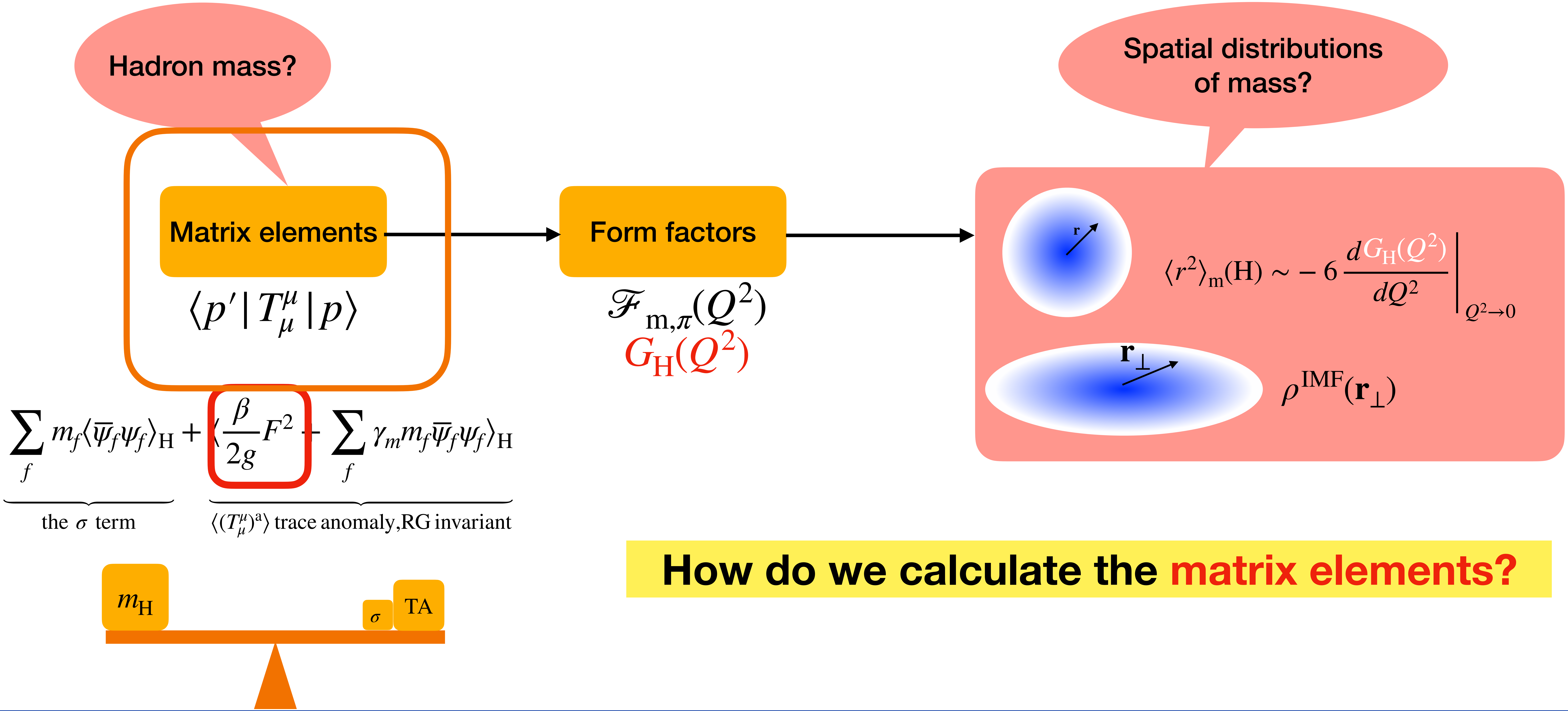
$$\rho_{\mathrm{H}}^{\mathrm{IMF}}(\mathbf{r}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{r}_\perp} \tilde{G}_{\mathrm{H}}(Q^2) \Big|_{\substack{P_z \to \infty \\ \mathbf{P}\cdot\mathbf{\Delta}=0}}$$



$\Delta = p - p' \qquad \mathrm{P} = p + p'$

C. Lorcé, H. Moutarde, and A. P. Trawiński, Eur. Phys. J. C 79, 89 (2019), arXiv:1810.09837 [hep-ph]

From the form factor to spatial distribution



How do we calculate the matrix elements?

The path integral in the Minkowski space

Input parameters

$m_q, \ldots, \alpha_s, \ldots$

- Observables from the path integral (Minkowski):

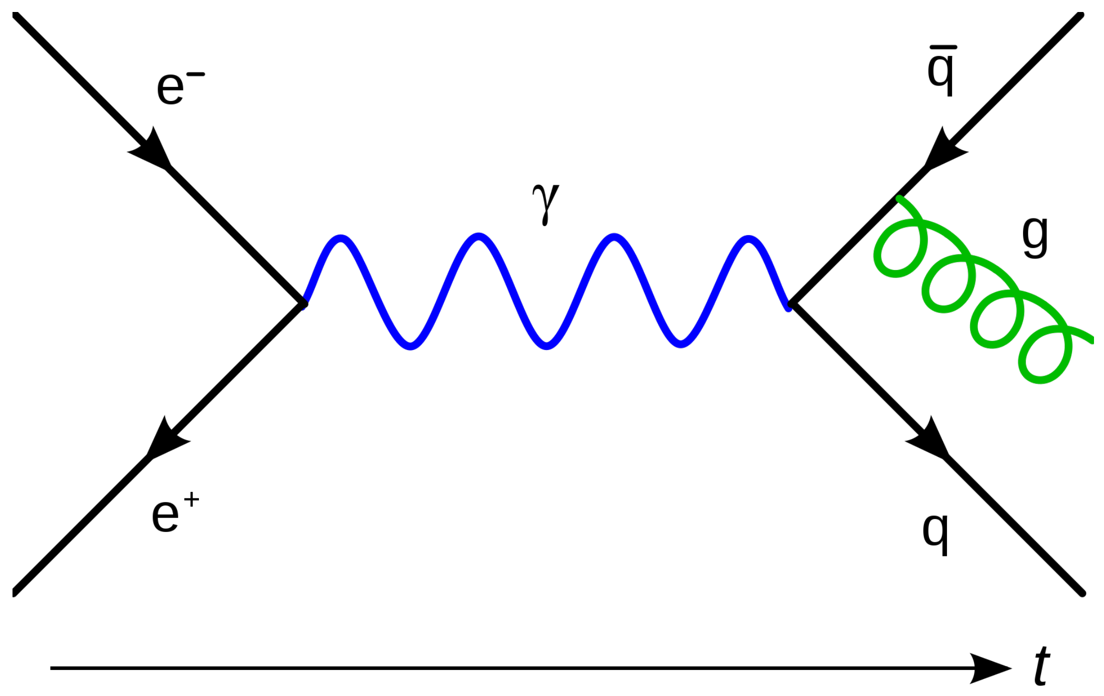
$$\langle O \rangle = \frac{\int \mathscr{D}\phi O e^{iS_{\mathrm{M}}(\phi)}}{\int \mathscr{D}\phi e^{iS_{\mathrm{M}}(\phi)}} \longleftarrow Z(J) = \int \mathscr{D}\phi e^{i\int d^4x[\mathscr{L}_0 + \mathscr{L}_1 + J\phi]}$$

E.g. n-point correlation functions

$$\langle 0 | \mathrm{T}\phi(x_1)\ldots\phi(x_n) | 0 \rangle \longleftarrow \frac{\delta}{i\delta x_1}\cdots\frac{\delta}{i\delta x_n} Z(J)\Big|_{J=0}$$

$\downarrow$

S-matrix, scattering amplitudes, …



By Joel Holdsworth (Joelholdsworth) - Non-Derived SVG of Radiate gluon.png, originally the work of SilverStar at Feynmann-diagram-gluon-radiation.svg, updated by joelholdsworth., Public Domain, https://commons.wikimedia.org/w/index.php?curid=1764161

Feynman diagrams
Feynman rules
+
Perturbative expansions

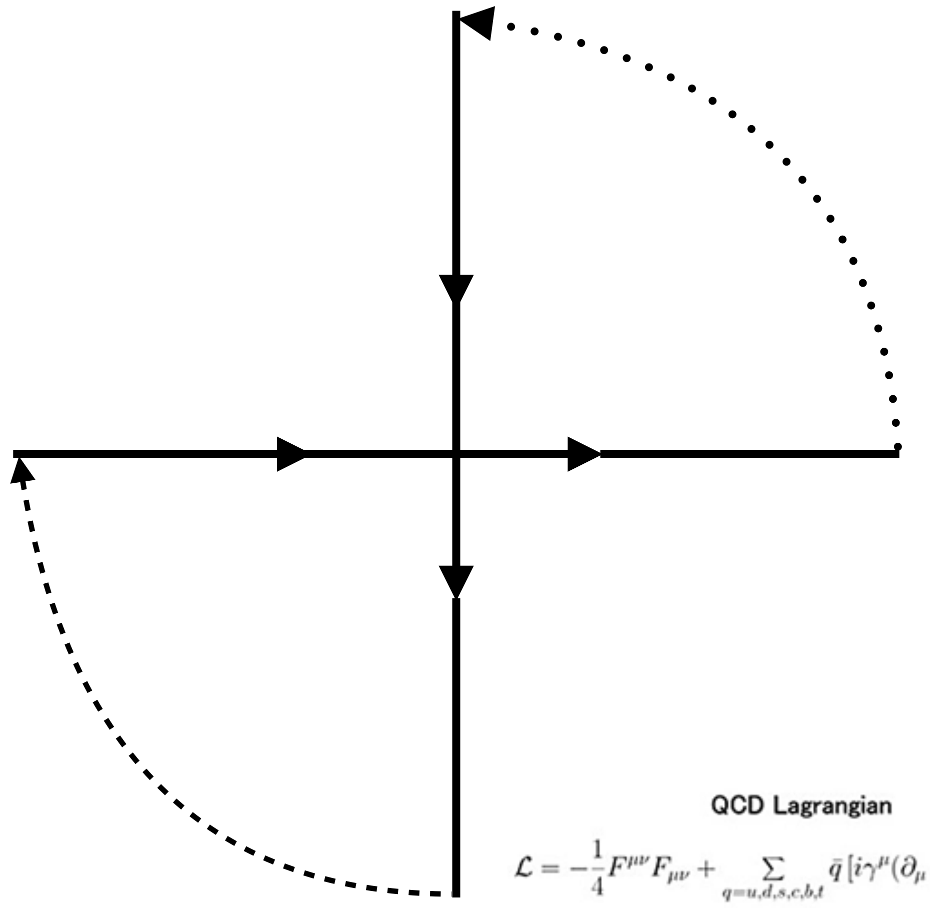
- When the couplings are strong …… $\alpha_s \sim 1$
- Can we calculate without perturbative expansions/non-perturbatively?

The path integral in the Euclidean space

- Observables from the path integral (Minkowski):

$$\langle O \rangle = \frac{\int \mathcal{D}\phi O e^{iS_{\rm M}(\phi)}}{\int \mathcal{D}\phi e^{iS_{\rm M}(\phi)}} \longleftarrow Z_M(J) = \int \mathcal{D}\phi e^{i\int d^4x \mathscr{L}_M}$$

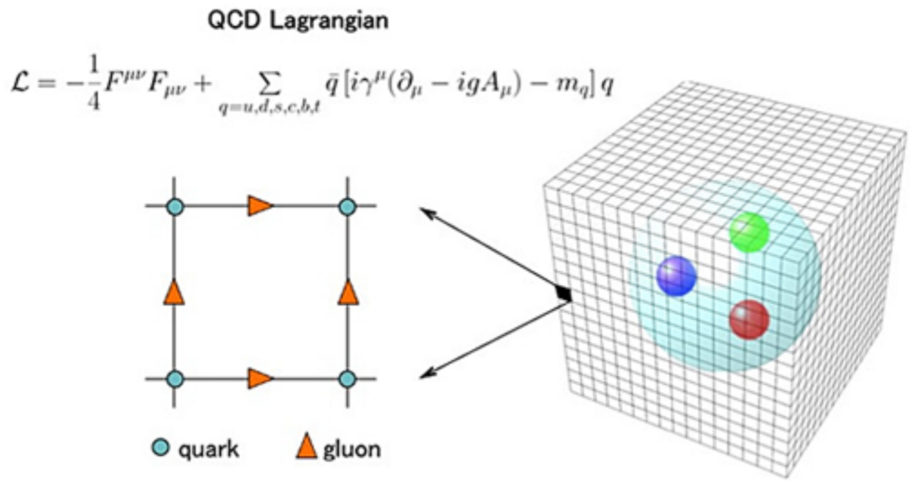
Wick rotation $t \to -i\tau$, $k^0 \to ik_0^E$



- Observables from the path integral (Euclidean):

$$\langle O \rangle = \frac{\int \mathcal{D}\phi O e^{-S_{\rm E}(\phi)}}{\int \mathcal{D}\phi e^{-S_{\rm E}(\phi)}} \longleftarrow Z_E(J) = \int \mathcal{D}\phi e^{-\int d^4x_E \mathscr{L}_E}$$

(partition function in statistical mechanics)



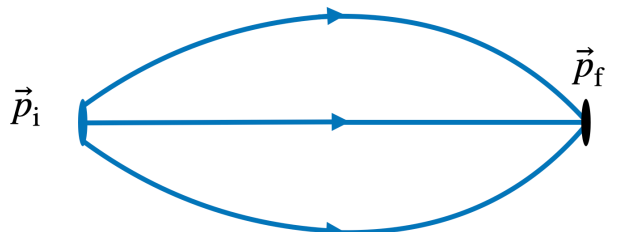
- Ensemble average: with distribution $\propto e^{-S_E(\phi)}$

$$\langle O \rangle \simeq \frac{1}{N}\sum_N O_i \pm \mathcal{O}(\sqrt{N})$$

E.g. n-point correlation functions

$$\langle 0 | \mathrm{T}\phi(x_1)\ldots\phi(x_n) | 0 \rangle$$

Wick's theorem



→ hadron spectrum, matrix elements …

- Discretization of space and time in a finite volume: lattice
- Generate with Monte-Carlo methods with lattice actions $S^{\rm lat}(\phi)$

Non-perturbative

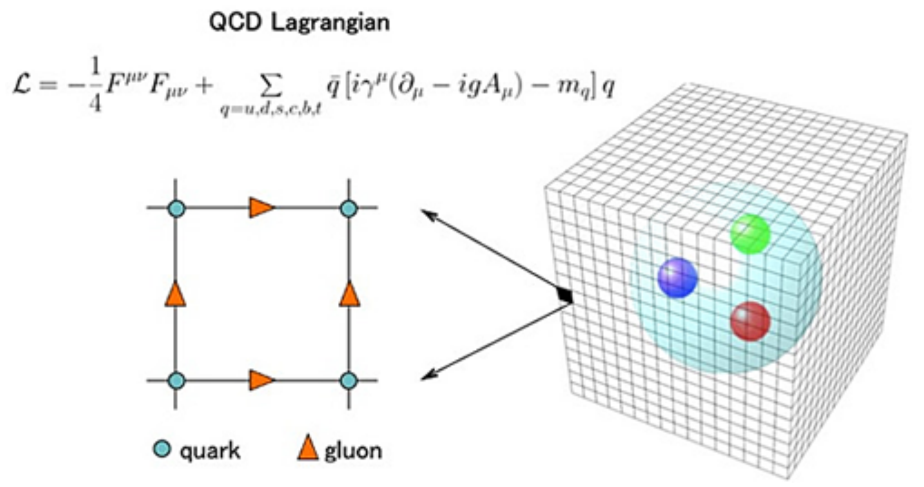
Input parameters
$a, m_q, \ldots, \alpha_s, \ldots$

Numerical setup with lattice QCD

- Observables from the path integral (Euclidean):

$$\langle O \rangle = \frac{\int \mathscr{D}\phi O e^{-S_{\mathrm{E}}(\phi)}}{\int \mathscr{D}\phi e^{-S_{\mathrm{E}}(\phi)}} \longleftarrow \quad Z_E(J) = \int \mathscr{D}\phi e^{-\int d^4 x_E \mathscr{L}_E}$$

(partition function in statistical mechanics)



- Ensemble average: with distribution $\propto e^{-S_{\mathrm{E}}(\phi)}$

$$\langle O \rangle \simeq \frac{1}{N} \sum_N O_i \pm \mathscr{O}(\sqrt{N})$$

- Generate with Monte-Carlo methods with lattice actions $S_{\mathrm{E}}^{\mathrm{lat}}(\phi)$:
  2+1-flavor domain-wall fermion configurations with Iwasaki gauge action

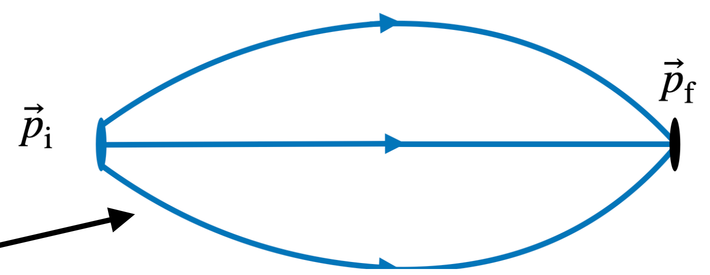
Input parameters
$a, m_q, \ldots, \alpha_s, \ldots$

Ensemble	$L^3 \times T$	a (fm)	L(fm)	m_π (MeV)	N_{conf}
24I	$24^3 \times 64$	0.1105(3)	2.65	340	788

E.g. n-point correlation functions

$$\langle 0 | \mathrm{T}\phi(x_1) \ldots \phi(x_n) | 0 \rangle$$

↓ Wick's theorem



$$S_F(y, j, b; x, i, a) = (D_{\mathrm{ov}}^{-1})_{x,i,a}^{y,j,b}$$

Renormalization → hadron spectrum, matrix elements …, at $m_{\mathrm{v},q}$

Overlap fermion with valence quark masses $m_{\mathrm{v},q}$

Renormalization on the lattice

The trace anomaly emerges with the lattice regulation after renormalization:

$$T^\mu_\mu = \underbrace{\sum_f m_f \overline{\psi}_f \psi_f}_{\text{the } \sigma \text{ term}} + \underbrace{\frac{\beta(g)}{2g} F^2 + \sum_f \gamma_m(g) m_f \overline{\psi}_f \psi_f}_{\langle (T^\mu_\mu)_a \rangle \text{ trace anomaly,RG invariant}}$$

S. Caracciolo, G. Curci, P. Menotti, and A. Pelissetto, Annals of Physics 197, 119 (1990)

H. Makino and H. Suzuki, Progress of Theoretical and Experimental Physics 2014, 063B02 (2014)

M. Dalla Brida, L. Giusti, and M. Pepe, JHEP 04, 043 (2020), arXiv:2002.06897 [hep-lat]

Renormalization method: based on the mass sum rule

F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

- $\frac{\beta(g)}{2g}$ and $\gamma_m(g)$ are independent of the hadron state
- Solve the mass sum rule equations for pseudo-scalar(π) and vector meson(ρ) at one valence mass:

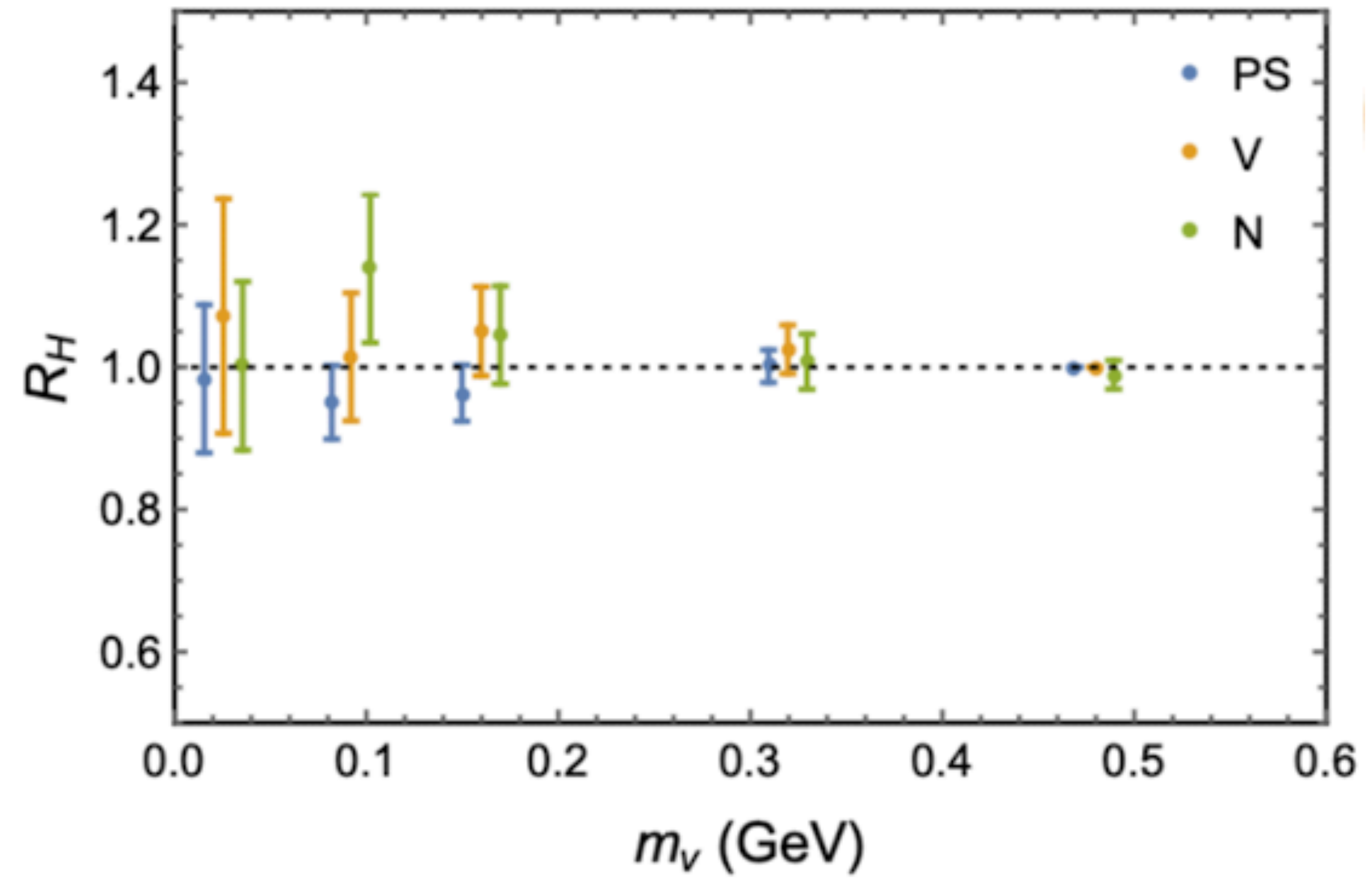
$$M_{\rm PS} - (1+\gamma_m)\langle H_m \rangle_{\rm PS} - \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm PS} = 0,$$

$$M_{\rm V} - (1+\gamma_m)\langle H_m \rangle_{\rm V} - \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm V} = 0$$

and obtain the bare $\frac{\beta(g)}{2g}$ and $\gamma_m(g)$

- Verify the assumptions: sum rule satisfied for other masses Mixing with lower dimensional operators is negligible

$$R_H = \left[(1+\gamma_m)\langle H_m \rangle_{\rm H} + \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm H}\right] / m_{\rm H} \sim 1$$

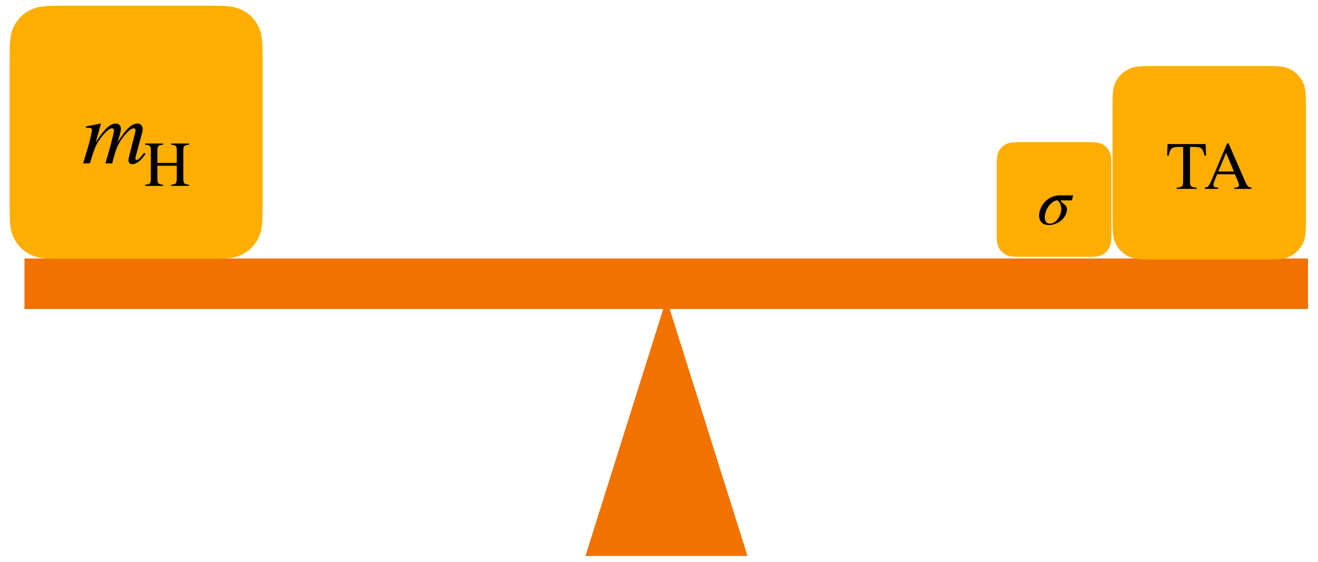


Renormalization on the lattice

The trace anomaly terms need renormalization:

$$T^{\mu}_{\mu} = \underbrace{\sum_f m_f \overline{\psi}_f \psi_f}_{\text{the } \sigma \text{ term}} + \underbrace{\boxed{\frac{\beta(g)}{2g}} F^2 + \sum_f \gamma_m(g) m_f \overline{\psi}_f \psi_f}_{\langle (T^{\mu}_{\mu})_a \rangle \text{ trace anomaly,RG invariant}}$$

Renormalization method: based on the mass sum rule



- $\frac{\beta(g)}{2g}$ and $\gamma_m(g)$ are independent of the hadron state
- Solve the mass sum rule equations for pseudo-scalar(π) and vector meson(ρ) at $m_{\rm v} a \sim 0.3$:

$$M_{\rm PS} - (1+\gamma_m)\langle H_m \rangle_{\rm PS} - \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm PS} = 0,$$

$$M_{\rm V} - (1+\gamma_m)\langle H_m \rangle_{\rm V} - \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm V} = 0$$

and obtain the bare $\frac{\beta(g)}{2g} = -0.08(1)$

F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

$\sim \mathcal{O}(m_{\rm v}^2 a^2)$
discretization error?

Other systematic errors?

- For the nucleon, $\langle H_m \rangle_{\rm N}$ is small.
- Solve the mass sum rule equations for nucleon at $m_{\rm v} a \sim 0.016$:

$$M_{\rm N} - \langle H_m \rangle_{\rm N} - \frac{\beta(g)}{2g}\langle F^2 \rangle_{\rm N} \simeq 0$$

and obtain the bare $\frac{\beta(g)}{2g} = -0.129(6)$

Numerical setup with lattice QCD

- Observables from the path integral (Euclidean):

$$\langle O \rangle = \frac{\int \mathscr{D}\phi O e^{-S_E(\phi)}}{\int \mathscr{D}\phi e^{-S_E(\phi)}} \longleftarrow Z_E(J) = \int \mathscr{D}\phi e^{-\int d^4x_E \mathscr{L}_E}$$

(partition function in statistical mechanics)

- Ensemble average: with distribution $\propto e^{-S_E(\phi)}$

$$\langle O \rangle \simeq \frac{1}{N} \sum_N O_i \pm \mathscr{O}(\sqrt{N})$$

- Generate with Monte-Carlo methods with lattice actions $S_E^{\text{lat}}(\phi)$:
  2+1-flavor domain-wall fermion configurations with Iwasaki gauge action

Input parameters
$a, m_q, \dots, \alpha_s, \dots$

Ensemble	$L^3 \times T$	a (fm)	L(fm)	m_π (MeV)	N_{conf}
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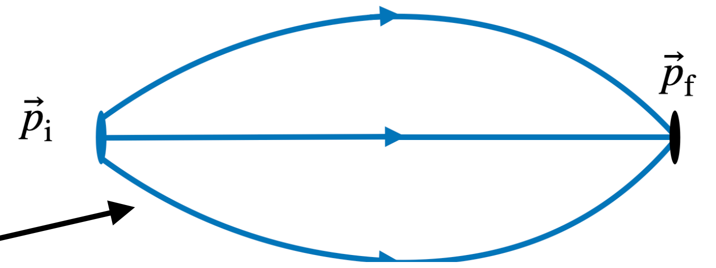
$$\langle 0 | \mathrm{T}\phi(x_1)\dots\phi(x_n) | 0 \rangle$$

↓ Wick's theorem



Renormalization → hadron spectrum, matrix elements …, at $m_{v,q}$

$$S_F(y, j, b; x, i, a) = (D_{\text{ov}}^{-1})_{x,i,a}^{y,j,b}$$

Overlap fermion with valence quark masses $m_{v,q}$

↓ **chiral extrapolation to $m_{v,q}^{\text{phys}}$**

$G_H(Q^2)$ $\langle r^2 \rangle_m(H)$ $\rho^{\text{IMF}}(\mathbf{r}_\perp)$ …, at $m_{v,q}^{\text{phys}}$

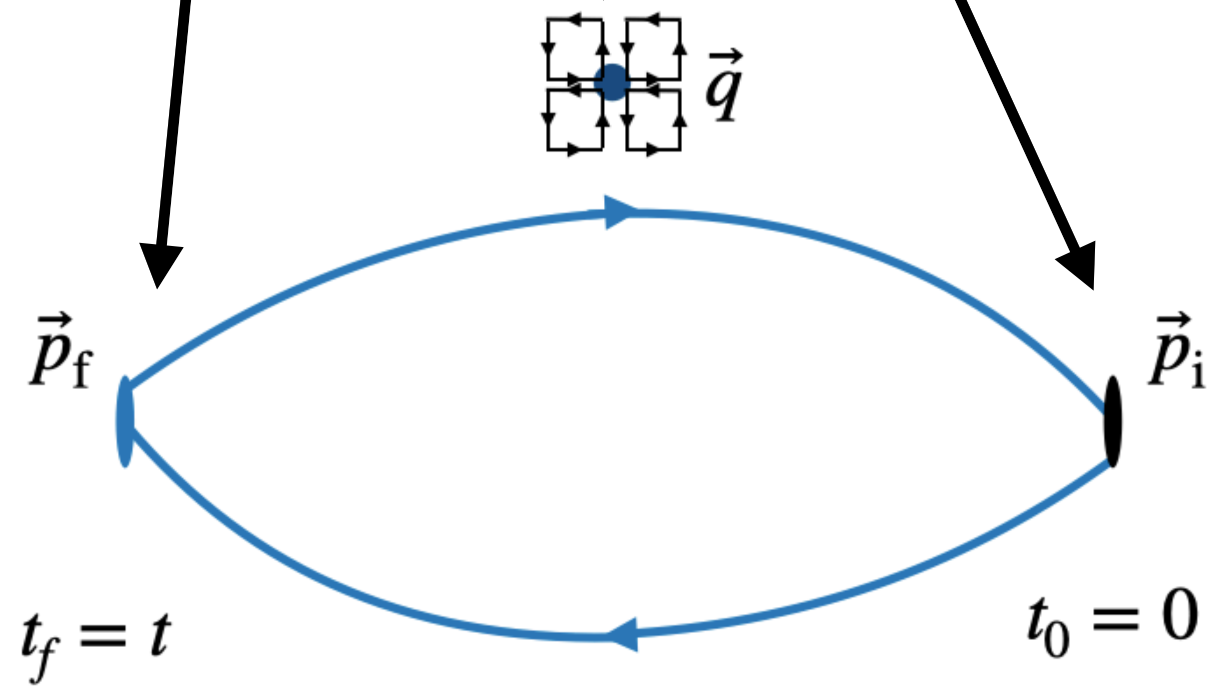
Trace anomaly form factors from lattice QCD

- What kind of observables do we measure on the lattice?
- To extract the form factors:
  - Three-point correlation functions:

interpolating operators

$$C_{\pi,\mathrm{3pt}}(t,\tau;\overrightarrow{p_i},\overrightarrow{p_f}) = \sum_{\vec{x}_f,\vec{z}} e^{-i\vec{p}_f\cdot\vec{x}_f} e^{i\vec{q}\cdot\vec{z}} \langle \chi_\pi(\vec{x}_f,t)\mathcal{O}(\vec{z},\tau)\chi_\pi^\dagger(\vec{0},0)\rangle$$

$$1 = \int \frac{d^3p}{(2\pi)^3} |p\rangle \frac{m}{E_p} \langle p|$$



$$\xrightarrow{t\gg\tau\gg 0} \langle \pi|\mathcal{O}|\pi\rangle Z_{\vec{p}_i} Z_{\vec{p}_f} \frac{m_\pi}{E_{p_i}} \frac{m_\pi}{E_{p_f}} e^{-E_i\tau - E_f(t-\tau)}$$

$m_\pi G_\pi(Q^2)$

Trace anomaly form factors from lattice QCD

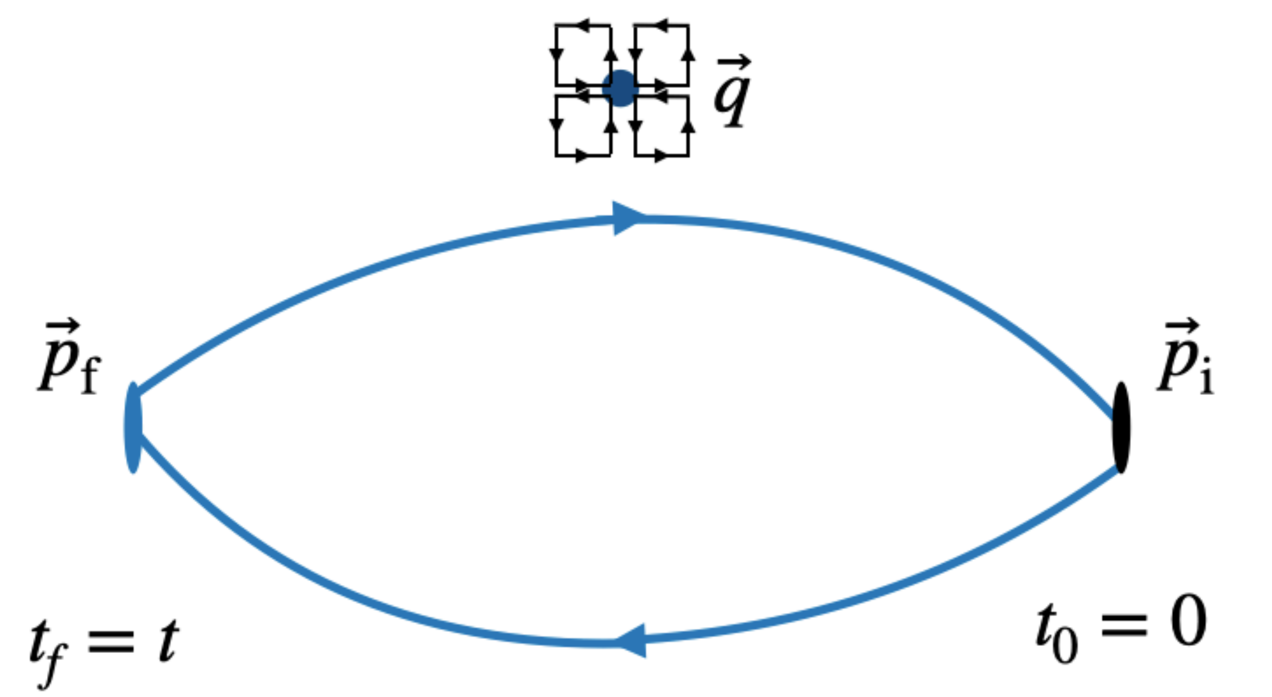
- What kind of observables do we measure on the lattice?

- To extract the form factors:

  - Three-point correlation functions:

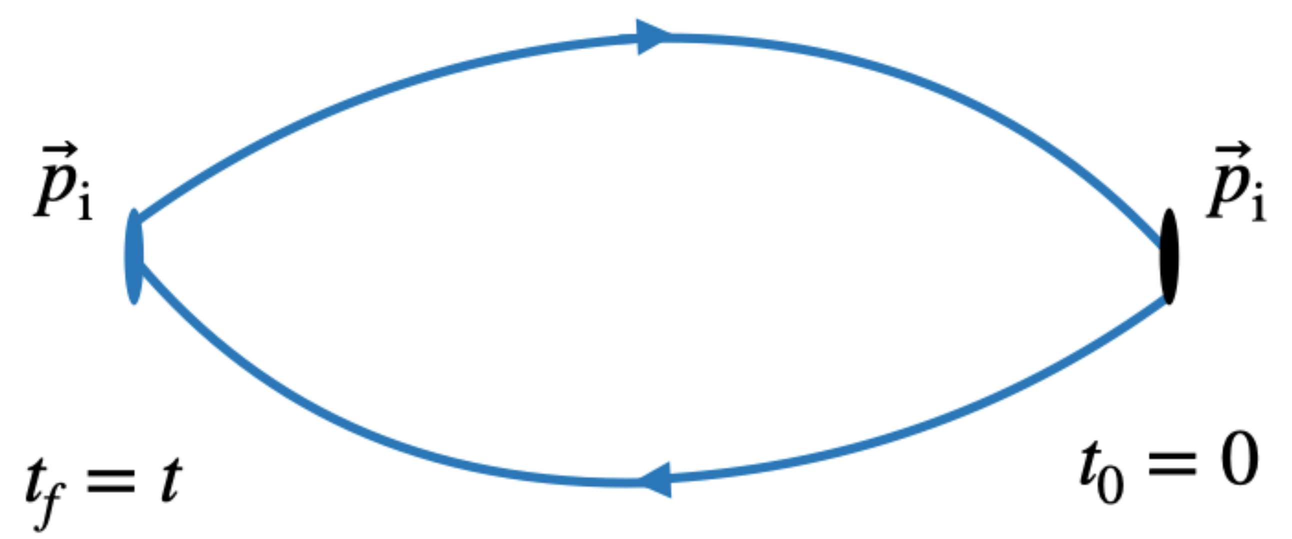
$$C_{\pi,3\mathrm{pt}}(t,\tau;\overrightarrow{p_i},\overrightarrow{p_f}) = \sum_{\vec{x}_f,\vec{z}} e^{-i\vec{p}_f\cdot\vec{x}_f} e^{i\vec{q}\cdot\vec{z}} \langle \chi_\pi(\vec{x}_f,t)\mathcal{O}(\vec{z},\tau)\chi_\pi^\dagger(\vec{0},0)\rangle$$

$$\xrightarrow{t\gg\tau\gg0} \underbrace{\langle\pi|\mathcal{O}|\pi\rangle}_{m_\pi G_\pi(Q^2)} Z_{\vec{p}_i} Z_{\vec{p}_f} \frac{m_\pi}{E_{p_i}} \frac{m_\pi}{E_{p_f}} e^{-E_i\tau - E_f(t-\tau)}$$



  - Two-point correlation functions:

$$C_{\pi,2\mathrm{pt}}(t) = \langle\chi_\pi(t)\chi_\pi^\dagger(0)\rangle \xrightarrow{t\gg0} \frac{m_\pi}{E_i} Z_{\vec{p}_i}^2 [e^{-E_i t} + e^{-E_i(T-t)}]$$



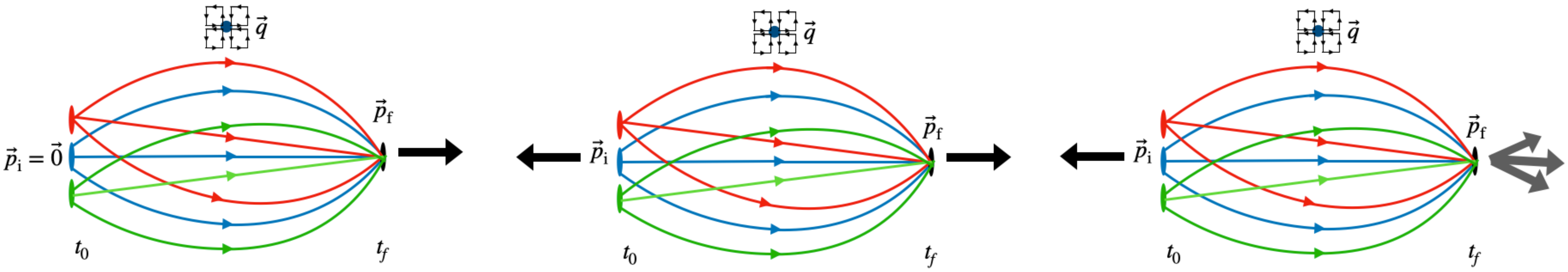
Numerical setup

- overlap valence fermions on
  one ensemble of 2+1-flavor domain-wall fermion configurations with Iwasaki gauge action

Ensemble	$L^3 \times T$	a (fm)	L(fm)	m_π (MeV)	$N_{\rm conf}$	$N_{\rm src}$
24I	$24^3 \times 64$	0.1105(3)	2.65	340	788	64×2

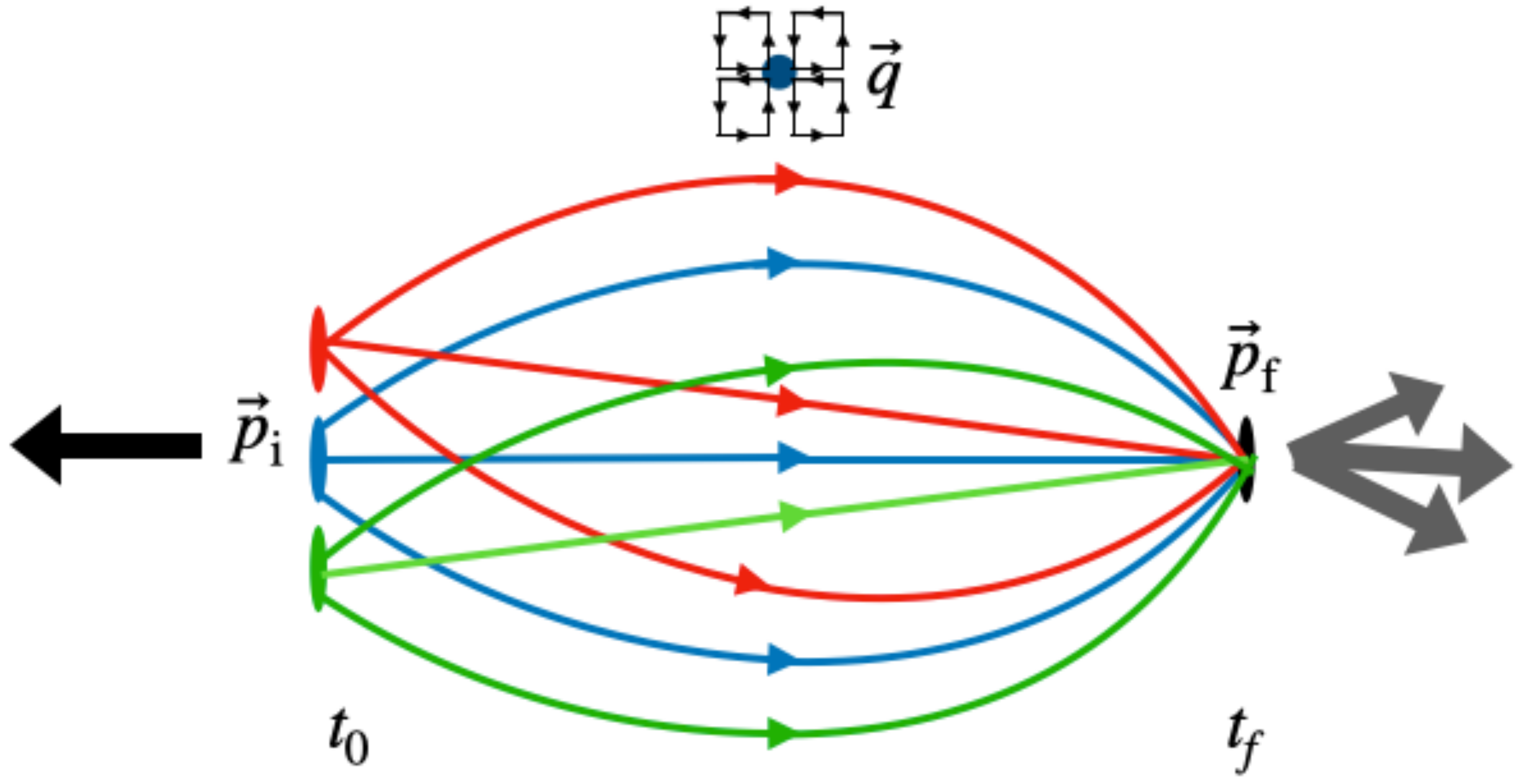
→ $G_{\rm H}(Q^2)$ at 7 valence pion masses

- source-at-rest: $|\vec{p}_{\rm i}| = 0$ with $\vec{q} = \vec{p}_{\rm f}$
- back-to-back: $\vec{p}_{\rm f} = -\vec{p}_{\rm i}$ with $\vec{q} = 2\vec{p}_{\rm f}$
- near-back-to-back: $\vec{p}_{\rm f} \neq -\vec{p}_{\rm i}, \vec{p}_{\rm f} \,\&\, -\vec{p}_{\rm i} \sim \vec{q}/2$



Joint fit of the two- and three point functions

Three-point functions

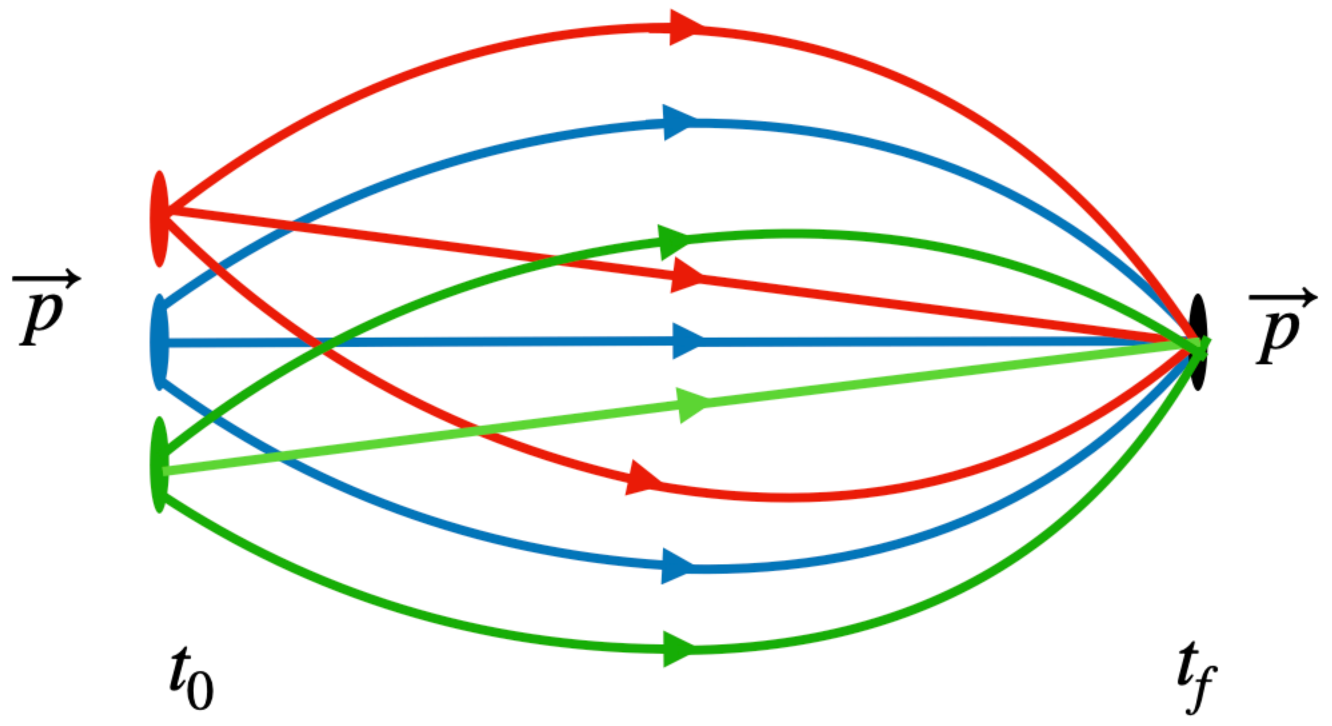


$$C_{\mathrm{H,3pt}}(\vec{p}_i, \vec{p}_f, t, \tau) = m_\mathrm{H}\, G_\mathrm{H}(Q^2)\, \mathscr{K}_{\mathrm{H,3pt}}(p_i, p_f) Z_{\vec{p}_i} Z_{\vec{p}_f} e^{-E_i\tau - E_f(t-\tau)}$$

$$+C_1 e^{-E_i^1\tau - E_f(t-\tau)} + C_2 e^{-E_i\tau - E_f^1(t-\tau)} + C_3 e^{-E_i^1\tau - E_f^1(t-\tau)}$$

3pt-2pt joint fit $\rightarrow G_\mathrm{H}(Q^2)$

Two-point functions

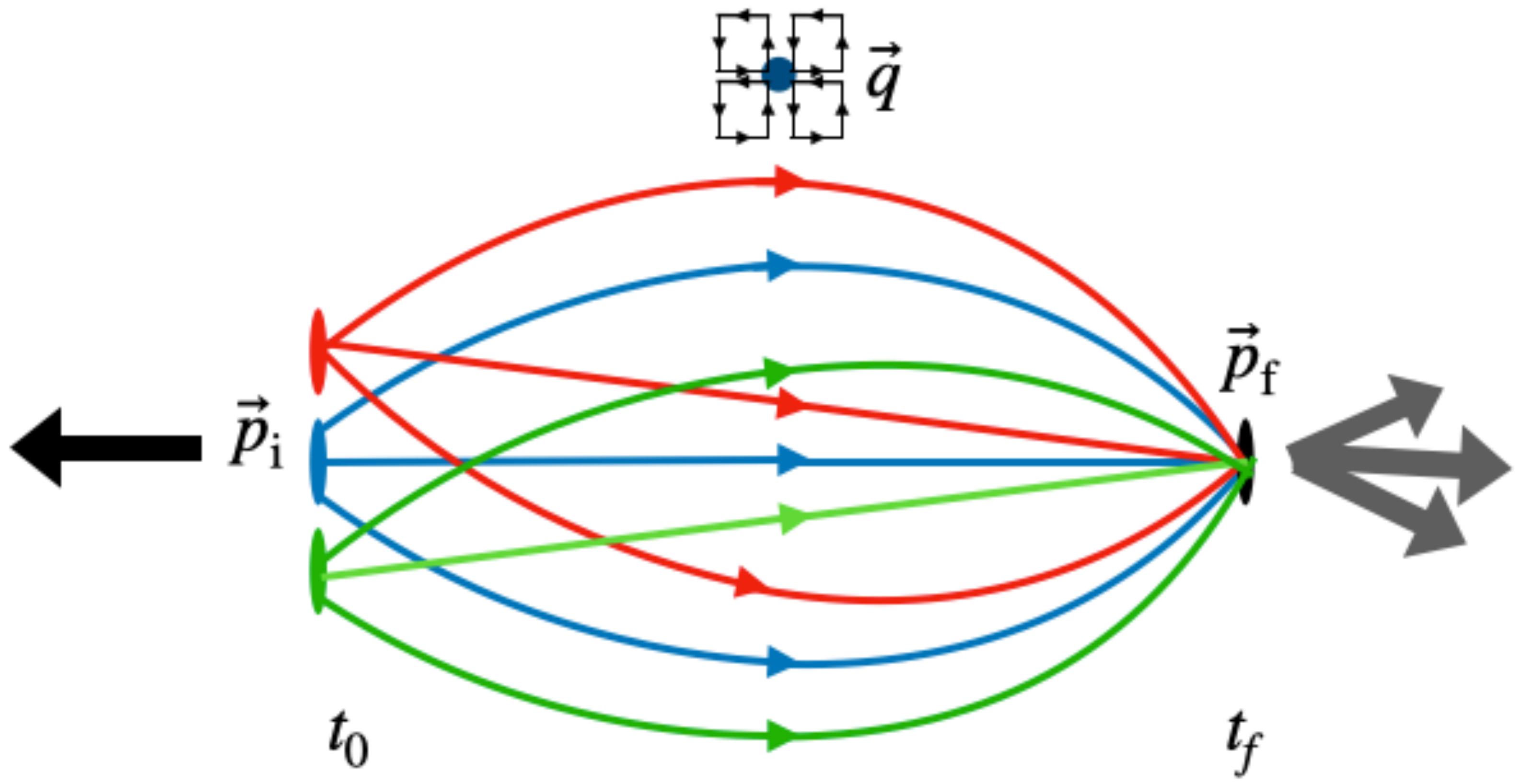


$$C_{\mathrm{H,2pt}}(t) = \mathscr{K}_{\mathrm{H,2pt}}(E_i) Z^2_{\vec{p}_i}[e^{-E_i t} + e^{-E_i(T-t)}] + A_1 e^{-E_i^1 t}$$

$$C_{\mathrm{H,2pt}}(t) = \mathscr{K}_{\mathrm{H,2pt}}(E_f) Z^2_{\vec{p}_f}[e^{-E_f t} + e^{-E_f(T-t)}] + A_1 e^{-E_f^1 t}$$

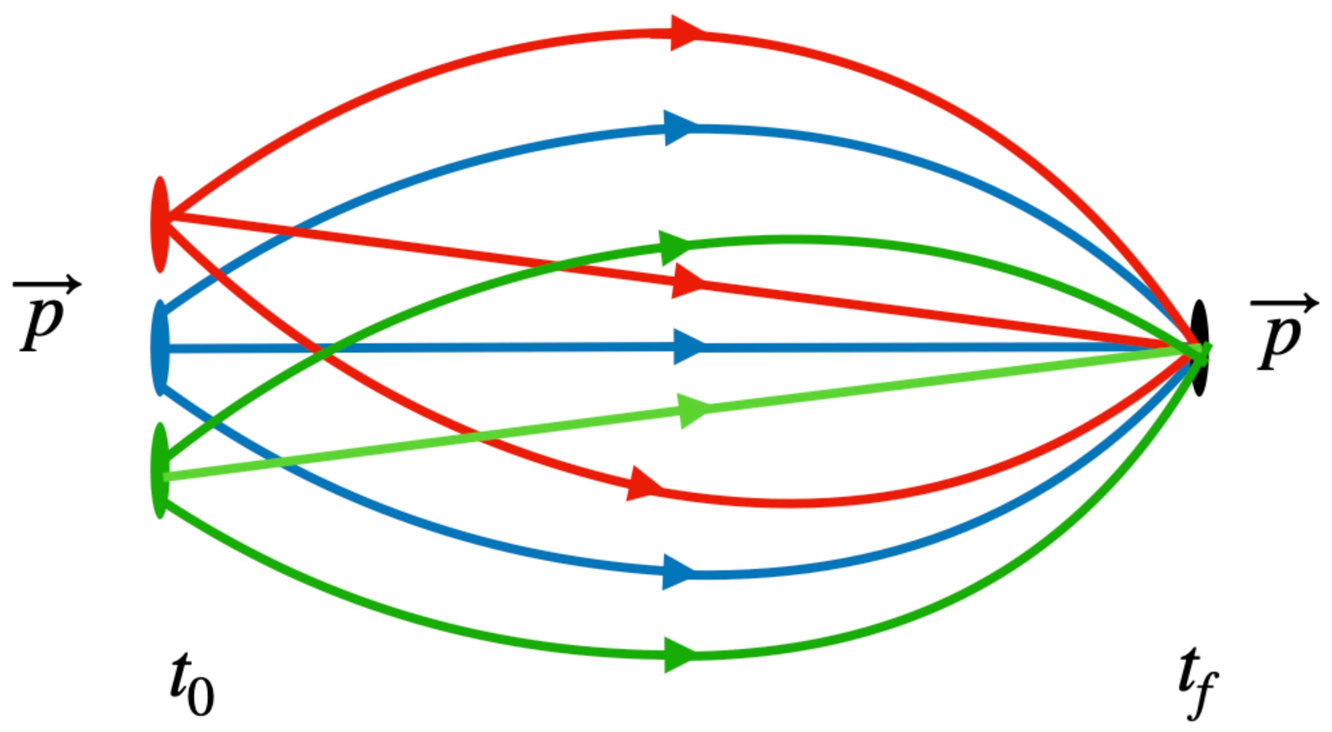
Ratio of the two- and three point functions

Three-point functions



$$R_{\text{sqrt}}(t,\tau;\vec{p}_i,\vec{p}_f) = \frac{C_{\text{H},3pt}(t,\tau;\vec{p}_i,\vec{p}_f)}{C_{\text{H},2pt}(t;\vec{p}_f)} \times \sqrt{\frac{C_{\text{H},2pt}(t-\tau;\vec{p}_i)C_{\text{H},2pt}(t;\vec{p}_f)C_{\text{H},2pt}(\tau;\vec{p}_f)}{C_{\text{H},2pt}(t-\tau;\vec{p}_f)C_{\text{H},2pt}(t;\vec{p}_i)C_{\text{H},2pt}(\tau;\vec{p}_i)}} \Bigg/ \left[\frac{m_{\text{H}}\mathscr{K}_{\text{H},3pt}(p_i,p_f)}{\sqrt{\mathscr{K}_{\text{H},2pt}(p_i)\mathscr{K}_{\text{H},2pt}(p_f)}}\right]$$

Two-point functions



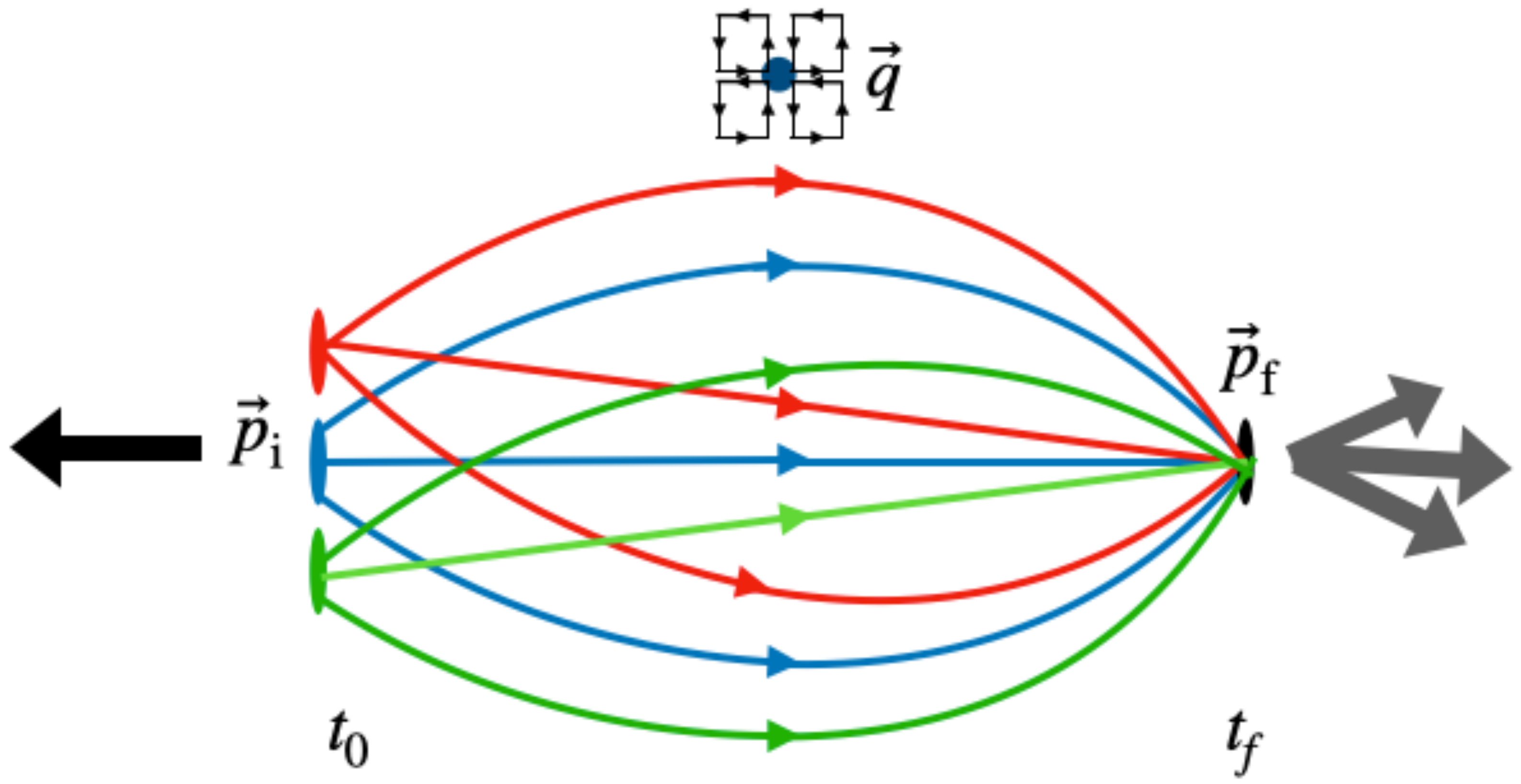
$$R_{\text{sqrt}} \sim G_{\text{H}}(Q^2) + C_1'' e^{-\Delta E_i^1 \tau} + C_2'' e^{-\Delta E_f^1 (t-\tau)} + C_3'' e^{-\Delta E_i^1 \tau - \Delta E_f^1 (t-\tau)}$$

$$\xrightarrow{t \gg \tau \gg 0} G_{\text{H}}(Q^2)$$

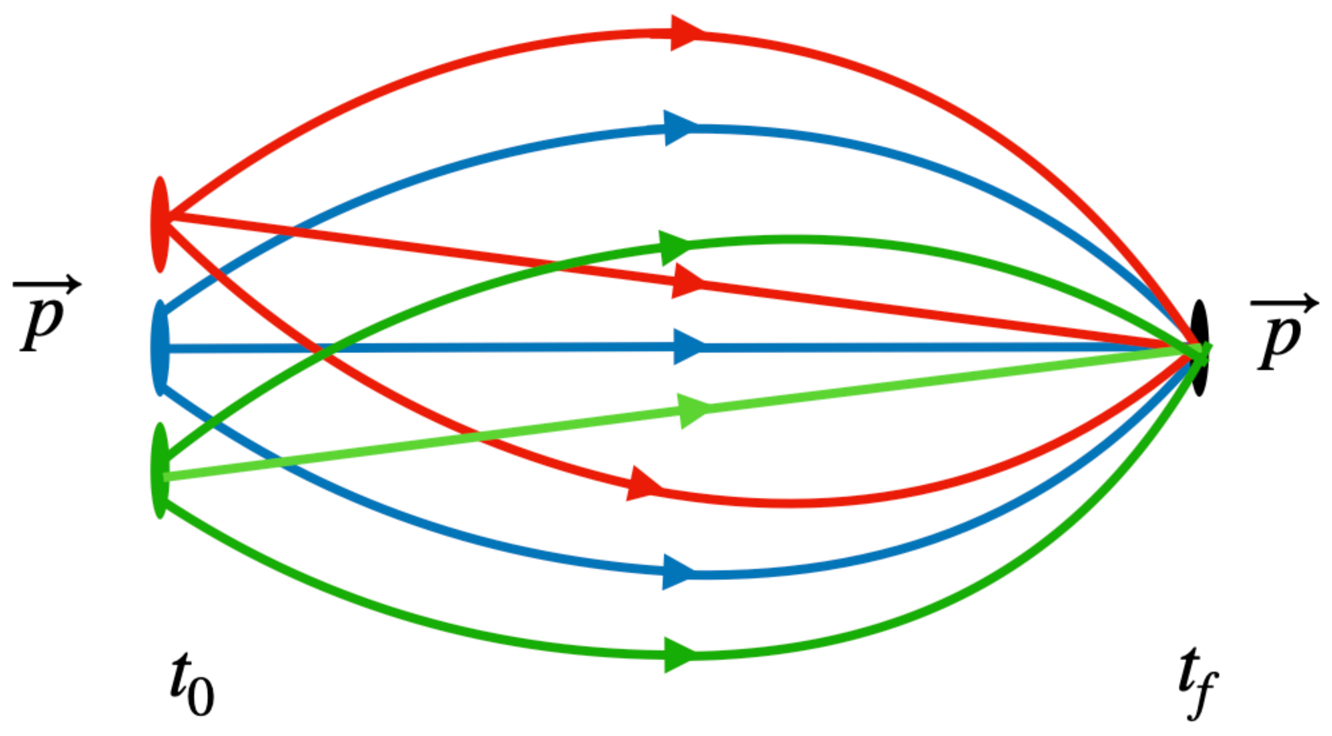
Compare it with the results from 3pt-2pt joint fit

Extract form factors from the two- and three point functions

Three-point functions



Two-point functions



$$R_{\rm sqrt} \sim G_{\rm H}(Q^2) + C_1'' e^{-\Delta E_i^1 \tau} + C_2'' e^{-\Delta E_f^1 (t-\tau)} + C_3'' e^{-\Delta E_i^1 \tau - \Delta E_f^1 (t-\tau)}$$

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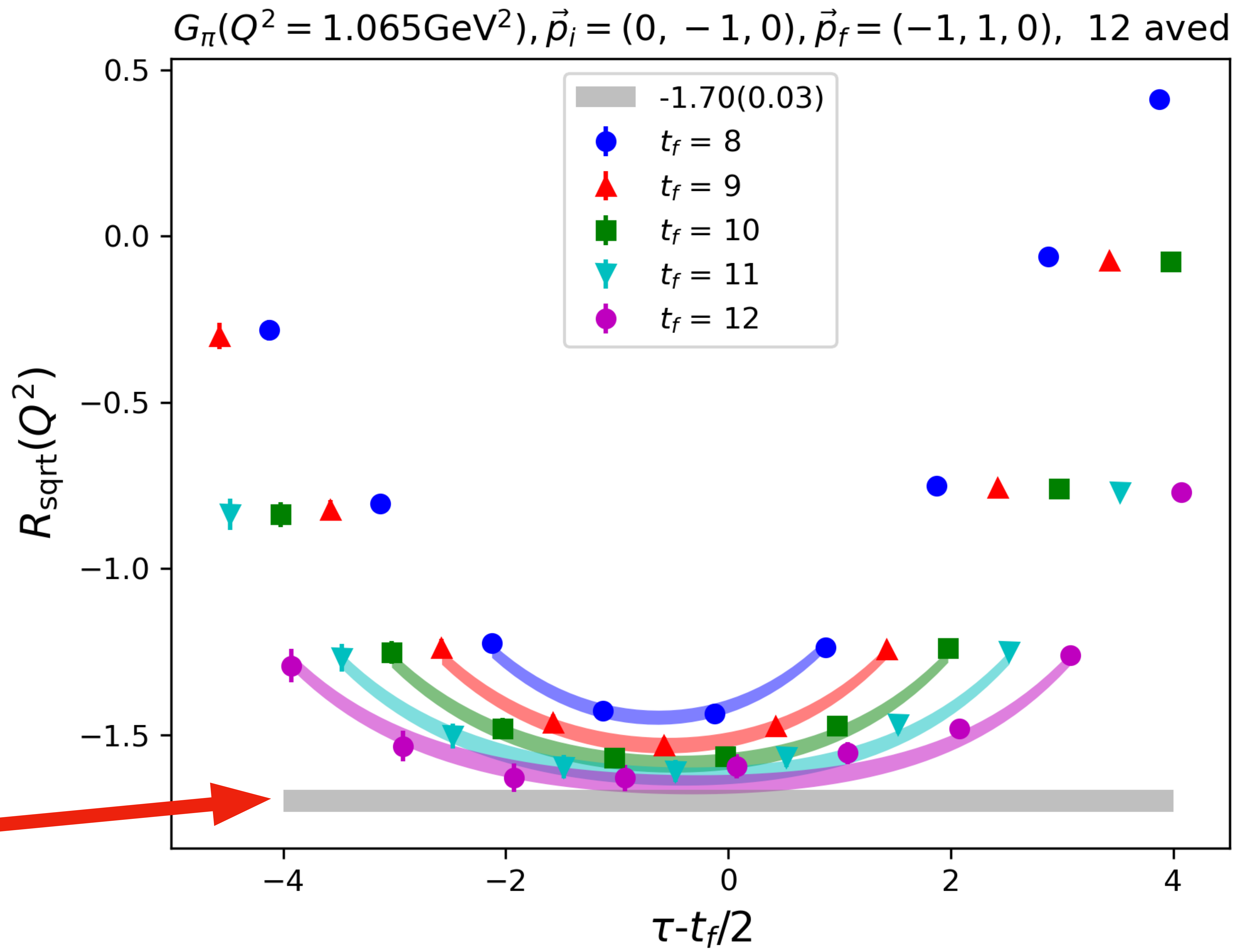

$G_{\rm H}(Q^2)$

Extract form factors from the two- and three point functions

- source-at-rest:
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- back-to-back:
  $\vec{p}_f = -\vec{p}_i$ with $\vec{q} = 2\vec{p}_f$
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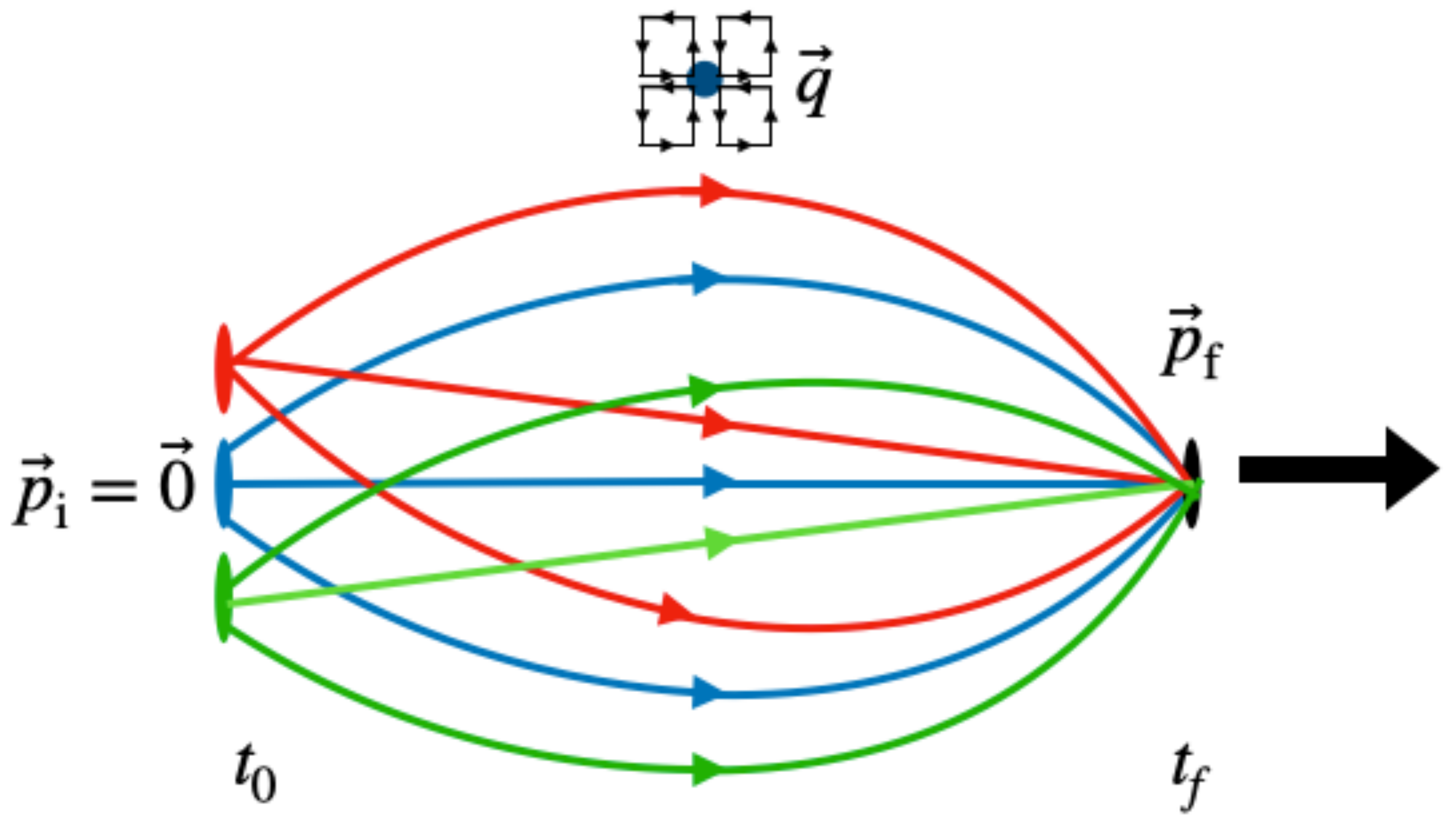

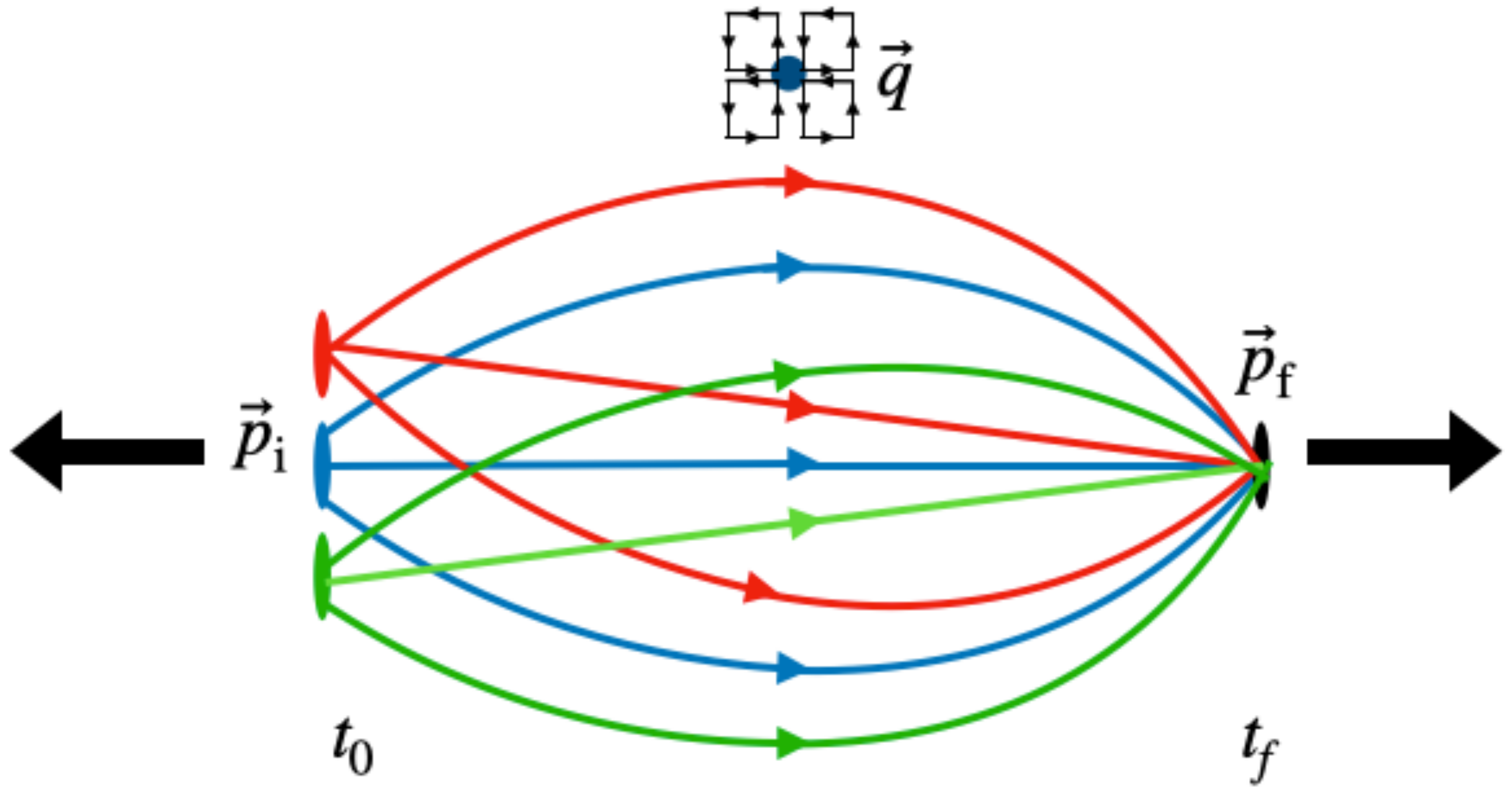

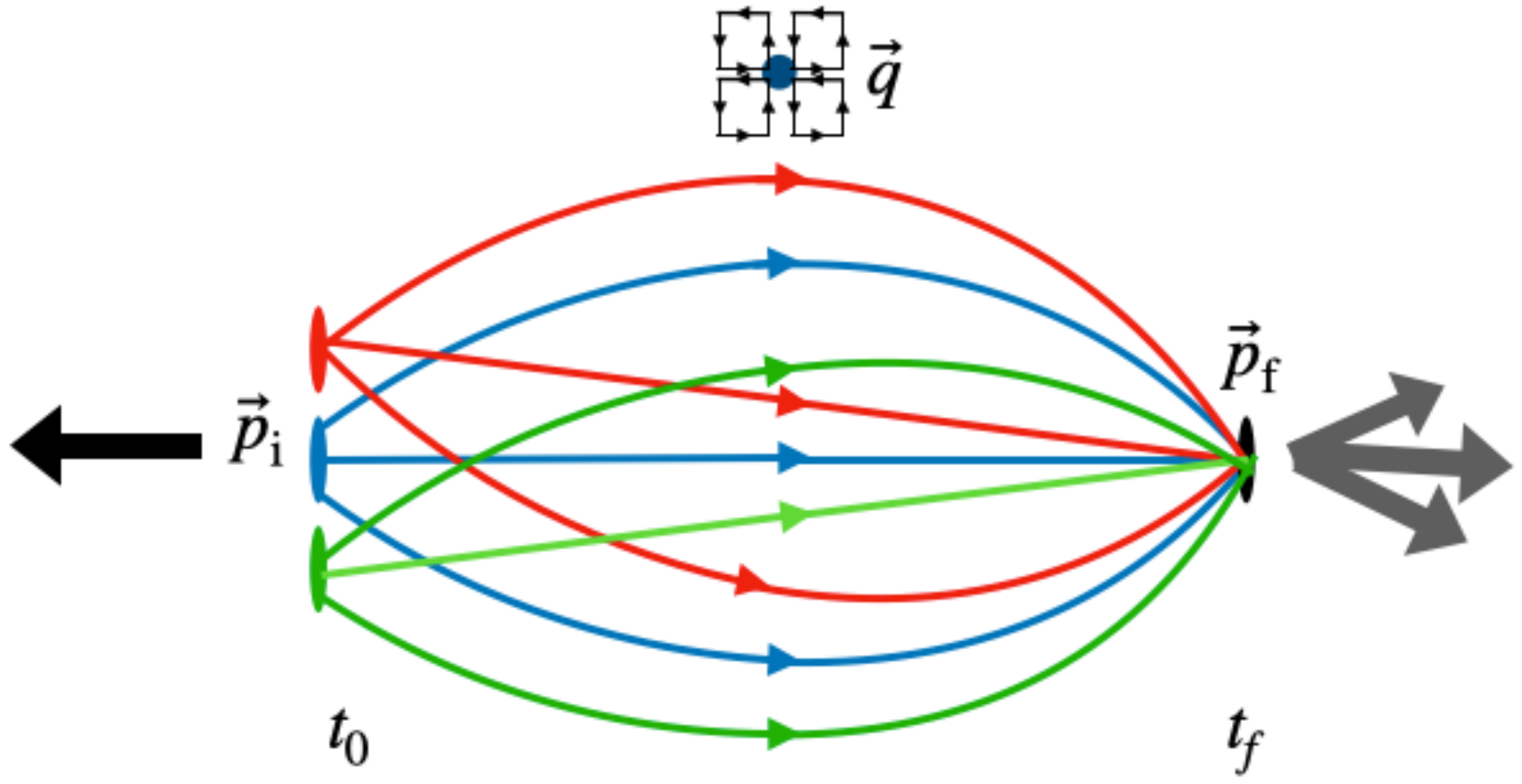

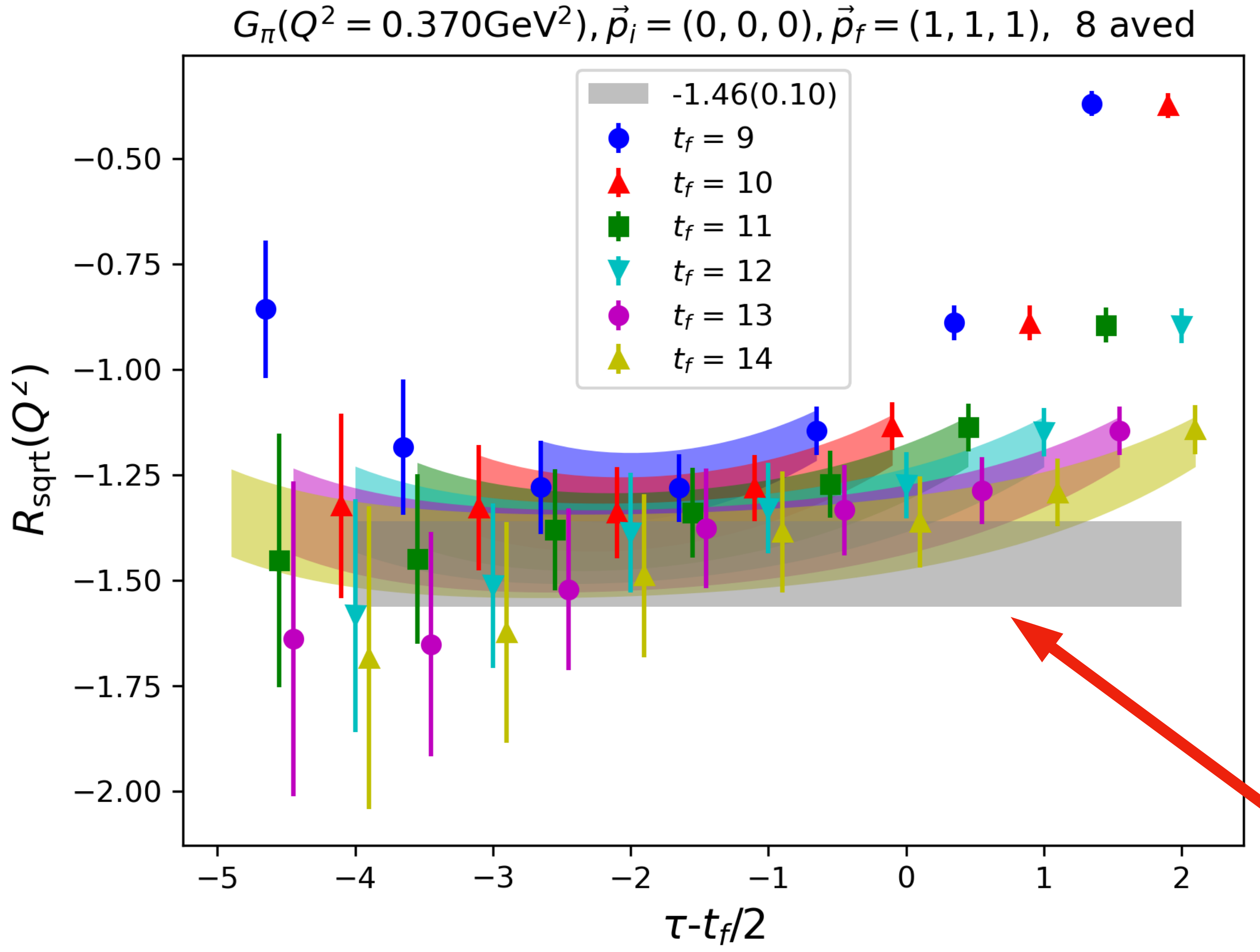

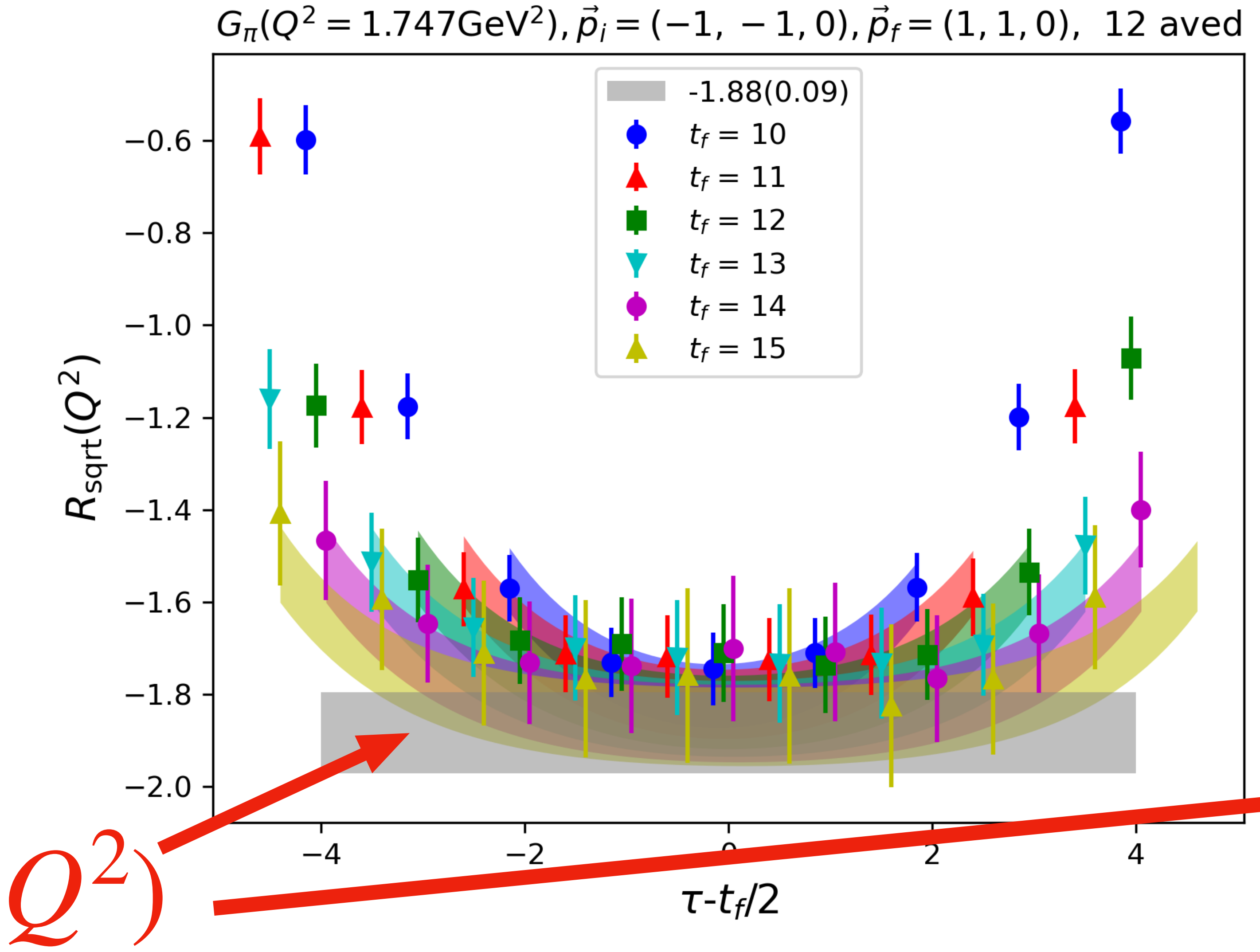

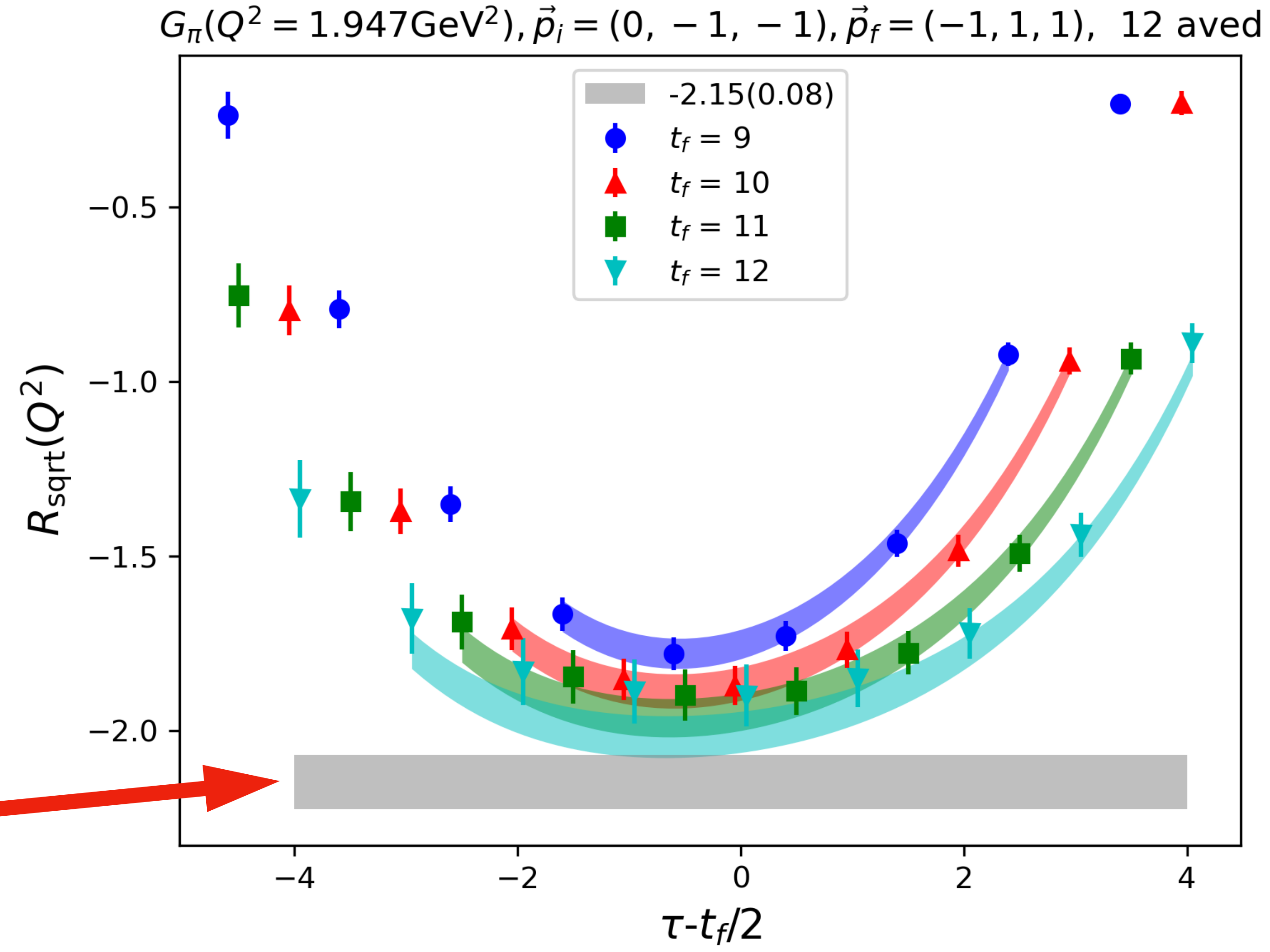

$G_H(Q^2)$

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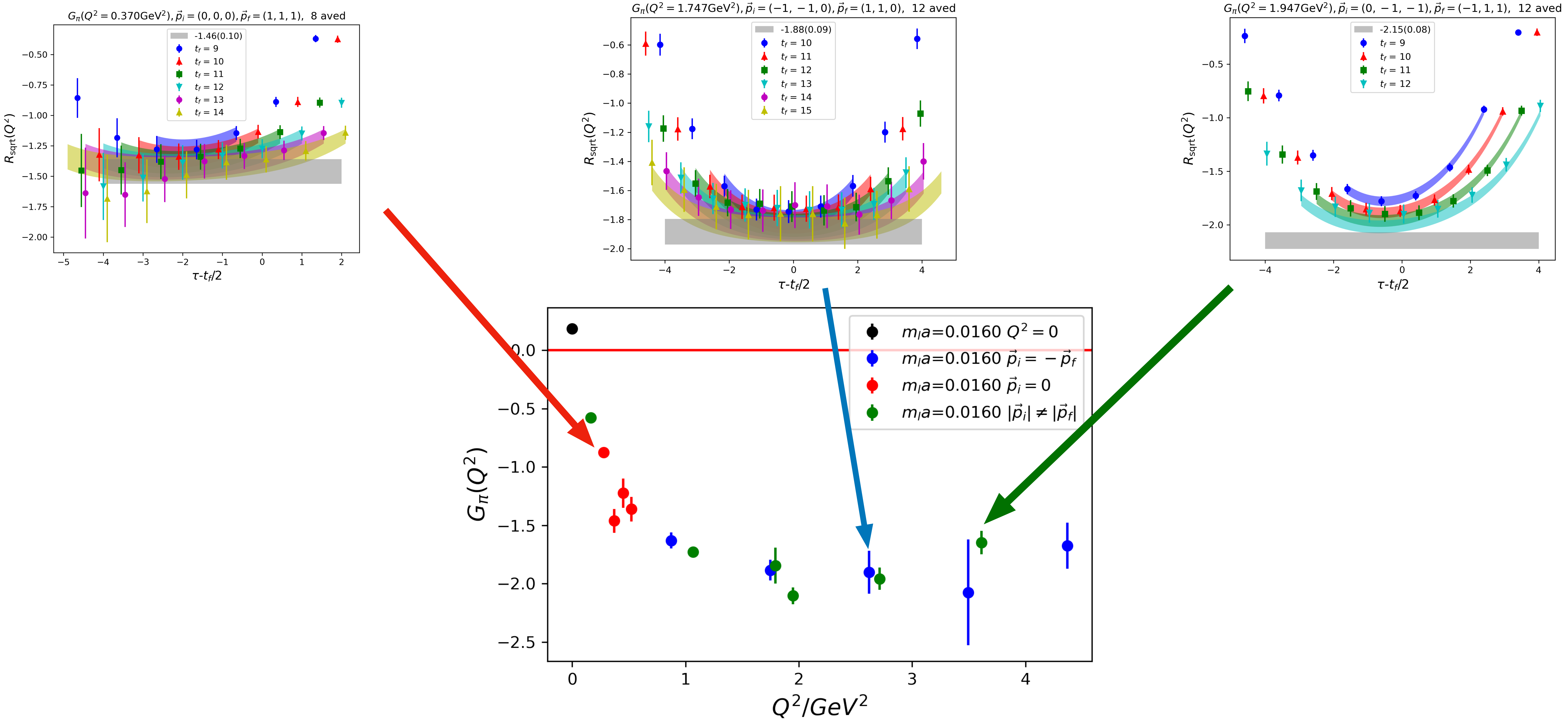


Outline

- Introduction
  - Scale invariance(?), trace anomaly and hadron mass
  - The “pion mass puzzle” & the mass distribution in the pion
  - Radius and spatial distribution of the trace anomaly
- Calculation of the QCD trace anomaly from lattice QCD (glue part)
  - Numerical setup
  - Extract form factors from 2-point, 3-point correlation functions
- ➔ Results
  - Trace anomaly form factors, mass radii, and spatial distributions
- Conclusion and outlook

Results

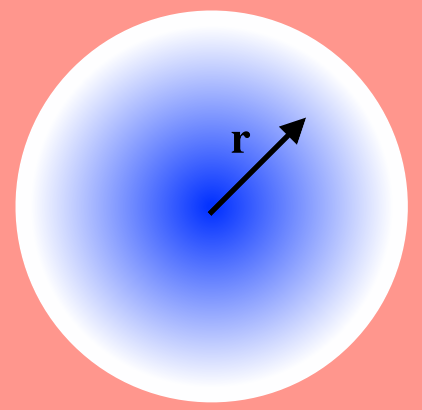
Hadron mass?

Matrix elements → **Form factors** → Spatial distributions of mass?

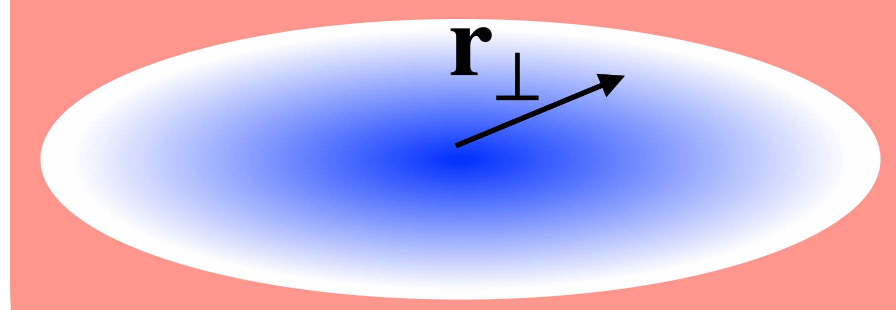
$$\langle p' | T^{\mu}_{\mu} | p \rangle$$

$$\mathscr{F}_{\mathrm{m},\pi}(Q^2)$$

$$G_{\mathrm{H}}(Q^2)$$

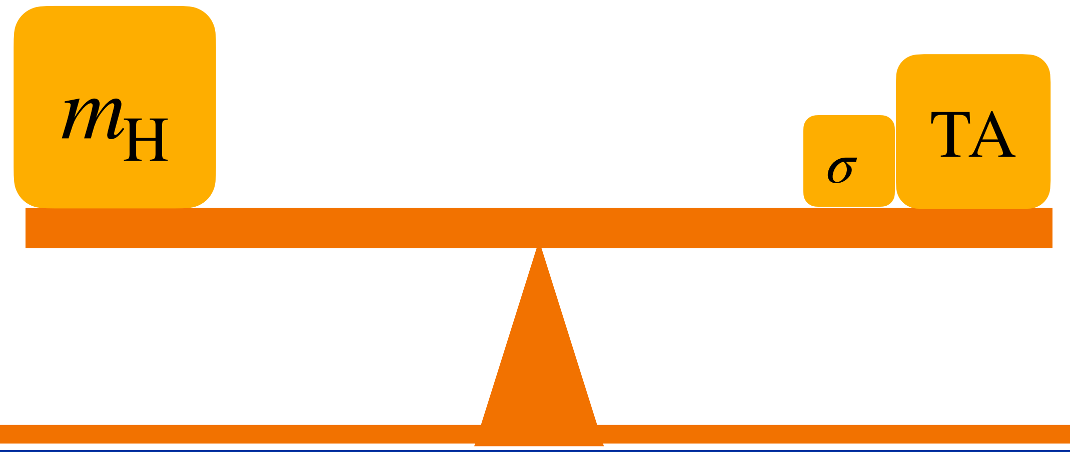


$$\langle r^2 \rangle_{\mathrm{m}}(\mathrm{H}) \sim -6 \left. \frac{dG_{\mathrm{H}}(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

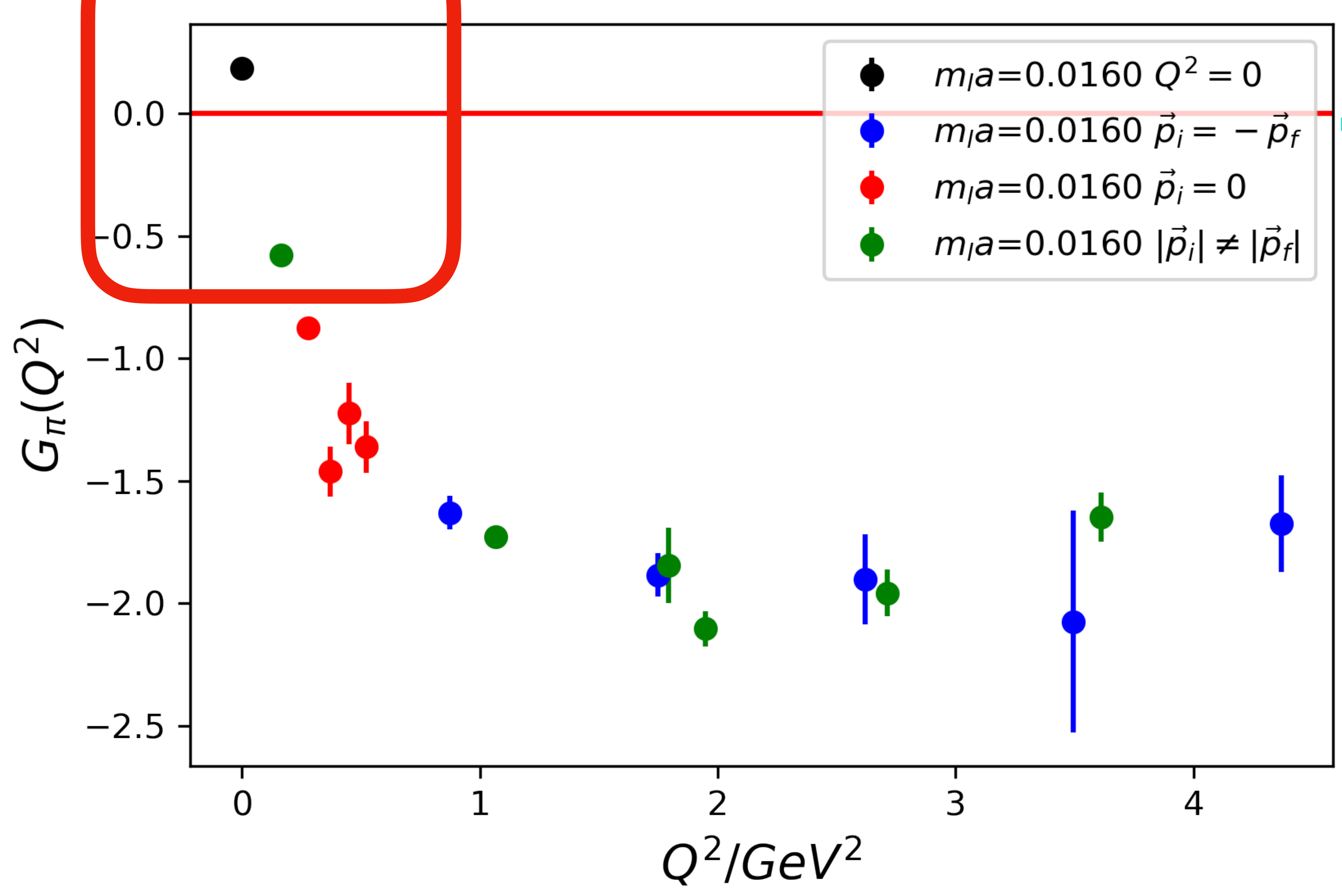


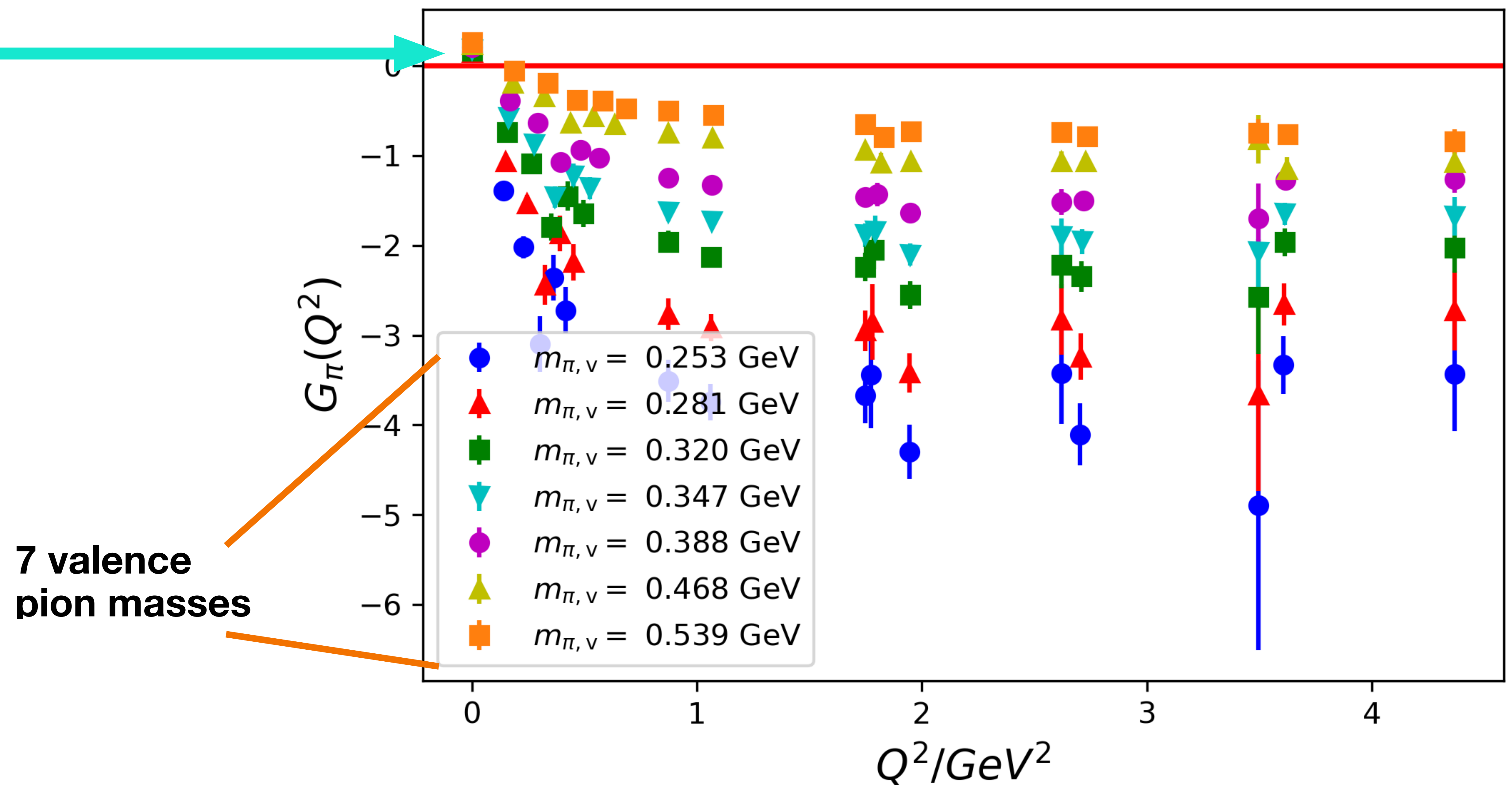
$$\rho^{\mathrm{IMF}}(\mathbf{r}_{\perp})$$

$$\underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\text{the } \sigma \text{ term}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_{\mathrm{H}}}_{\langle (T^{\mu}_{\mu})^{\mathrm{a}} \rangle \text{ trace anomaly, RG invariant}}$$



glue trace anomaly form factor of the pion preliminary





- **positive** at $Q^2 = 0\,\mathrm{GeV}^2$ (contribution to the pion mass from glue)
- **sign change** of glue trace anomaly form factor of the pion

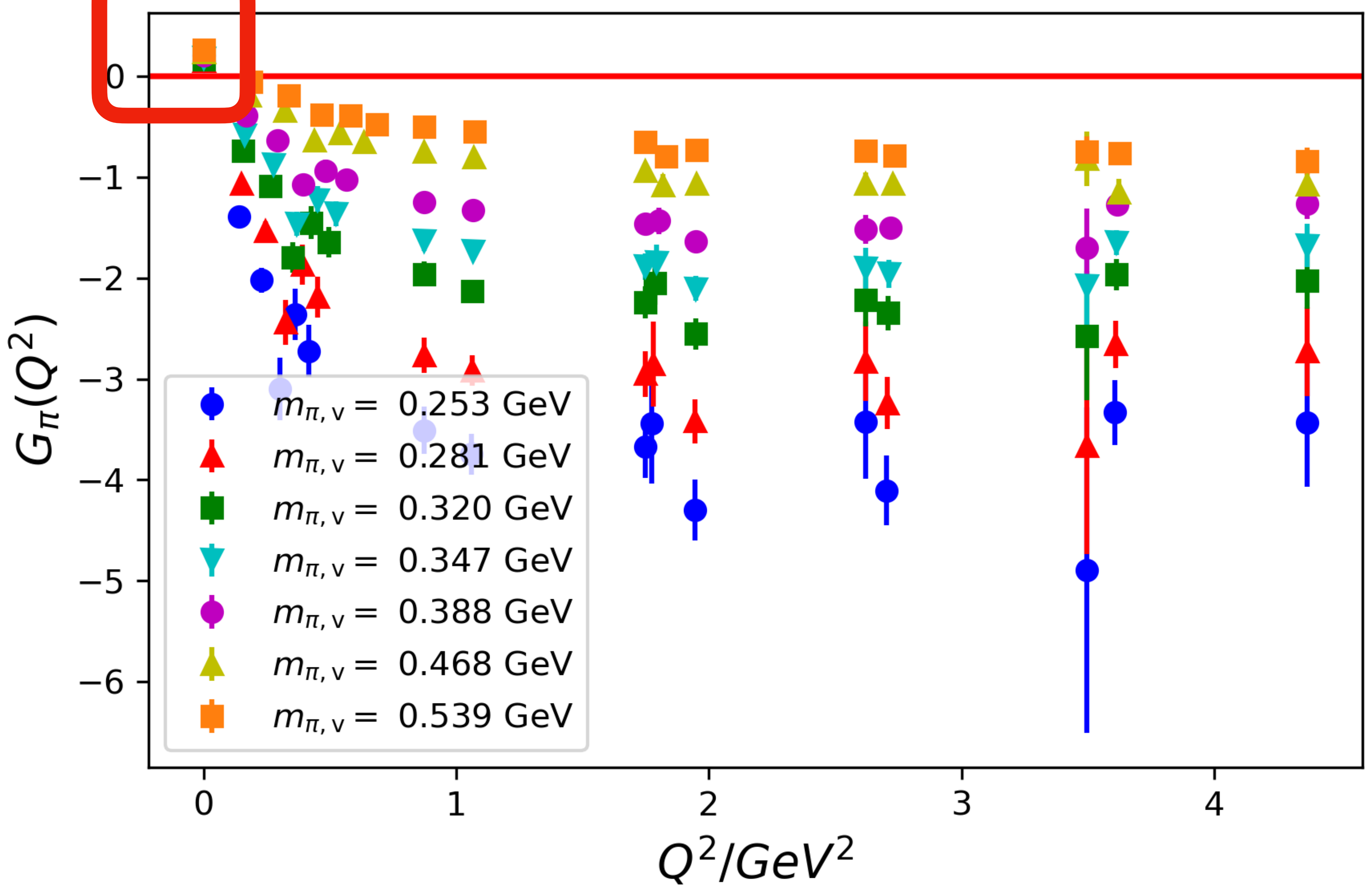
- The predictions from chiral perturbation theory at small Q^2 region:

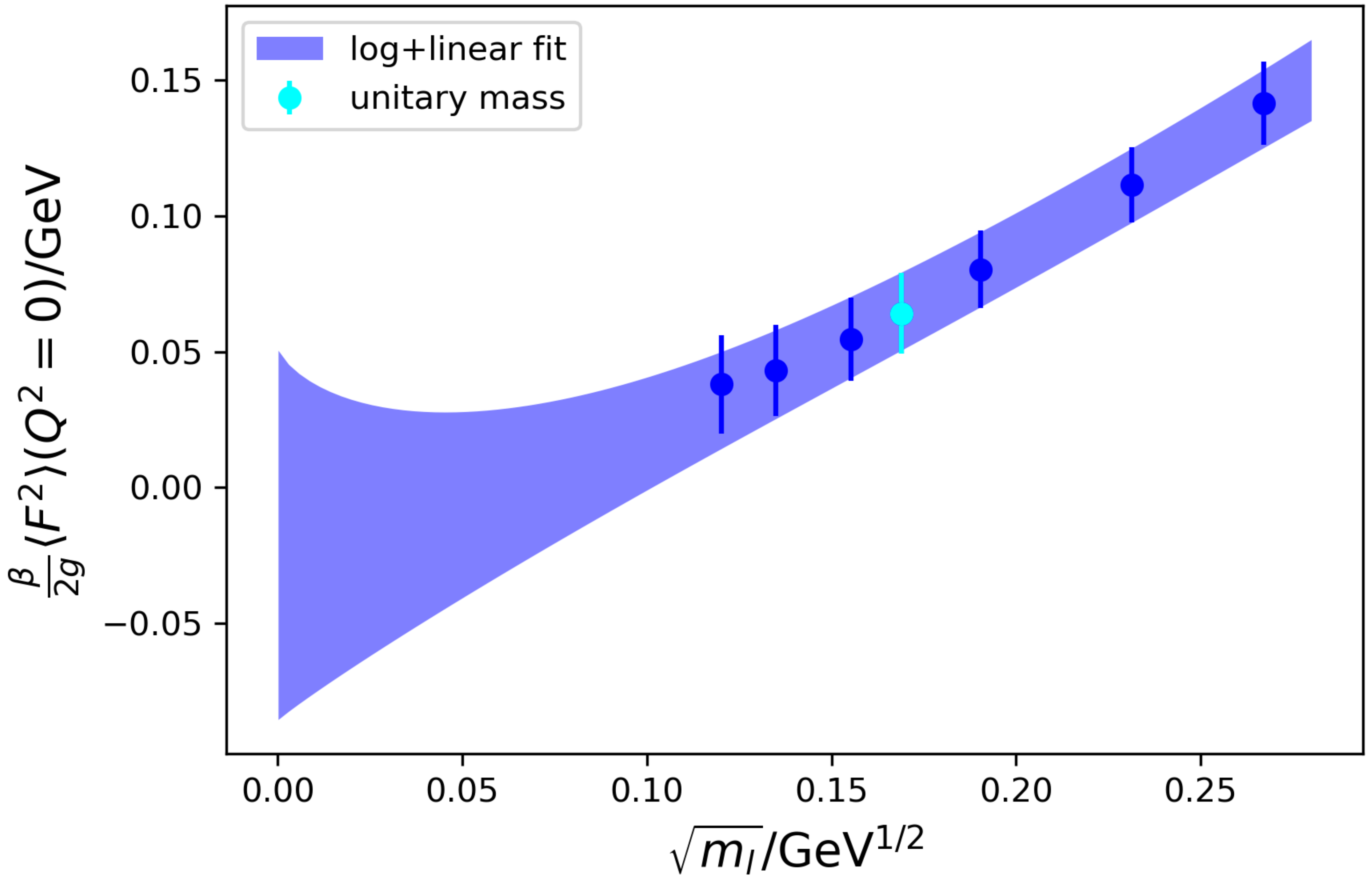
$$\mathcal{F}^{\mathrm{ChPT}}_{\mathrm{ta},\pi}(Q^2) \sim \frac{1}{2} - \frac{1}{2m_\pi^2}Q^2$$

[Novikov, Shifman, Z. Phys. C8, 43 (1981)]
[Chen, Phys.Rev. D57 (1998) 2837-2846]
[Y. Hatta, Private communication]

glue trace anomaly form factor of the pion preliminary

$$\underbrace{m_\pi}_{\propto \sqrt{m_f}} = \underbrace{\sum_f m_f \langle \overline{\psi}_f \psi_f \rangle_\pi}_{\text{the } \sigma \text{ term } \propto \sqrt{m_f}} + \underbrace{\langle \frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \overline{\psi}_f \psi_f \rangle_\pi}_{\langle (T^\mu_\mu)^a \rangle \text{ trace anomaly } \propto \sqrt{m_f}}$$





- **positive** at $Q^2 = 0\ \mathrm{GeV}^2$ (contribution to the pion mass from glue)

$$\langle \frac{\beta}{2g} F^2 \rangle \propto \sqrt{m_f}$$

glue part of the trace anomaly density of the pion preliminary

- Form factors $G_{\rm H}(Q^2)$ $\rightarrow \tilde{G}_{\rm H}(Q^2) = G_{\rm H}(Q^2)/G_{\rm H}(Q^2=0)$

Fit with functional form: z-expansion

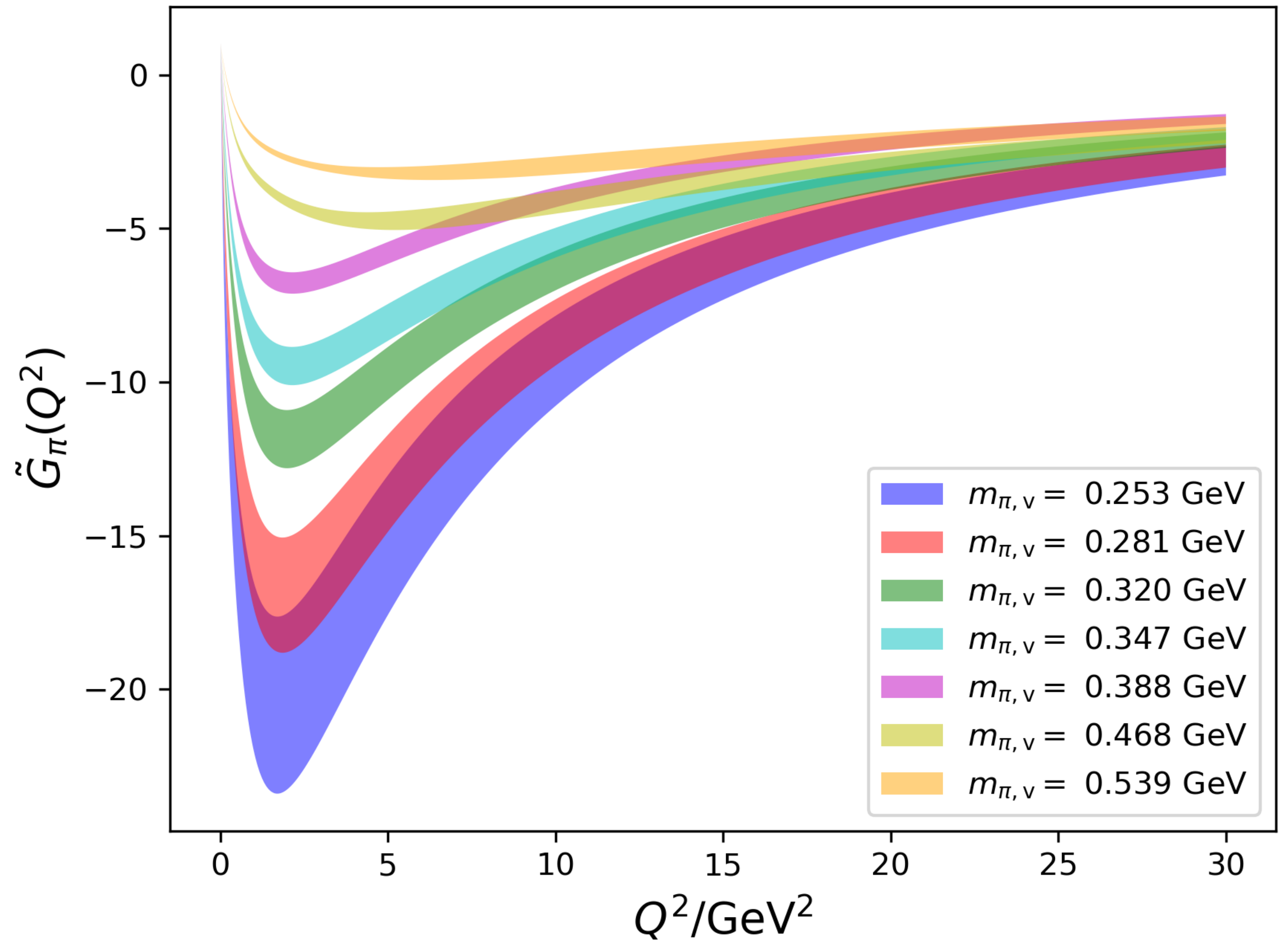
$$\tilde{G}_{\rm H}(Q^2) = \tilde{\mathscr{G}}_{\rm H}(z) = \sum_{k=0}^{k_{\rm max}} a_k z^k,$$

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}, t = -Q^2$$

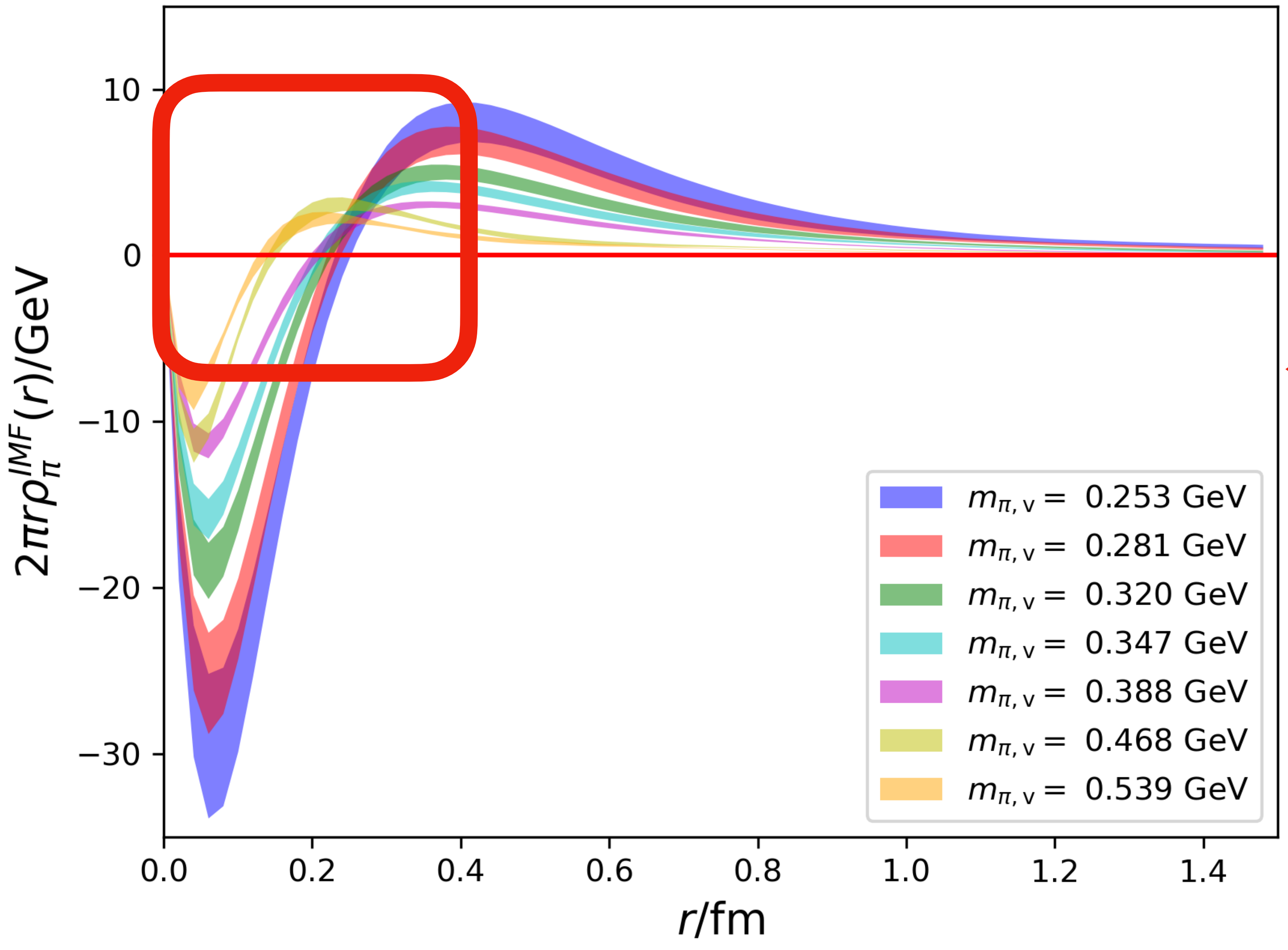
[R. J. Hill and G. Paz, Phys. Rev. D 82, 113005 (2010)]

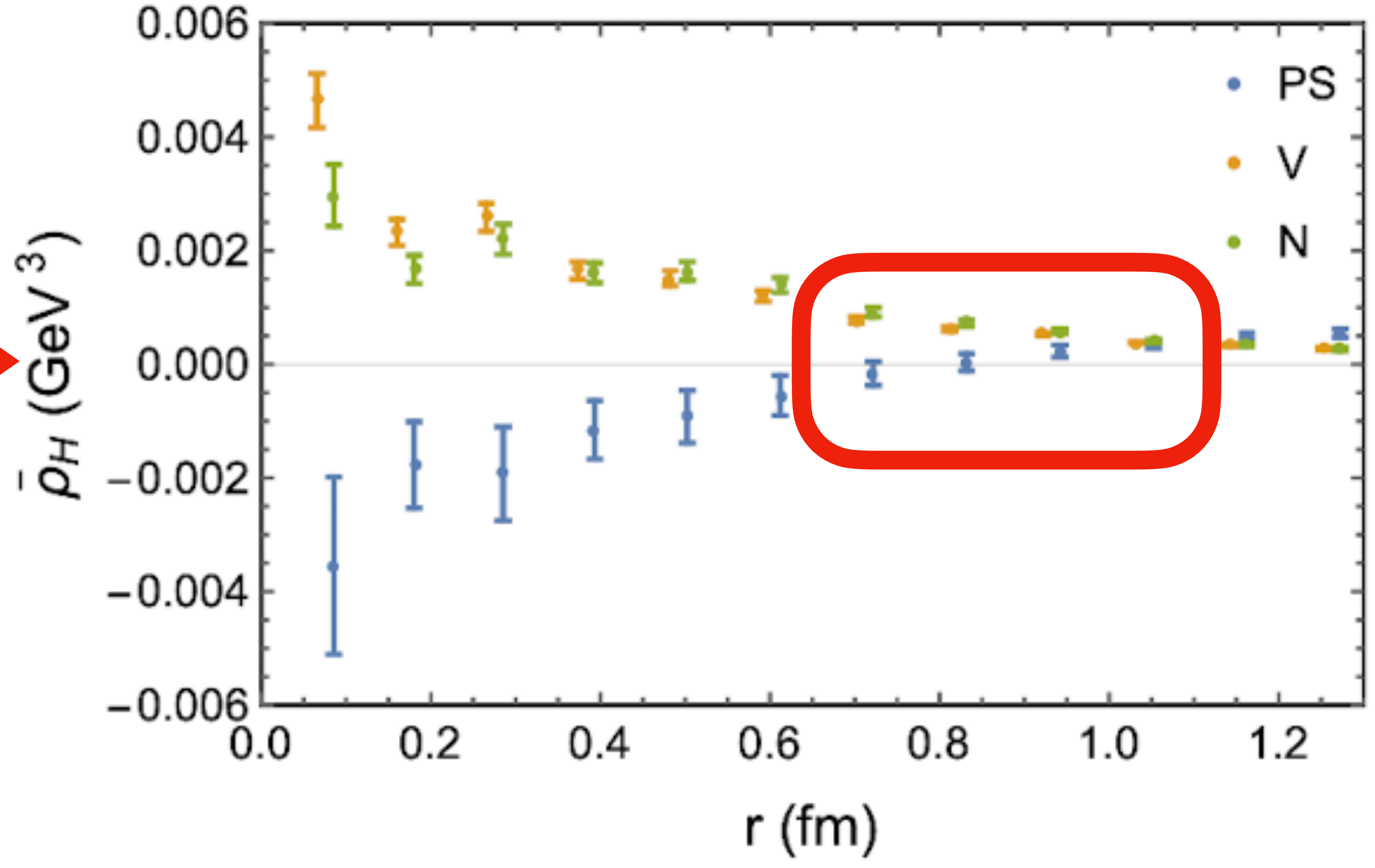
glue trace anomaly spatial distribution of the pion preliminary

Form factors



Spatial distribution





F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

2D spatial distribution in the infinite momentum frame (IMF)

$$P_z \to \infty \text{ and } \mathbf{P}\cdot\mathbf{\Delta} = 0$$

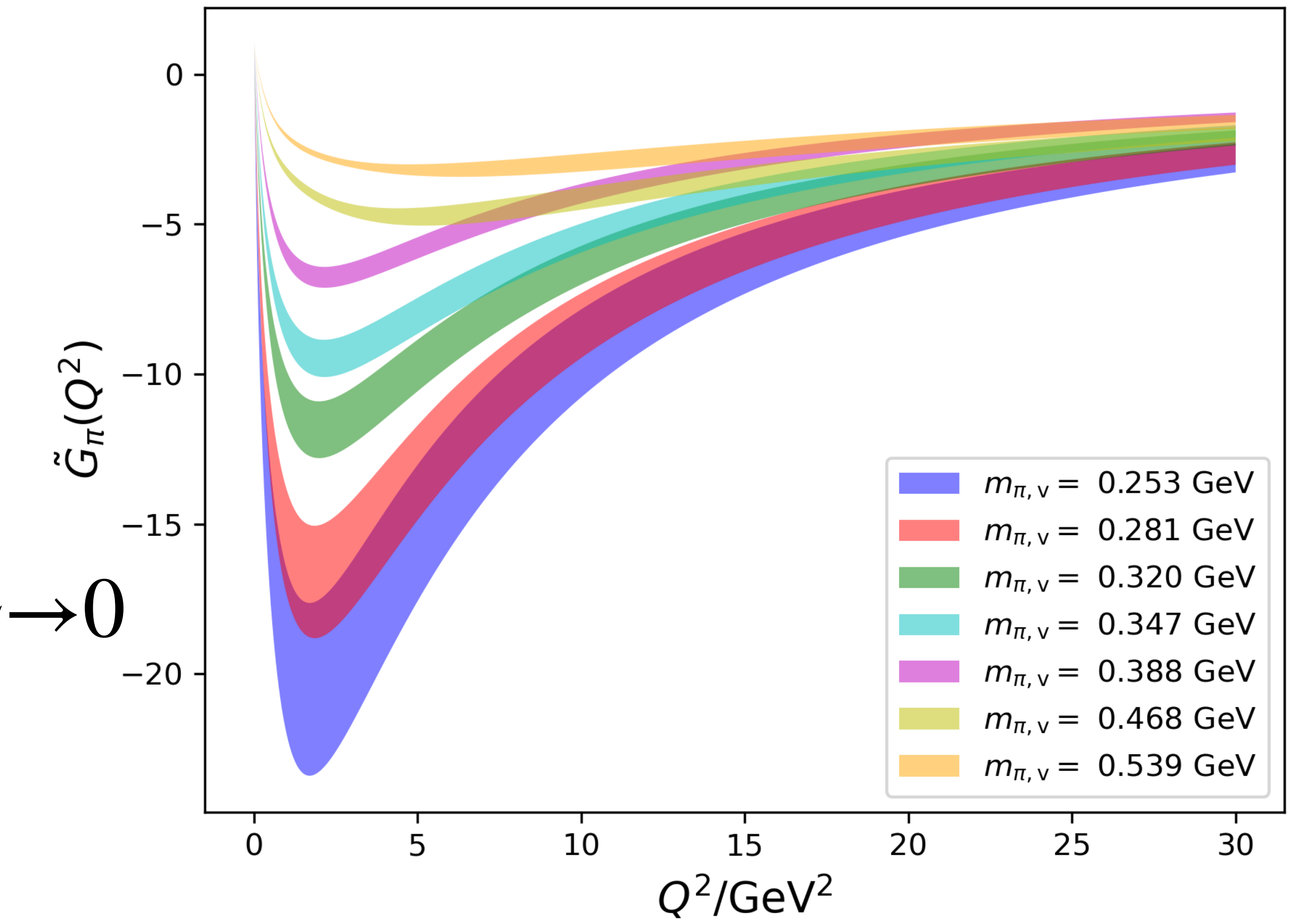
$$\rho_{\mathrm{H}}^{\mathrm{IMF}}(\mathbf{r}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}_\perp} \tilde{G}_{\mathrm{H}}(Q^2)\Big|_{\mathbf{P}\cdot\mathbf{\Delta}=0}^{P_z\to\infty}$$

- **sign change** of glue trace anomaly spatial distribution of the pion, negative at small r.

Radius of the glue trace anomaly of the pion preliminary

- Radius of the glue trace anomaly

$$\langle r^2\rangle_{\rm m}({\rm H}) = -6\frac{d\mathscr{F}_{\rm m,H}(Q^2)}{dQ^2}\Bigg|_{Q^2\to 0} \sim \langle r_g^2\rangle_{\rm ta}({\rm H}) = -6\frac{dG_{\rm H}(Q^2)}{dQ^2}\Bigg|_{Q^2\to 0}$$



- Lattice: chiral extrapolation to the physical pion mass using

$$\langle r_g^2\rangle_{\rm ta}(\pi) = a_\pi/m_\pi^2 + b_\pi + c_\pi \log\left(\frac{m_\pi^2}{m_{\pi,phy}^2}\right) + d_\pi m_\pi^2$$

$$\langle r_g^2\rangle_{\rm ta}^{\rm phy}(\pi) = 21.5(5.2)(11.7)\ {\rm fm}^2$$

Possibly large systematic errors here

- The predictions from chiral perturbation theory at small Q^2 region:

$$\langle r^2\rangle_{\rm m}^{\rm ChPT}(\pi) = \frac{3}{m_\pi^2} \simeq 6\ {\rm fm}^2$$

[Novikov, Shifman, Z. Phys. C8, 43 (1981)]
[Chen, Phys.Rev. D57 (1998) 2837-2846]
[Y. Hatta, Private communication]

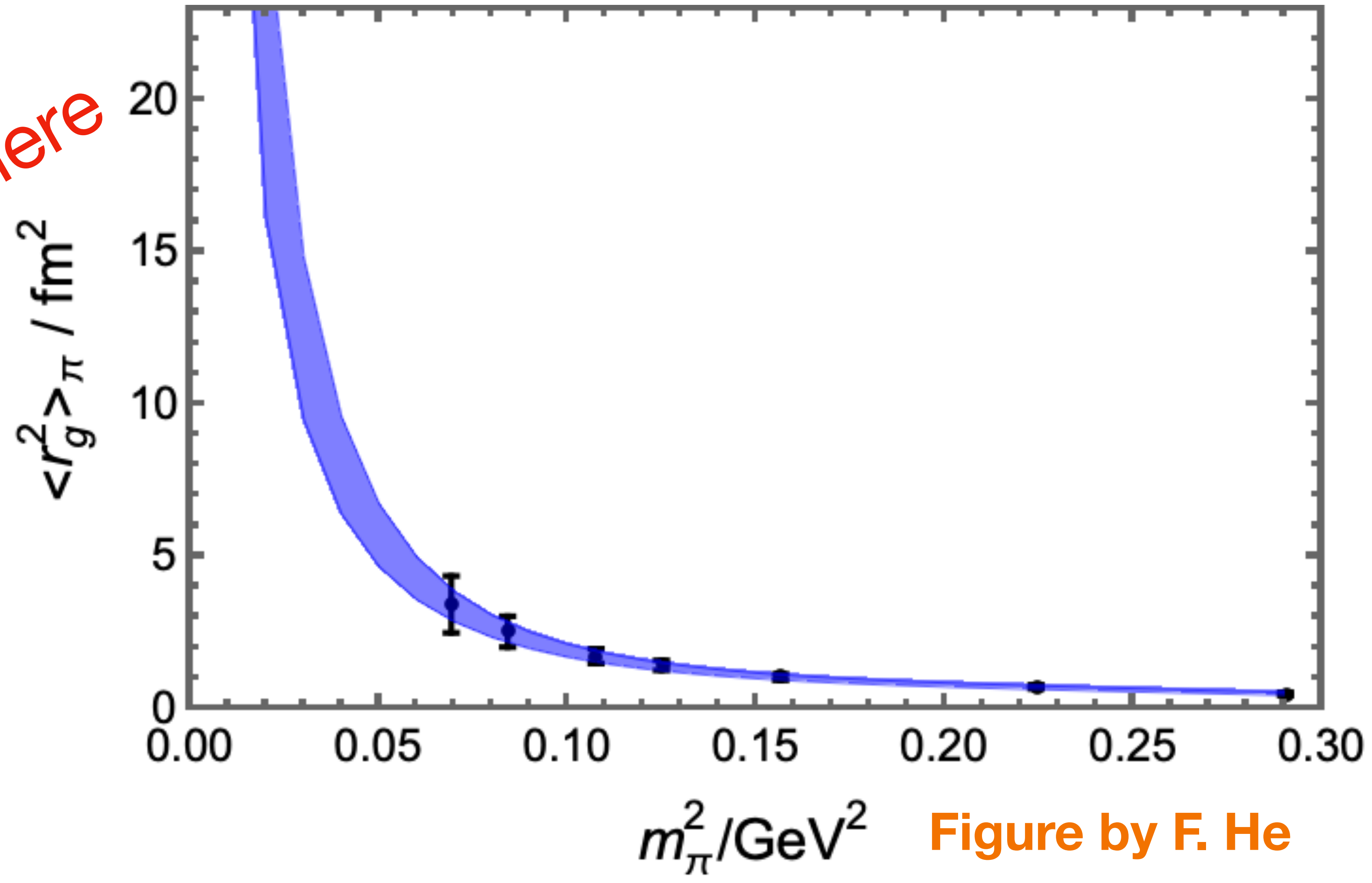
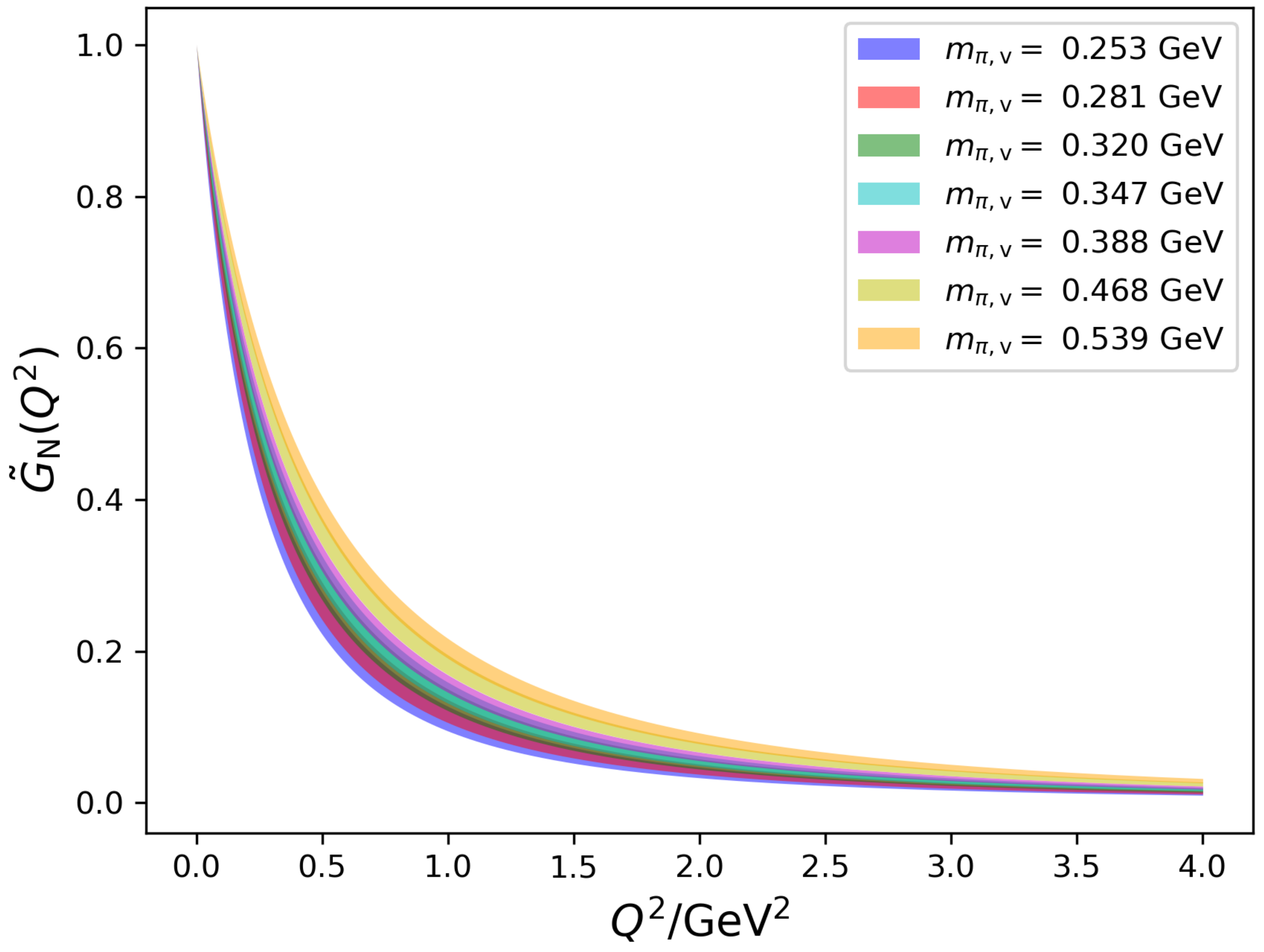


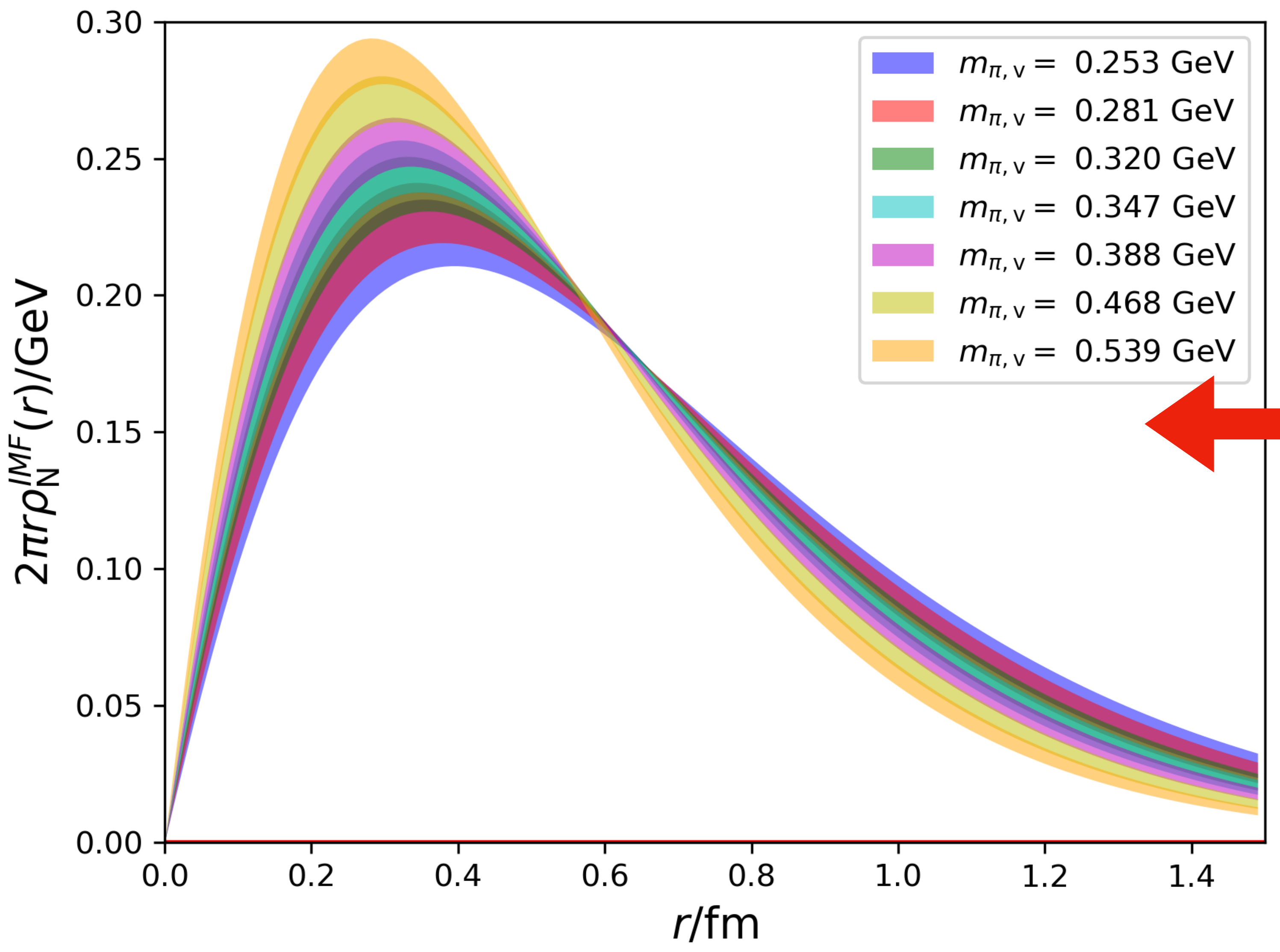
Figure by F. He

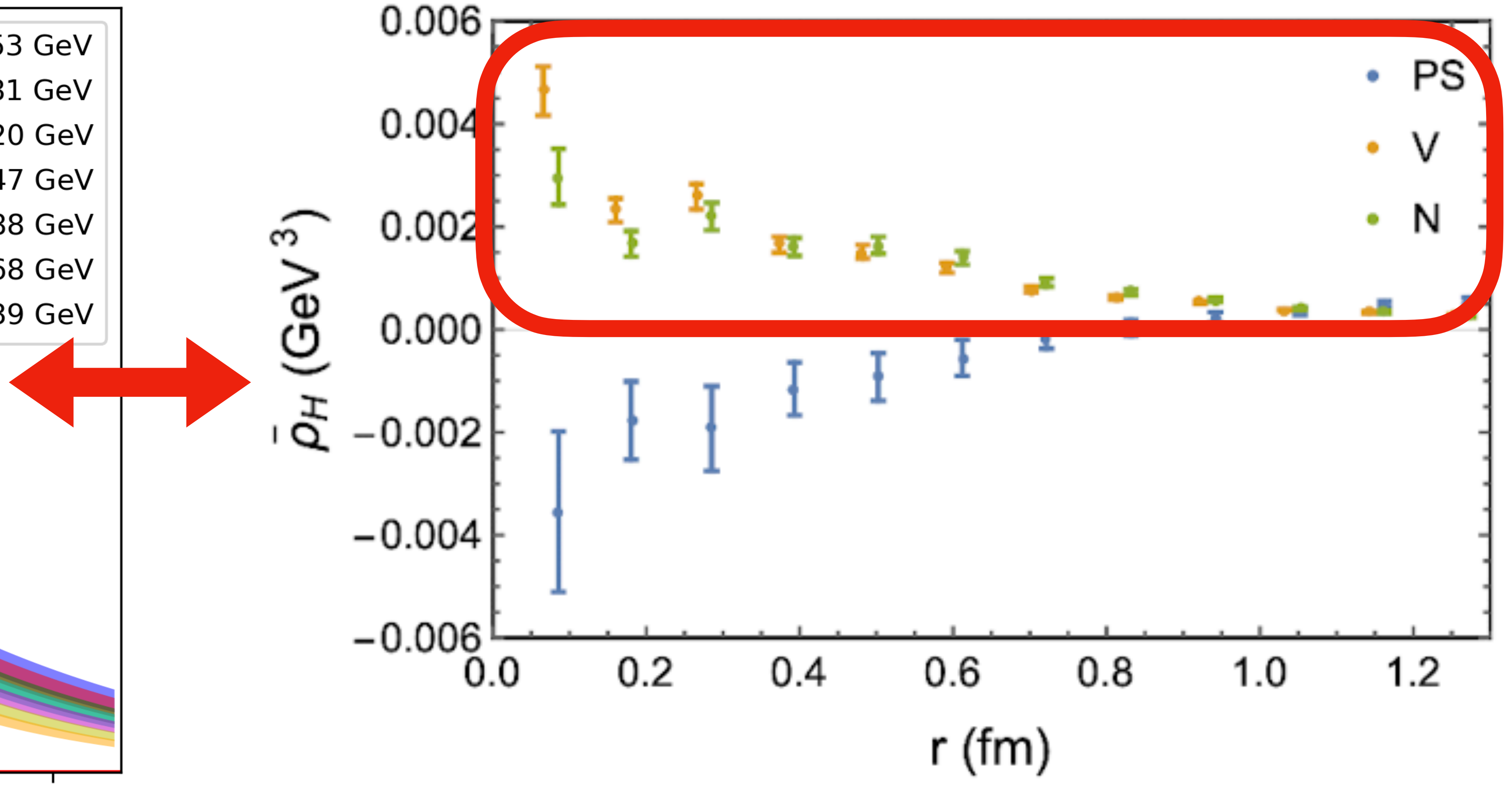
Results for the nucleon preliminary

Form factors



Spatial distribution





F. He, P. Sun and Y.B. Yang (χQCD) (PRD 2021, 2101.04942)

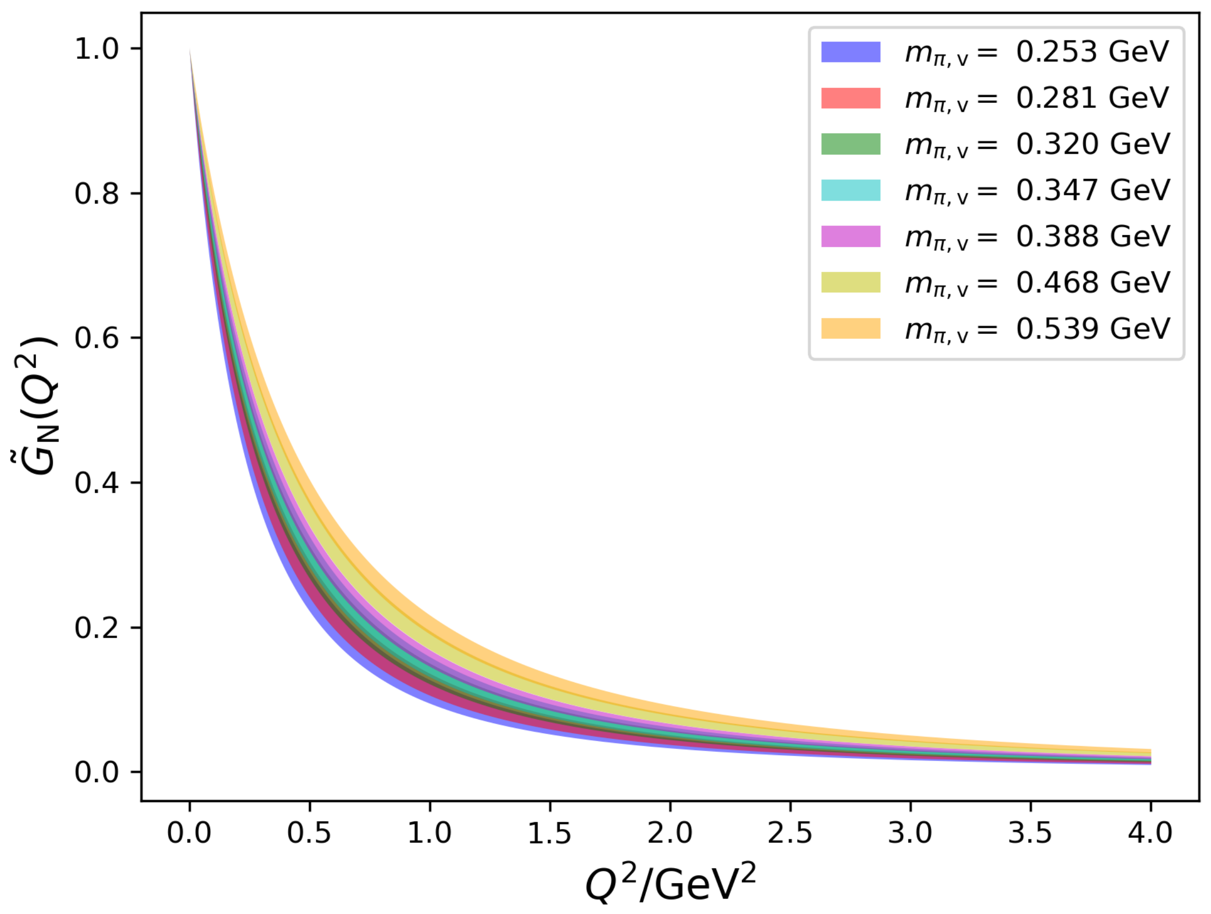
2D spatial distribution in the infinite momentum frame (IMF) $P_z \to \infty$ and $\mathbf{P} \cdot \mathbf{\Delta} = 0$

- **NO sign change** of glue trace anomaly form factor of the nucleon
- **NO sign change** of glue trace anomaly spatial distribution of the nucleon, always positive.

Mass radius of the nucleon preliminary

- Radius of the trace anomaly(glue part)

$$\langle r^2\rangle_{\rm m}({\rm H}) = -6\,\frac{d\mathscr{F}_{\rm m,H}(Q^2)}{dQ^2}\Bigg|_{Q^2\to 0} \sim \langle r_g^2\rangle_{\rm ta}({\rm H}) = -6\,\frac{dG_{\rm H}(Q^2)}{dQ^2}\Bigg|_{Q^2\to 0}$$



- Lattice: chiral extrapolation to the physical pion mass using

$$\langle r_g^2\rangle_{\rm ta}({\rm N}) = a_{\rm N} + b_{\rm N}\, m_\pi^2 + c_{\rm N}\, m_\pi^2 \log\left(\frac{m_\pi^2}{m_{\pi,phy}^2}\right)$$

$$R_{\rm m} \simeq R_g \equiv \sqrt{\langle r_g^2\rangle_{\rm ta}^{\rm phys}({\rm N})} = 0.89(10)(07)\,{\rm fm}$$

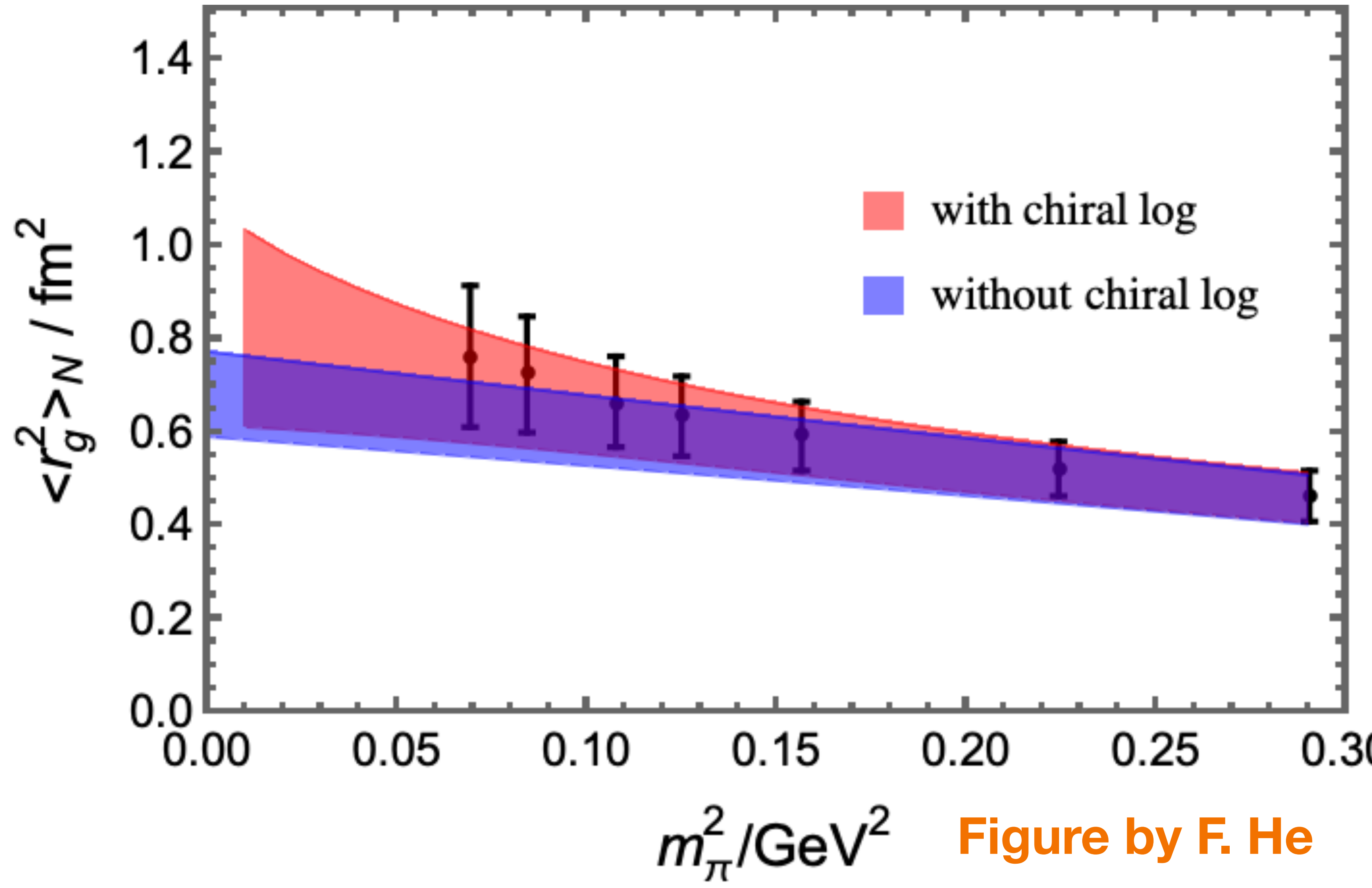


Figure by F. He

Trace anomaly form factors and GFF

X. Ji, arXiv:2102.07830 [hep-ph]

K.-F. Liu, arXiv:2302.11600 [hep-ph]

Trace Anomaly Form Factors

$$T^\mu_\mu = \sum_f m_f \overline{\psi}_f \psi_f + \underbrace{\left[\sum_f m_f \gamma_m(g) \overline{\psi}_f \psi_f + \frac{\beta(g)}{2g} F^2 \right]}_{(T^\mu_\mu)^a \text{ trace anomaly, RG invariant}}$$

Energy-momentum Tensor

$$T^{\mu\nu} = \hat{T}^{\mu\nu} + \bar{T}^{\mu\nu}$$

Trace part: $\frac{\eta_{\mu\nu}}{d} T^\alpha_\alpha$ → $A_{q,g}$ $\bar{C}_{q,g}$

Traceless part: $(T^{\mu\nu} - \frac{\eta_{\mu\nu}}{d} T^\alpha_\alpha)$ → $A_{q,g}$ $B_{q,g}$ $D_{q,g}$

Tong, et al., Physics Letters B 823, 136751 (2021)

Pefkou, et al., Phys. Rev. D 105, 054509 (2022)

Hackett, et al., (arXiv:2307.11707)

Y. Hatta, arXiv:1810.05116 [hep-ph]

Gravitational Form Factors

moments of Generalized Parton Distribution (GPD)

$$\langle P' | (T^{\mu\nu}_{q,g}) | P \rangle / 2m_N = \bar{u}(P') \Big[A_{q,g}(Q^2) \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g}(Q^2) \frac{\bar{P}^{(\mu} i\sigma^{\nu)\alpha} q_\alpha}{2m_N} + D_{q,g}(Q^2) \frac{q^\mu q^\nu - g^{\mu\nu} q^2}{m_N} + \bar{C}_{q,g}(Q^2) m_N \eta^{\mu\nu} \Big] u(P)$$

$$1 = \int \frac{d^3p}{(2\pi)^3} |p\rangle \frac{m}{E_p} \langle p|, \quad |p\rangle = \sqrt{\frac{E_p}{m}} a_p^+ |\Omega\rangle$$

$$|p\rangle = \sqrt{2E_p} a_p^+ |\Omega\rangle$$

$$\langle p', s' | T^\mu_\mu | p, s \rangle = m_N \mathscr{F}_{m,N}(Q^2) \bar{u}(p', s') u(p, s)$$

$$\langle P' | T^\mu_\mu | P \rangle / 2m_N = \bar{u}(P') \Big[(A(Q^2) m_N - B(Q^2) \frac{Q^2}{4m_N} + 3D(Q^2) \frac{Q^2}{m_N} \Big] u(P)$$

$$\mathscr{F}_{m,H}(Q^2) = \mathscr{F}_{ta,H}(Q^2) + \mathscr{F}_{\sigma,H}(Q^2)$$

$$\langle r^2 \rangle_{ta} = -6 \left(\frac{dA(Q^2)}{dQ^2} + \frac{3D(0)}{M^2} - \frac{d\mathscr{F}_\sigma(Q^2)}{dQ^2} \right)$$

Mass radius of the nucleon preliminary

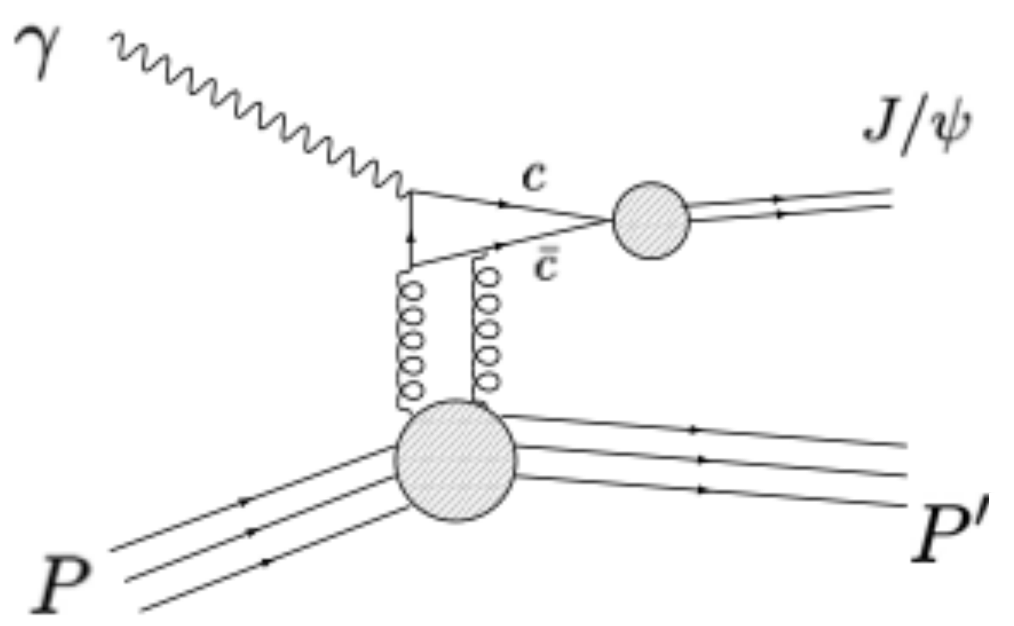
- Our result with lattice QCD

$$R_{\rm m} \simeq R_g \equiv \sqrt{\langle r_g^2 \rangle_{\rm ta}^{\rm phys}(\rm N)} = 0.89(10)(07)\,\rm fm$$

- A recent lattice calculation of the quark and glue GFF

$$R_{\rm m} \equiv \sqrt{\langle r^2 \rangle_{\rm m}^{\rm GFF}(\rm N)} = 1.038(98)\,\rm fm$$

Pefkou, et al., Phys. Rev. D 105, 054509 (2022) Pefkou, Private communication
Hackett, et al., (arXiv:2307.11707)



$$\mathcal{M}_{\gamma P\to\psi P}(t) = -Qe\, c_2\, \frac{16\pi^2 M}{b}\, \langle P'|T|P\rangle.$$

- A holographic GFF calculation with lattice input:

$$R_{\rm m} \simeq R_g = 0.926(8)\,\rm fm$$

K. A. Mamo and I. Zahed, Phys. Rev. D 106, 086004 (2022), arXiv:2204.08857 [hep-ph]

- A direct dipole fit to the recent GlueX Collaboration (experimental) data:

$$R_{\rm m} \equiv \sqrt{\langle R_{\rm m}^2 \rangle} \simeq R_g = 0.55(3)\,\rm fm$$

D. E. Kharzeev, Phys. Rev. D 104, 054015 (2021),arXiv:2102.00110
A. Ali et al. (GlueX), Phys. Rev. Lett. 123, 072001 (2019), arXiv:1905.10811

- Using the quark (from lattice) and the glue GFF from fitting the near-threshold J/Ψ production at $\xi > 0$

$$R_{\rm m} \simeq R_g = 1.20(13)\,\rm fm$$

Y. Guo, X. Ji, Y. Liu, and J. Yang, Phys. Rev. D 108, 034003 (2023), arXiv:2305.06992 [hep-ph]

Conclusion and outlook

- We have calculated the trace anomaly form factors of the EMT(glue part) with lattice QCD to reveal the mass distribution within hadrons:

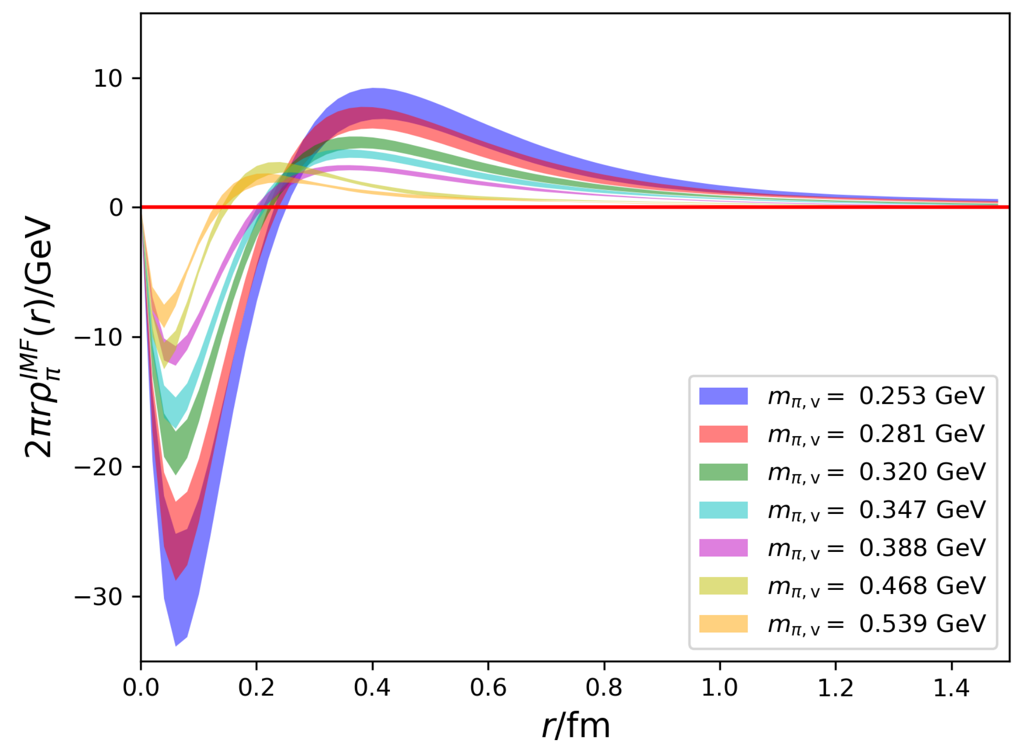
  - For the pion, we find a unique **sign change** in both $G_\pi(Q^2)$ and $\rho_\pi^{\mathrm{IMF}}(\mathbf{r}_\perp)$.

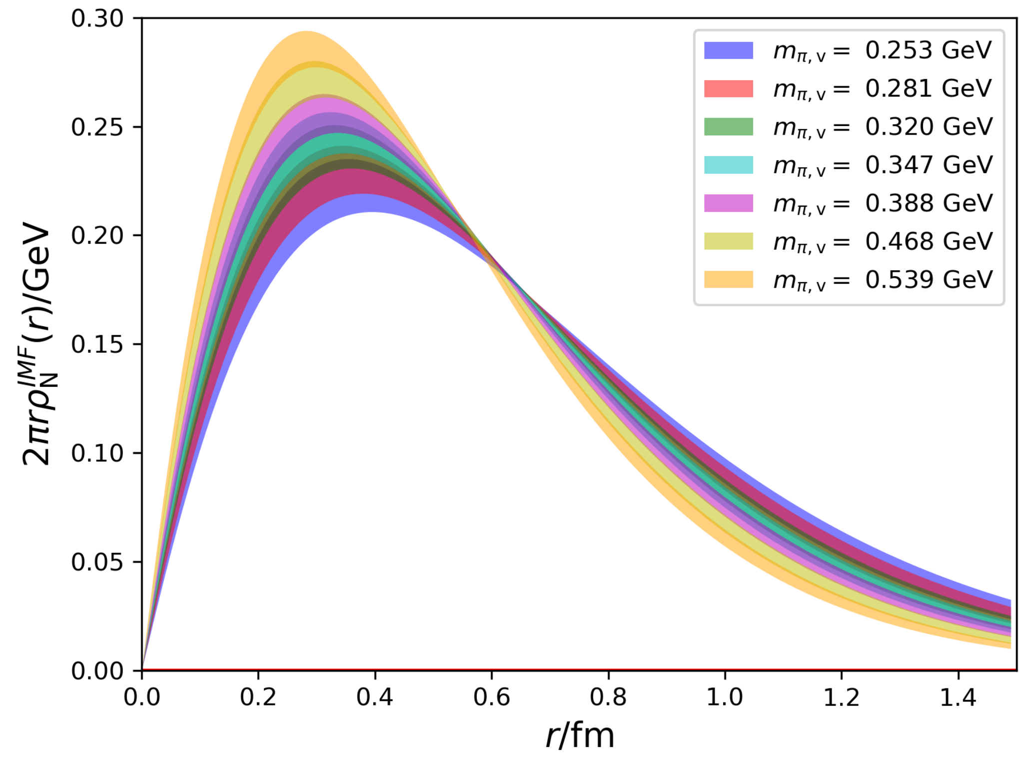
$$\langle r_g^2 \rangle_{\mathrm{ta}}^{\mathrm{phys}}(\pi) = 21.5(5.2)(11.7)\,\mathrm{fm}^2$$

Possibly large systematic errors here

  - For the nucleon, we find **no sign change** in both $G_{\mathrm{N}}(Q^2)$ and $\rho_{\mathrm{N}}^{\mathrm{IMF}}(\mathbf{r}_\perp)$.

$$R_{\mathrm{m}} \simeq R_g \equiv \sqrt{\langle r_g^2 \rangle_{\mathrm{ta}}^{\mathrm{phys}}(\mathrm{N})} = 0.89(10)(07)\,\mathrm{fm}$$





Outlook

- In the future, we will include the quark part for the pion, nucleon and ρ meson.

- Calculations on lattice ensembles with physical quark masses and smaller lattice spacings for extrapolations to the continuum limit.

$$T^\mu_\mu = \underbrace{\sum_f m_f \bar{\Psi}_f \Psi_f}_{\text{the } \sigma \text{ term}} + \underbrace{\frac{\beta}{2g} F^2 + \sum_f \gamma_m m_f \bar{\Psi}_f \Psi_f}_{\langle (T^\mu_\mu)_{\mathrm{a}} \rangle \text{ trace anomaly,RG invariant}}$$

m_π^{phys} $\quad a \to 0$

Thanks for your attention!