Inclusive semileptonic B-decays from lattice QCD

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JLAB theory seminar
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Based on "*Lattice QCD study of inclusive semileptonic decays of heavy mesons*" [hep-lat] 2203.11762

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Outline

1. Introduction
   - Exclusive semileptonic decays

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1. Introduction
   - Exclusive semileptonic decays
2. Inclusive decays formalism
3. Inclusive decays on the lattice
   - HLT algorithm
   - Lattice computation
4. OPE comparison

Outline

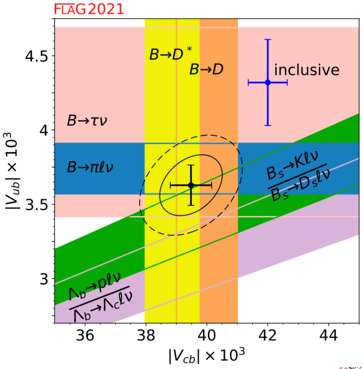
Anomalies & tensions

Some interesting anomalies in Flavour Physics:

Lepton Flavour Universality Violation ($\simeq 3\sigma$), $g-2$ (4σ?)

A persistent tension

The tension between the exclusive and inclusive determination of the CKM parameter V_{cb} is $\simeq 2.7\sigma$



The CKM matrix

Fundamental parameters

The CKM matrix is a 3×3 unitary matrix which can be parametrised by three mixing angles and one CP-violating phase. CKM elements are fundamental parameters of SM.

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

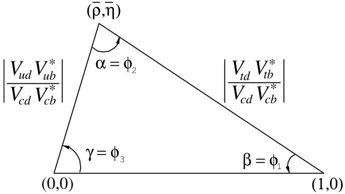


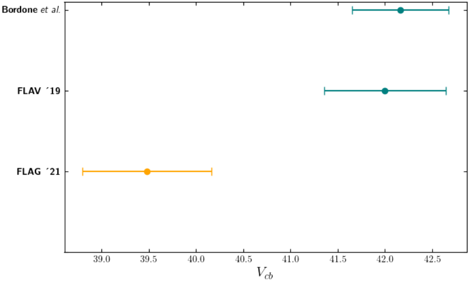
Figure: [PDG, CKM review]

V_{cb} problem

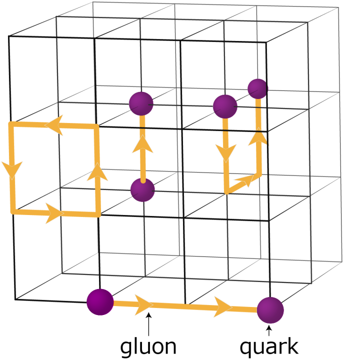
$$|V_{cb}|_{inc.} = 42.00(64)10^{-3}$$ [HFLAV '19]

$$|V_{cb}|_{inc.} = 42.16(51)10^{-3}$$ [Bordone, Capdevila, Gambino '21]

$$|V_{cb}|_{exc.} = 39.48(68)10^{-3}$$ [FLAG '21]



Lattice QCD



- systematically improvable tool for non-perturbative QCD calculations
- work in a discretised $4D$ Euclidean spacetime in a finite volume $L^3 \times T$
- The action is interpreted as the Boltzmann weight factor which allows for MC simulations
- Extract physics from Correlation functions of operators

$$\langle O \rangle = \mathcal{Z}^{-1} \int [DU] e^{-S_{QCD}[U]} O[U] \approx \frac{1}{N} \sum_{i=1}^{N} O[U_i]$$

Gauge ensembles

On the lattice we can generate lots of different gauge configs collected in what is called an *ensemble* $\{U_i\}$. On this ensemble one can compute Euclidean correlators

2-Pt Correlator

For example, a correlator involving two operators can be written as:

$$\sum_n \langle 0|\hat{O}_2|n\rangle\langle n|\hat{O}_1|0\rangle e^{-tE_n} = \mathcal{Z}^{-1}\int [DU]e^{-S[U]}\hat{O}_1\hat{O}_2(U_i)$$

From the correlators one can extract the relevant physics

ETMC ensemble

B55.32 ensemble

Uses the Twisted Mass QCD action with $N_f = 2+1+1$ sea quarks.

$L^3 \times T$	N_{cnfg}	a		
$32^3 \times 64$	150	0.0815(30)		
m_{ud}^{phys}	m_s^{phys}	m_c^{phys}	m_π	
3.70(17)	99.6(4.3)	1176(39)	375(13)	(MeV)
$a\mu_s$	$a\mu_c$	$a\mu_b$		
0.021	0.25	0.5		

Unphysically light B_s mass

This corresponds to $m_b(\overline{\mathrm{MS}} = 2\ \mathrm{GeV}) = 2.4\ \mathrm{GeV}$ which gives a meson mass $M_{B_s} = 3.08(11)\ \mathrm{GeV}$

Exclusive semileptonic B-decays

Exclusive strategy

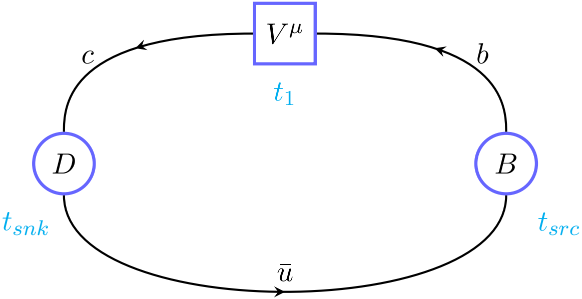
V_{cb} can be extracted from exclusive semileptonic decays:
$\frac{d\Gamma}{dw} \propto |V_{cb}|^2 |\mathcal{G}(w)|^2$
One can express the relevant matrix element of the decay in terms of Form Factors

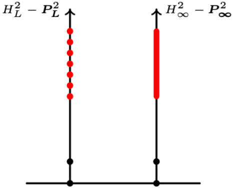
For example:

$$\langle D(p_D)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_D^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu\right] + f_0(q^2)\frac{M_B^2 - M_D^2}{q^2} q^\mu$$

3-Pt function

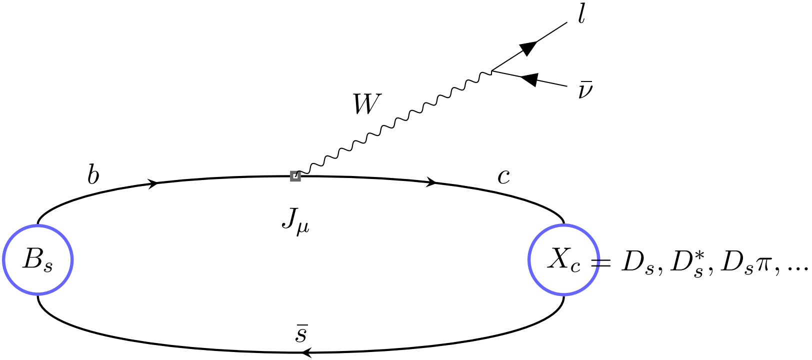
In lattice QCD we can extract the Form Factors fitting the three-point functions as a funtion of lattice time. Then one can use the experimental result for the Exclusive decay rate together with these form factors to obtain an estimate of V_{cb}





Inclusive $B_s \to X_c l\nu$

$$J_\mu = (V - A)_\mu = \bar{c}\gamma_\mu(1 - \gamma_5)b$$



Decay rate

we start by considering the differential decay rate:

$$\frac{d\Gamma}{dq^2 dq^0 dE_l} = \frac{G_F^2 |V_{cb}|^2}{8\pi^2} L_{\mu\nu} W^{\mu\nu}$$

with:

$$W_{\mu\nu}(w, \boldsymbol{q}) = \frac{(2\pi)^3}{2m_{B_s}} \langle \bar{B}_s(\mathbf{0}) | J_\mu^\dagger(0) \delta(\hat{H} - w) \delta^3(\hat{P} + \boldsymbol{q}) J_\nu | \bar{B}_s(\mathbf{0}) \rangle$$

Integrating over E_l:

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{\boldsymbol{q}^2_{max}} d\boldsymbol{q}^2 \sqrt{\boldsymbol{q}^2} \sum_{l=0}^{2} (\sqrt{\boldsymbol{q}^2})^{2-l} Z^{(l)}(\boldsymbol{q}^2)$$

Introducing the kernel

$$\Theta^{(l)}(w_{max} - w) = (w_{max} - w)^l \theta(w_{max} - w)$$

We can write

$$Z^{(l)}(\boldsymbol{q}^2) = \int_0^\infty dw \Theta^{(l)}(w_{max} - w) T^{(l)}(w, \boldsymbol{q}^2)$$

with:

$$T^{(0)} = W^{00} + \sum_{i,j=1}^{3} \hat{n}_i \hat{n}_j W^{ij} - \sum_{i=1}^{3} \hat{n}_i (W^{0i} + W^{i0})$$

Connection to hadronic tensor

$$W_{\mu\nu}(w, \boldsymbol{q}) \;=\; \frac{(2\pi)^3}{2m_{B_s}} \langle \bar{B}_s(\mathbf{0}) | J_\mu^\dagger(0) \delta(\hat{H} - w) \delta^3(\hat{P} + \boldsymbol{q}) J_\nu | \bar{B}_s(\mathbf{0}) \rangle$$

$\updownarrow$

$$M_{\mu\nu}(t; \boldsymbol{q}) = e^{-m_{B_s} t} \int d^3x \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2m_{B_s}} \langle \bar{B}_s(\mathbf{0}) | J_\mu^\dagger(\boldsymbol{x}, t) J_\nu(0, \mathbf{0}) | \bar{B}_s(\mathbf{0}) \rangle$$

Inverse Problem

LQCD

$$M_{\mu\nu}(t, \boldsymbol{q}) = \int d^3x \frac{e^{i\boldsymbol{q}\cdot\boldsymbol{x}}}{2m_{B_s}} \langle \bar{B}_s(\boldsymbol{0})| J_\mu^\dagger(\boldsymbol{0}, 0) e^{-t\hat{H} + i\hat{\boldsymbol{P}}\cdot\boldsymbol{x}} J_\nu(\boldsymbol{0}, 0) |\bar{B}_s(\boldsymbol{0})\rangle$$

$$= \frac{(2\pi)^3}{2m_{B_s}} \langle \bar{B}_s(\boldsymbol{0})| J_\mu^\dagger(\boldsymbol{0}, 0) e^{-t\hat{H}} \delta^3(\hat{\boldsymbol{P}} + \boldsymbol{q}) J_\nu(\boldsymbol{0}, 0) |\bar{B}_s(\boldsymbol{0})\rangle$$

$$= \int_0^\infty dw \, W_{\mu\nu}(w, \boldsymbol{q}) \, e^{-wt}$$

WHAT WE WANT

Smooth function

On the lattice we have to introduce the lattice spacing a:

$$G^{(l)}(a\tau;\boldsymbol{q}) = \int_0^\infty dw T^{(l)}(w,\boldsymbol{q})e^{-aw\tau}$$

If a function is infinitely differentiable (smooth) then we can approximate it numerically:

$$f(w) = \sum_\tau^\infty g_\tau e^{-aw\tau}$$

So that:

$$\sum_\tau^\infty g_\tau G^{(l)}(a\tau;\boldsymbol{q}) = \int_0^\infty dw T^{(l)}(w,\boldsymbol{q}) f(w)$$

Smeared kernel

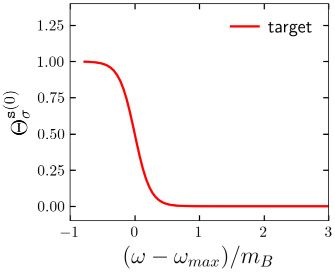
Problem

The θ-function is definetly NOT smooth

Solution

We need to introduce a *smeared* kernel θ_σ

$$\Theta_\sigma^{(l)}(w_{max} - w) = (w_{max} - w)^l \theta_\sigma(w_{max} - w)$$
$$= m_{B_s}^l \sum_{\tau=1}^{\infty} g_\tau^{(l)}(w_{max}, \sigma) e^{-aw\tau}$$



Smeared spectral density

$$\sum_{\tau}^{\infty} g_\tau(w_{max},\sigma) G^{(l)}(a\tau;\boldsymbol{q}) = \int_0^\infty dw T^{(l)}(w,\boldsymbol{q}) \Theta_\sigma^{(l)}(w_{max}-w)$$

$$Z^{(l)}(\boldsymbol{q}^2) = \int_0^\infty dw \Theta^{(l)}(w_{max}-w) T^{(l)}(w,\boldsymbol{q}^2)$$

$$Z_\sigma^{(l)}(\boldsymbol{q}^2) = \int_0^\infty dw \Theta_\sigma^{(l)}(w_{max}-w) T^{(l)}(w,\boldsymbol{q}^2) = \sum_{\tau}^{\infty} g_\tau(w_{max},\sigma) G^{(l)}(a\tau;\boldsymbol{q})$$

HLT algorithm

The details can be found in:
[Hansen, Lupo, Tantalo '19, Phys. Rev. D 99, 094508]
[hep-lat/1903.06476]
we want to reconstruct:

$$\Theta_{\sigma}^{(l)}(w_{max} - w) = \sum_{\tau=1}^{\tau_{max}} g_{\tau} e^{-aw\tau}$$

$$W_{\lambda}[g] = (1-\lambda)\frac{A[g]}{A[0]} + \lambda B[g]$$

$$\left.\frac{\partial W_{\lambda}[g]}{\partial g_{\tau}}\right|_{g_{\tau}=g_{\tau}^{\lambda}} = 0$$

$A[g^\lambda]$ is the reconstruction bias

$$A[g] = a \int_{E_0}^{\infty} dw \left\{ \Theta_\sigma^{(l)} - \sum_{\tau=1}^{\tau_{max}} g_\tau e^{-aw\tau} \right\}^2$$

$B[g^\lambda]$ is the statistical variance

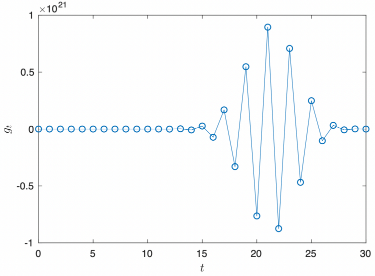
$$B[g] = \sum_{\tau,\tau'=1}^{\tau_{max}} g_\tau g_{\tau'} \frac{Cov[G^{(l)}(a\tau), G^{(l)}(a\tau')]}{[G^{(l)}(0)]^2}$$

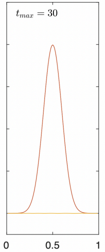
g_τ^λ defines the approximation of the kernel

$$\Theta_\sigma^\lambda = \sum_{\tau=1}^{\tau_{max}} g_\tau^\lambda e^{-aw\tau}$$

the best choice for λ can be found via:

$$\left.\frac{\partial W(\lambda)}{\partial \lambda}\right|_{\lambda=\lambda_\star} = 0$$



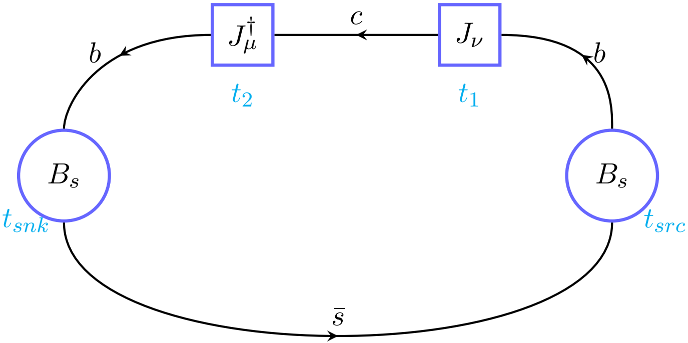


$$Z_\sigma^{(l)\lambda_\star}(\boldsymbol{q}^2) = \int_0^\infty dw \Theta_\sigma^{(l)\lambda_\star}(w_{max} - w) T^{(l)}(w, \boldsymbol{q}^2)$$
$$= \sum_\tau^\infty g_\tau^{\lambda_\star}(w_{max}, \sigma) G^{(l)}(a\tau; \boldsymbol{q})$$

Euclidean 4-point Correlator

[Gambino,Hashimoto'20, Phys. Rev. Lett. 125, 032001, hep-lat/2005.13730]

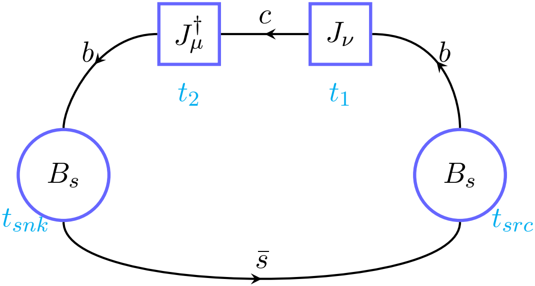
$$C_{\mu\nu}(t_{snk}, t_2, t_1, t_{src}; \boldsymbol{q}) = \int d^3x e^{i\boldsymbol{q}\cdot\boldsymbol{x}} T\langle 0|\tilde{\phi}_{B_s}(\mathbf{0}, t_{snk}) J_\mu^\dagger(\boldsymbol{x}, t_2) J_\nu(\mathbf{0}, t_1) \tilde{\phi}_{B_s}^\dagger(\mathbf{0}, t_{src})|0\rangle$$

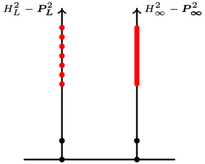


Euclidean 4-point Correlator

[Gambino,Hashimoto'20, Phys. Rev. Lett. 125, 032001, hep-lat/2005.13730]

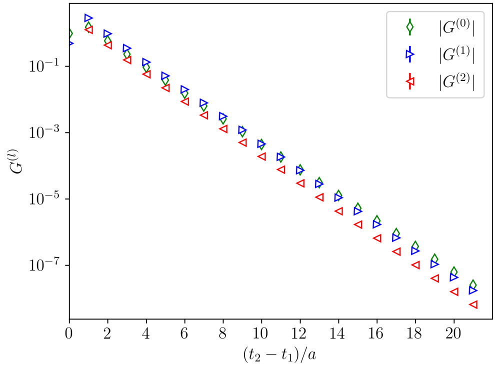
$$C_{\mu\nu}(t_{snk}, t_2, t_1, t_{src}; \boldsymbol{q}) = \int d^3x e^{i\boldsymbol{q}\cdot\boldsymbol{x}} T\langle 0|\tilde{\phi}_{B_s}(\mathbf{0}, t_{snk}) J^\dagger_\mu(\boldsymbol{x}, t_2) J_\nu(\mathbf{0}, t_1)$$
$$\tilde{\phi}^\dagger_{B_s}(\mathbf{0}, t_{src})|0\rangle$$





G correlators

$$G^{(l)}(t_2 - t_1; \boldsymbol{q}) = M_{\mu\nu}(t_2 - t_1; \boldsymbol{q}) = Z_{B_s} \lim_{\substack{t_{\text{snk}} \to +\infty \\ t_{\text{src}} \to -\infty}} \frac{C_{\mu\nu}(t_{snk}, t_2, t_1, t_{src}; \boldsymbol{q})}{C(t_{snk} - t_2)C(t_1 - t_{src})}$$



Correlators and spectral density

LQCD ← $G^{(l)}(t, \boldsymbol{q}^2)$

$$G^{(l)}(t, \boldsymbol{q}^2) = \int_0^\infty dw\, T^{(l)}(w, \boldsymbol{q}^2)\, e^{-wt}$$

WHAT WE WANT ← $T^{(l)}(w, \boldsymbol{q}^2)$

$$\sum_\tau^\infty g_\tau(w_{max}, \sigma) G^{(l)}(a\tau; \boldsymbol{q}) = \int_0^\infty dw T^{(l)}(w, \boldsymbol{q}) \Theta_\sigma^{(l)}(w_{max} - w)$$

$$\Theta_\sigma^\lambda = \sum_{\tau=1}^{\tau_{max}} g_\tau^\lambda e^{-aw\tau}$$

Three kernels

$$\theta_\sigma^{\mathsf{s}}(x) = \frac{1}{1+e^{-\frac{x}{\sigma}}}\,, \qquad \theta_\sigma^{\mathsf{s1}}(x) = \frac{1}{1+e^{-\sinh\left(\frac{x}{r^{\mathsf{s1}}\sigma}\right)}}\,,$$

$$\theta_\sigma^{\mathsf{e}}(x) = \frac{1+\mathsf{erf}\left(\frac{x}{r^{\mathsf{e}}\sigma}\right)}{2}\,,$$

Smeared kernel reconstruction

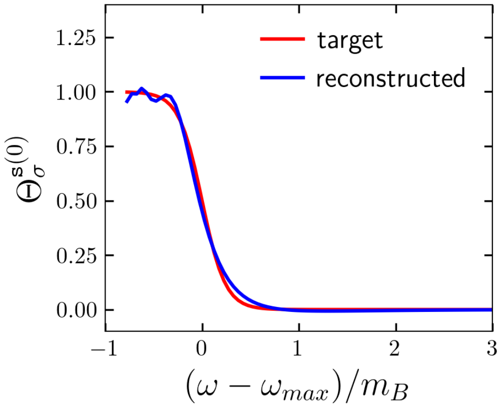


Figure: $\lambda = \lambda_\star$

Smeared Kernel λ

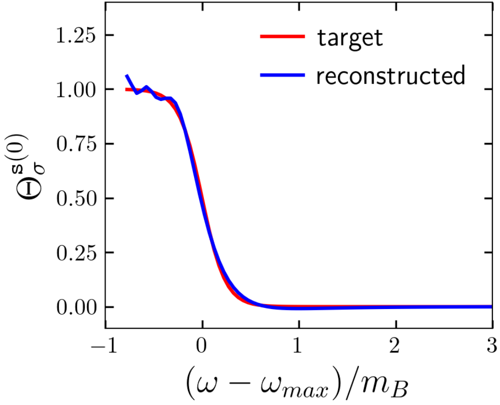


Figure: $\lambda < \lambda_\star$

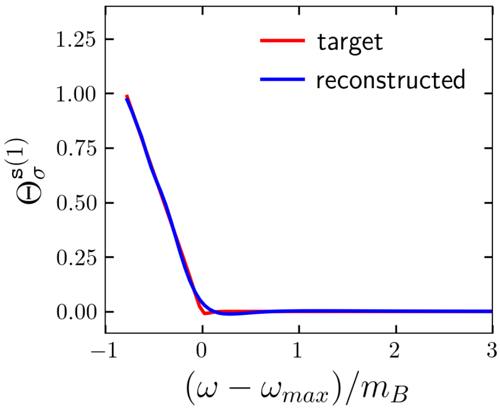


Figure: $\lambda = \lambda_\star$

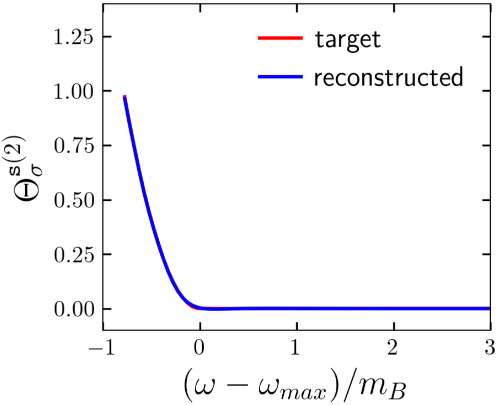


Figure: $\lambda = \lambda_\star$

Modified Sigmoid

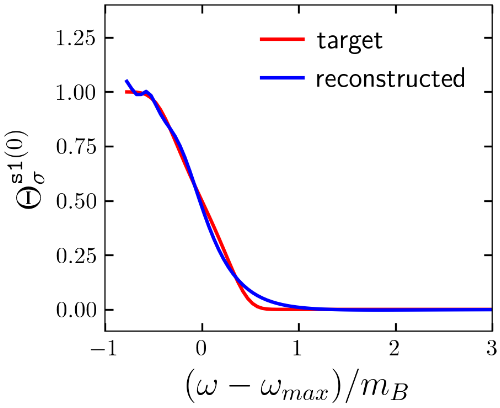


Figure: $\lambda = \lambda_{\star}$

Error Function

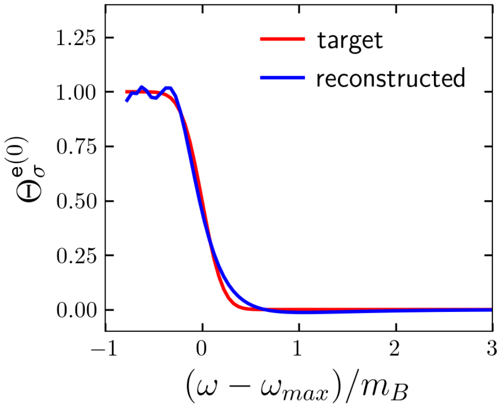
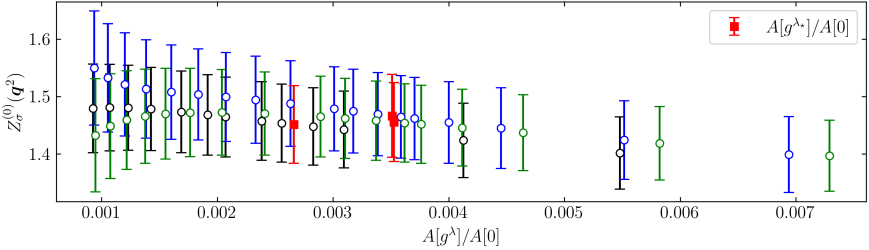


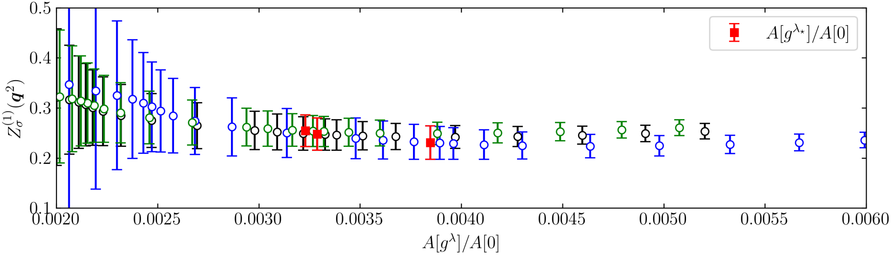
Figure: $\lambda = \lambda_\star$

$$Z_{\sigma}^{(l)\lambda_\star}(\boldsymbol{q}^2) = \int_0^\infty dw \Theta_{\sigma}^{(l)\lambda_\star}(w_{max} - w) T^{(l)}(w, \boldsymbol{q}^2)$$
$$= \sum_{\tau}^{\infty} g_{\tau}^{\lambda_\star}(w_{max}, \sigma) G^{(l)}(a\tau; \boldsymbol{q})$$

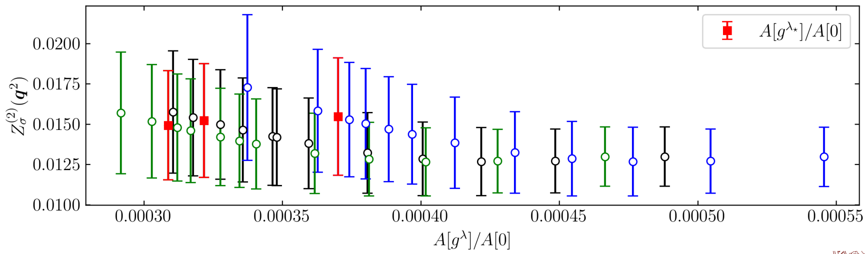
Smeared Spectral Density

Now we can apply the smeared kernel to $Z_\sigma^{(l)}(\boldsymbol{q})$ which is obtained by applying the coefficients g_τ^λ to the correlator $G^{(l)}(t, \boldsymbol{q}^2)$ and perform the integral over the allowed phase space





$A[g^{\lambda_\star}]/A[0]$
$Z_\sigma^{(1)}(\boldsymbol{q}^2)$
0.5
0.4
0.3
0.2
0.1
0.0020
0.0025
0.0030
0.0035
0.0040
0.0045
0.0050
0.0055
0.0060
$A[g^\lambda]/A[0]$

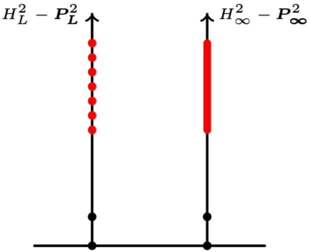


$A[g^{\lambda_\star}]/A[0]$
$Z_\sigma^{(2)}(\boldsymbol{q}^2)$
0.0200
0.0175
0.0150
0.0125
0.0100
0.00030
0.00035
0.00040
0.00045
0.00050
0.00055
$A[g^\lambda]/A[0]$

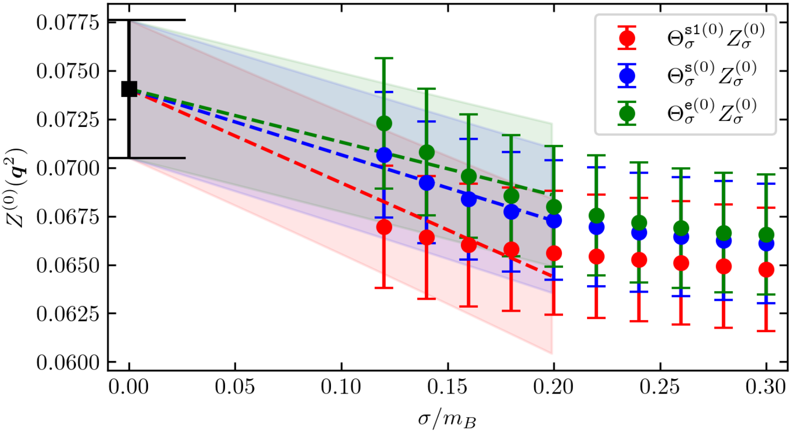
$\sigma \to 0$ limit

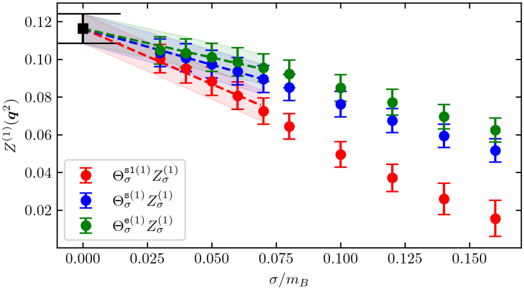
$$Z^{(l)}(\boldsymbol{q}^2) = \lim_{\sigma \to 0} \left(\lim_{V \to \infty} \right) \int_0^\infty dw T^{(l)}(w, \boldsymbol{q}^2) \Theta_\sigma^{(l)}(w_{max} - w)$$

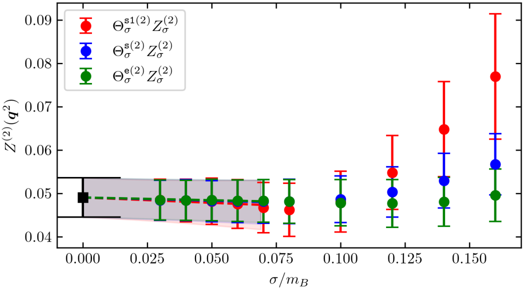
$$= \lim_{\sigma \to 0} \left(\lim_{V \to \infty} \right) m_{B_s}^l \sum_\tau^\infty g_\tau^{(l)}(w_{max}, \sigma) G^{(l)}(a\tau, \boldsymbol{q})$$



Extrapolation to $\sigma = 0$




$Z^{(1)}(\boldsymbol{q}^2)$
0.12
0.10
0.08
0.06
0.04
0.02
0.000
0.025
0.050
0.075
0.100
0.125
0.150
σ/m_B
$\Theta^{s1(1)}_\sigma Z^{(1)}_\sigma$
$\Theta^{s(1)}_\sigma Z^{(1)}_\sigma$
$\Theta^{e(1)}_\sigma Z^{(1)}_\sigma$

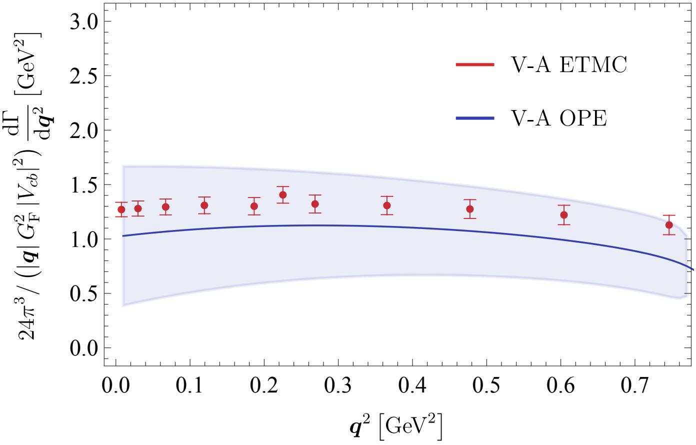

$Z^{(2)}(\boldsymbol{q}^2)$
0.09
0.08
0.07
0.06
0.05
0.04
0.000
0.025
0.050
0.075
0.100
0.125
0.150
σ/m_B
$\Theta^{s1(2)}_\sigma Z^{(2)}_\sigma$
$\Theta^{s(2)}_\sigma Z^{(2)}_\sigma$
$\Theta^{e(2)}_\sigma Z^{(2)}_\sigma$

Differential decay rate

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{\boldsymbol{q}^2_{max}} d\boldsymbol{q}^2 \sqrt{\boldsymbol{q}^2} \sum_{l=0}^{2} (\sqrt{\boldsymbol{q}^2})^{2-l} Z^{(l)}(\boldsymbol{q}^2)$$

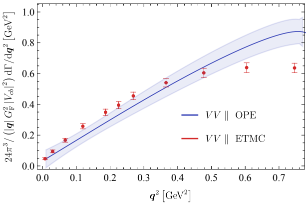
Differential Decay Rate comparison

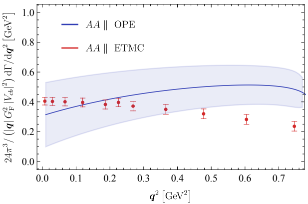
After we take the $\sigma \to 0$ limit, we can confront our results with the analytic results from the OPE

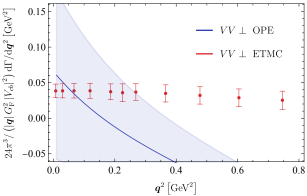


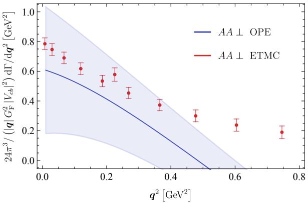
Channel decomposition

One can decompose the contribution to the differential decay rate into $V_\parallel$, $V_\perp$, $A_\parallel$, $A_\perp$





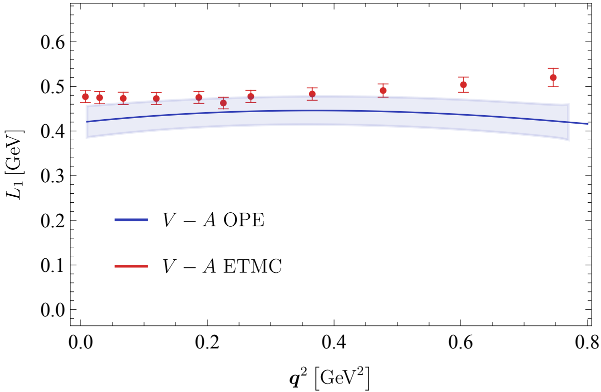




Lepton Energy Moment

We also looked at the Lepton Energy Moment defined as

$$L_1(\boldsymbol{q}^2) = \frac{\int dq_0 dE_l E_l \left[\frac{d\Gamma}{d\boldsymbol{q}^2 dq_0 dE_l}\right]}{\int dq_0 dE_l \left[\frac{d\Gamma}{d\boldsymbol{q}^2 dq_0 dE_l}\right]}$$



Final Results

The last thing to do is to perform the integration over $\boldsymbol{q}^2$

	ETMC	OPE
$\Gamma/\lvert V_{cb}^2\rvert \times 10^{13}$ (GeV)	$0.987(60)$	$1.20(46)$
$\langle E_\ell \rangle$ (GeV)	$0.491(15)$	$0.441(43)$
$\langle E_\ell^2 \rangle$ (GeV ^{2})	$0.263(16)$	$0.207(49)$
$\langle E_\ell^2 \rangle - \langle E_\ell \rangle^2$(GeV^2)	$0.022(16)$	$0.020(8)$
$\langle M_X^2 \rangle$ (GeV^2)	$3.77(9)$	$4.32(56)$

Table: Total width and moments in the ETMC case.

Future prospects

- Perform the analysis with several values of lattice spacing, volumes and $b-$quark masses in order to perform the extrapolations: $a \to 0$, $V \to \infty$, $m_b \to m_b^{phys}$
- Apply this method to other Semileptonic decays for which we have good experimental results for Branching ratios, such as those involving the $D_{(s)}-$meson
- Extend this method to semileptonic decays needed to estimate V_{ub} such as $B \to X_u l\nu$
- Calculate quantities needed for an accurate estimate of the inclusive value of V_{cb}

BACKUP SLIDES

Spectral representation

In the rest frame of the B_s meson, one can write the Hadronic Tensor as:

$$W^{\mu\nu}(p,q) = \sum_{X_c} (2\pi)^3 \delta^{(4)}(p-q-r) \frac{1}{2E_{B_s}(\boldsymbol{p})} \langle \bar{B}_s(\boldsymbol{p}) | J^{\mu\dagger}(0) | X_c(\boldsymbol{r}) \rangle \langle X_c(\boldsymbol{r}) | J^{\nu}(0) | \bar{B}_s(\boldsymbol{p}) \rangle$$

Which can also be written with respect to the QCD Hamiltonian and momentum operators, in the *spectral representation*

$$W_{\mu\nu}(w,\boldsymbol{q}) = \frac{(2\pi)^3}{2m_{B_s}} \langle \bar{B}_s(\boldsymbol{0}) | J_\mu^\dagger(0) \delta(\hat{H} - w) \delta^3(\hat{P} + \boldsymbol{q}) J_\nu(0) | \bar{B}_s(\boldsymbol{0}) \rangle$$

Tensor Decomposition

Accoding to Lorentz invariance and time-reversal symmetry, the Hadronic Tensor can be decomposed as follows

$$W^{\mu\nu}(p,q) = -g^{\mu\nu}W_1(w,\boldsymbol{q}^2) + \frac{p^\mu p^\nu}{m_{B_s}^2}W_2(w,\boldsymbol{q}^2) - \frac{i\varepsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{m_{B_s}^2}W_3(w,\boldsymbol{q}^2) + \frac{q^\mu q^\nu}{m_{B_s}^2}W_4(w,\boldsymbol{q}^2) + \frac{p^\mu q^\nu + p^\nu q^\mu}{m_{B_s}^2}W_5(w,\boldsymbol{q}^2)$$

For convenience we will redefine these components w.r.t. a different basis:

$$\hat{\boldsymbol{n}} = \frac{\boldsymbol{q}}{\sqrt{\boldsymbol{q}^2}} \qquad \boldsymbol{\varepsilon}^{(a)} \cdot \hat{\boldsymbol{n}} = 0 \qquad \boldsymbol{\varepsilon}^{(a)} \cdot \boldsymbol{\varepsilon}^{(b)} = \delta^{ab}$$

$$Y^{(1)} = -\sum_{a=1}^{2} \sum_{i,j=1}^{3} \varepsilon_i^{(a)} \varepsilon_j^{(a)} W^{ij}$$

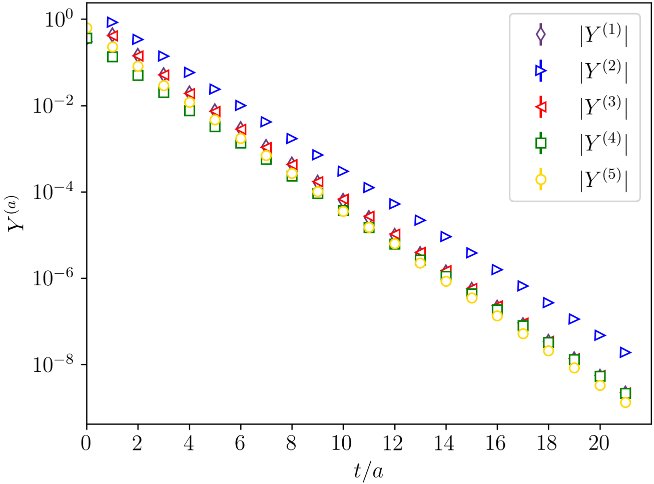
$$Y^{(2)} = W^{00}$$

$$Y^{(3)} = \sum_{i,j=1}^{3} \hat{n}^i \hat{n}^j W^{ij}$$

$$Y^{(4)} = \sum_{i=1}^{3} \hat{n}^i (W^{0i} + W^{i0})$$

$$Y^{(5)} = \frac{i}{2} \sum_{i,j,k=1}^{3} \epsilon^{ijk} \hat{n}^k W^{ij}$$

Y correlators

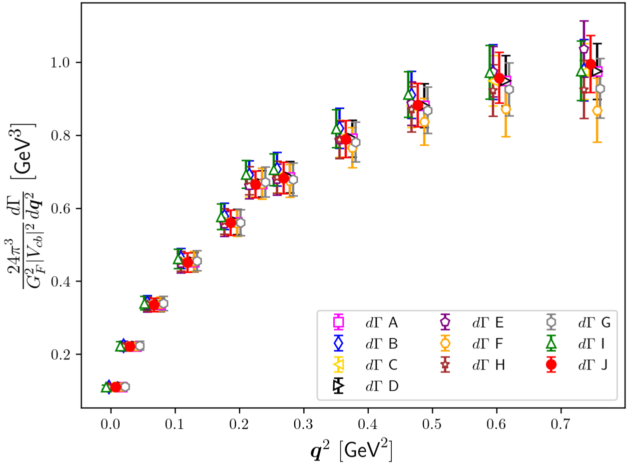


$$T^{(0)} = Y^{(2)} + Y^{(3)} - Y^{(4)}$$

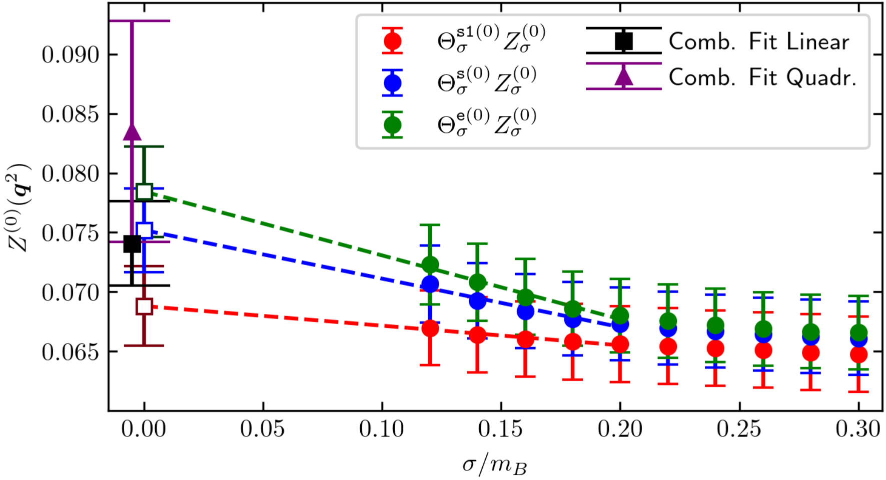
$$T^{(1)} = 2Y^{(3)} - 2Y^{(1)} - Y^{(4)}$$

$$T^{(2)} = Y^{(3)} - Y^{(1)}$$

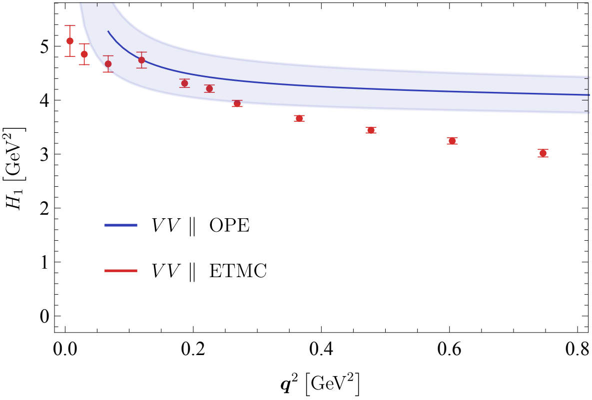
Systematics

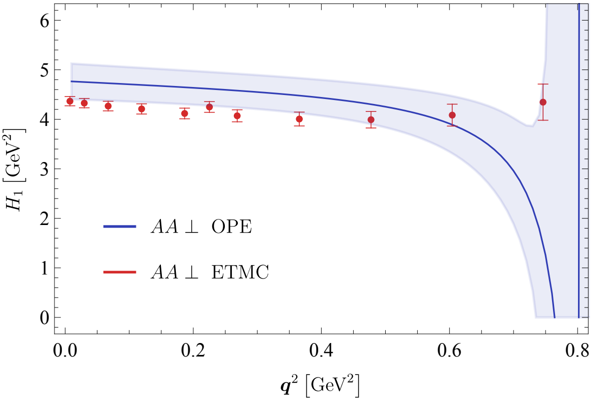


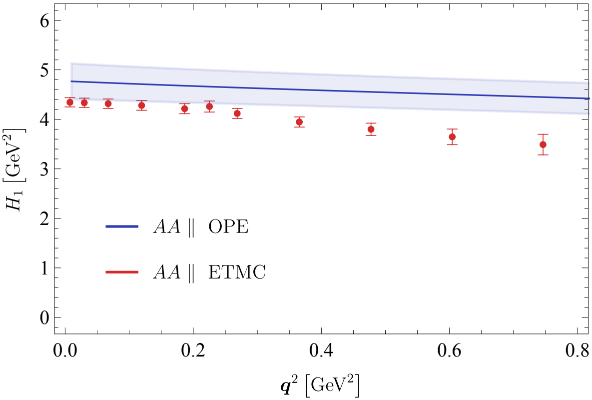
Extrapolation



Hadronic moments




6
5
4
3
2
1
0
H_1 [GeV^2]
0.0
0.2
0.4
0.6
0.8
q^2 [GeV^2]
$AA \perp$ OPE
$AA \perp$ ETMC



6
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0
H_1 [GeV^2]
0.0
0.2
0.4
0.6
0.8
q^2 [GeV^2]
$AA \parallel$ OPE
$AA \parallel$ ETMC