Determination of the Nuclear Spectral Function from $(e, e'p)$ data

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Theory Seminar
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PREAMBLE

⋆ Experiment E12-14-012, approved by PAC42 in 2014, measured the $^{40}_{18}\mathrm{Ar}$, $^{48}_{22}\mathrm{Ti}(e, e'p)$ cross sections in Jefferson Lab Hall A. The data, collected in 2017, will allow an accurate determination of the proton spectral functions of the nuclear targets.

Measurement of the Spectral Function of ^{40}Ar through the $(e, e'p)$ reaction

Proposal (PR12-14-012) submitted to the
Jefferson Lab Program Advisory Committee PAC 42
July 2014

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Publications

- H. Dai *et al.*, Phys. Rev. C **98**, 014617 (2018)
- H. Dai *et al.*, Phys. Rev. C **99**, 054608 (2019)
- M. Murphy *et al.*, Phys. Rev. C **100**, 054606 (2019)
- L. Gu *et al.*, Phys. Rev. C **103**, 034604 (2021)
- L. Jiang *et al.*, Phys. Rev. D **105**, 112002 (2022)

Outline

- ⋆ Why $(e, e'p)$?
  - Nucleon knock out as a tool to shed light on nuclear dynamics
  - Factorisation of the $(e, e'p)$ cross section and nuclear spectral function
  - Corrections arising from the occurrence of final state interactions
- ⋆ Why Argon and Titanium targets?
  - Neutrino energy reconstruction in accelerator based searches of neutrino oscillations
  - Neutrino and antineutrino interactions in liquid argon detectors
- ⋆ Status of the E12-14-012 analysis
- ⋆ Summary and prospects

Why $(e, e'p)$?

The Paradigm of Nuclear Theory

- ⋆ To a remarkably large extent, atomic nuclei can be described as non relativistic systems consisting of point-like particles, whose dynamics are dictated by the Hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk}$$

  - v_{ij} provides an accurate descritpion of the observed properties of the two-nucleon system, in both bound and scettering states, and reduces to Yukawa's one-pion-exchange potential at large distances
  - the inclusion of v_{ijk}—effectively taking into account the occurrence of processes involving the internal structure of the nucleon—is needed to explain the ground-state energies of the three-nucleon systems
  - v_{ij} is spin and isospin dependent, non spherically symmetric, and strongly repulsive at short distance
  - nuclear interactions can not be treated in perturbation theory in the basis of eigenstates of the non interacting system

The Mean-Field Approximation

- Nuclear systematics offers ample evidence supporting the further assumption, underlying the *nuclear shell model*, that the potentials appearing in the Hamiltonian can be eliminated in favour of a mean field

$$H \to H_{MF} = \sum_i \left[\frac{\mathbf{p}_i^2}{2m} + U_i \right] \quad , \quad U_i \sim \langle \sum_{j>i} v_{ij} + \sum_{k>j>i} v_{ijk} \rangle$$

$$\left[\frac{\mathbf{p}_i^2}{2m} + U_i \right] \phi_{\alpha_i} = \epsilon_{\alpha_i} \phi_{\alpha_i} \quad , \quad \alpha \equiv \{n, \ell, j\}$$

- For proposing and developing the nuclear shell model, E. Wigner, M. Goeppert Mayer and J.H.D. Jensen have been awarded the 1963 Nobel Prize in Physics

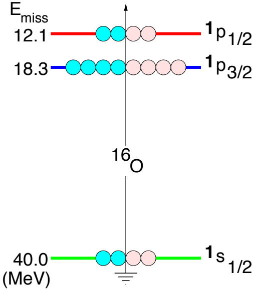
The Nuclear Ground State

- According to the shell model, in the nuclear groud state protons and neutrons occupy the A lowest energy eigenstates of the mean field Hamiltonian

$$H_{MF}\Psi_0 = E_0\Psi_0 \quad , \quad \Psi_0 = \frac{1}{A!}\det\{\phi_\alpha\} \quad , \quad E_0 = \sum_{\alpha\in\{F\}} \epsilon_\alpha$$

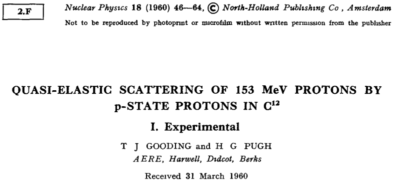
- Ground state of ^{16}O: $Z = N = 8$

$$(1S_{1/2})^2 \;\; , \;\; (1P_{3/2})^4 \;\; , \;\; (1P_{1/2})^2$$



Nucleon Knockout Reactions

- Nucleon knockout reactions, in which the outgoing nucleon and the scattered beam particle are detected in coincidence, have been readily recognized as a powerful tool for investigating the validity of the shell model



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Nuclear Physics **18** (1960) 46—64, © *North-Holland Publishing Co., Amsterdam*

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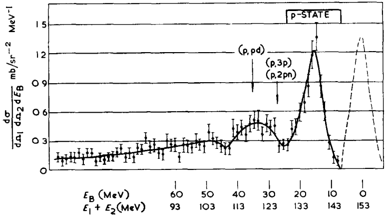
QUASI-ELASTIC SCATTERING OF 153 MeV PROTONS BY p-STATE PROTONS IN C^{12}

I. Experimental

T J GOODING and H G PUGH

AERE, Harwell, Didcot, Berks

Received 31 March 1960



- Early attempts with proton beams were plagued by the strong distortion of both the incoming and outgoing particles

FROM PROTON TO ELECTRON BEAMS

- Later the 1960s, it was argued that much cleaner information could be obtained from electron-nucleus scattering in the kinematical region corresponding to momentum transfer $|\mathbf{q}| \ll d^{-1}$—$d \sim 1.5\mathrm{fm}^{-1}$ being the average nucleon-nucleon distance in the target nucleus —in which the reaction predominantly involves individual nucleons

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Nuclear Physics **32** (1962) 139—151; © *North-Holland Publishing Co., Amsterdam*

QUASI-FREE ELECTRON-PROTON SCATTERING (I)

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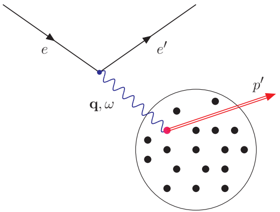
Received 6 July 1961

Abstract: It is shown that, from angular and energy correlation measurements on electron-proton pairs emerging from the scattering of high energy (300-1000 MeV) electrons on nuclei, detailed information on the energy levels and structures of the upper and lower shells of light and medium nuclei could be obtained. A calculation in which the distortion of the outgoing proton wave is taken into account has been performed for C^{12}. As compared to the result for zero distortion, the absolute magnitude of the correlation cross section is reduced, but the shape of its angular distribution is practically unchanged. Consequently the observed energy and angular correlations would immediately give both the binding energy and the momentum distribution of the nuclear proton in the shell model state out of which it has been ejected. From an extrapolation to other nuclei of the calculated value of the reduction factor for the cross section, it is expected that this situation prevails at least up to nuclei with $A = 50$. Finally some corrections are qualitatively discussed.

THE $(e, e'p)$ REACTION

- Consider the process $e + A \to e' + p + (A - 1)$ in which both the scattered electron and the outgoing proton, carrying momentum $\mathbf{p}'$, are detected in coincidence

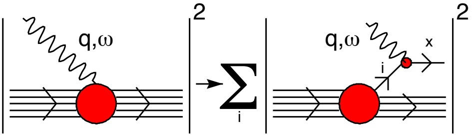


- In the absence of final state interactions (FSI), the initial energy and momentum of the knocked out nucleon can be identified with the *measured* missing momentum and energy, respectively

$$\mathbf{p}_m = \mathbf{p}' - \mathbf{q} \quad , \quad E_m = \omega - T_{\mathbf{p}'} - T_{A-1} \approx \omega - T_{\mathbf{p}'}$$

Factorisation of the Nuclear Cross Section

- for $\lambda \ll d_{\rm NN} \sim 1.5$ fm , the average NN distance in the target nucleus, nuclear scattering reduces to the incoherent sum of scattering processes involving individual nucleons



- Basic assumptions
  - $J^\mu_A(q) \approx \sum_i j^\mu_i(q)$ (single-nucleon coupling)
  - $|(A-1)_n, \mathbf{p}'\rangle \approx |(A-1)_n\rangle \otimes |\mathbf{p}'\rangle$ (factorization of the final state)
- As a zero-th order approximation, Final State Interactions (FSI) and processes involving two-nucleon Meson-Exchange Currents (MEC) are neglected

THE FACTORISED $(e, e'p)$ CROSS SECTION

⋆ Factorisation allows to rewrite the nuclear transition amplitude in the simple form

$$\langle (A-1)_n, \mathbf{p}' | J_A^\mu | 0 \rangle \to \sum_i M_n(\mathbf{p}' - \mathbf{q}) \langle \mathbf{p}' | j_i^\mu | \mathbf{p}' - \mathbf{q} \rangle$$

- The nuclear amplitude M_n is independent of momentum transfer
- The matrix element of the current between free-nucleon states can be computed using the fully relativistic expression

⋆ $(e, e'p)$ cross section

$$\frac{d\sigma_A}{dE_{e'} d\Omega_{e'} dE_p d\Omega_p} \propto \sigma_{ep} P(p_m, E_m)$$

⋆ The spectral function, trivially related to the two-point Green's function through $P(\mathbf{k}, E) = -\mathrm{Im}\, G(\mathbf{k}, E)/\pi$, is a fundamental quantity of many-body theory

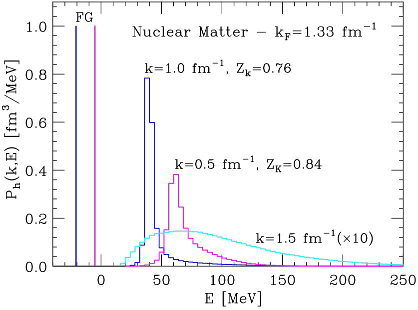
The Nuclear Spectral Function

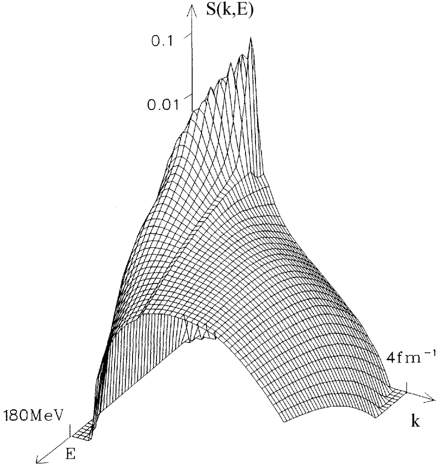
- The analytic structure of the two-point Green's function is reflected by the spectral function

$$P(\mathbf{k}, E) = \sum_{h \in \{F\}} Z_h |M_h(\mathbf{k})|^2 f_h(E - e_h) + P_B(\mathbf{k}, E)$$

- The spectral function yields the probability of removing a particle with momentum $\mathbf{k}$ from the target ground state leaving the residual system with excitation energy E
- In the mean field approximation underlying the nuclear shell model
  - Spectroscopic factors $Z_h \to 1$
  - Momentum dependence $M_h(\mathbf{k}) = \langle h | a_{\mathbf{k}} | 0 \rangle \to \phi_h(\mathbf{k})$, the momentum-space wave function of the single-particle state h
  - Energy distribution $F_h(E - e_h) \to \delta(E - e_h)$
  - Smooth contribution $P_B(\mathbf{k}, E) \to 0$: pure correlation effect

$P(\mathbf{k}, E)$ of Isospin-Symmetric Nuclear Matter





FSI: Distorted Wave Impulse Approximation (DWIA)

- In the presence of FSI, the *distorted* mean field spectral function can be written in the form

$$P^D_{MF}(\mathbf{p}_m, \mathbf{p}, E_m) = \sum_\alpha Z_\alpha |\phi^D_\alpha(\mathbf{p}_m, \mathbf{p})|^2 f_\alpha(E_m - E_\alpha)$$

with

$$\sqrt{Z_\alpha}\, \phi^D_\alpha(\mathbf{p}_m, \mathbf{p}) = \int d^3p_i\ \chi^\star_p(\mathbf{p}_i + \mathbf{q})\phi(\mathbf{p}_i)$$

where the *distorted wave* $\chi^\star_p(\mathbf{p}_i + \mathbf{q})$ includes the effects of FSI, described by a *complex* optical potential

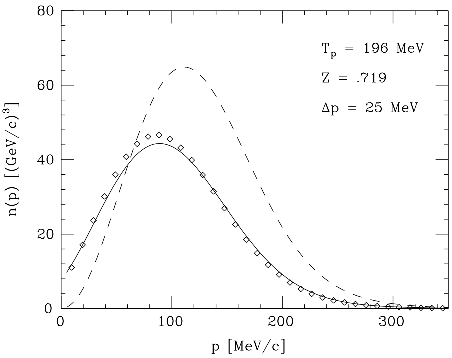
- The large body of existing $(e, e'p)$ data suggests that the effects of FSI can be strongly reduced measuring the cross section in *parallel kinematics*, that is with $\mathbf{p} \parallel \mathbf{q}$.
- in parallel kinematics, factorisatios of the nuclear cross section is also preserved to very high accuracy, the distorted momentum distribution only depends on missing momentum

$$n^D_\alpha(p_m) = Z_\alpha\ |\phi^D(p_m)|^2\ ,$$

and the effects FSI can be easily identified.

Distorted Momentum Distribution

- knock out of a p-shell protons in oxygen. Proton kinetic energy $T_p = 196$ MeV; parallel kinematics
- Distortion described by a *complex* optical potential (OP)



- FSI lead to a shift in missing momentum (real part of the OP), and a significant quenching, typically by a factor ~ 0.7 (imaginary part of the OP).

Spectral Function Measurements at Saclay

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Nuclear Physics **A262** (1976) 461—492; © *North-Holland Publishing Co., Amsterdam*

QUASI-FREE (e, e′p) SCATTERING ON ^{12}C, ^{28}Si, ^{40}Ca AND ^{58}Ni

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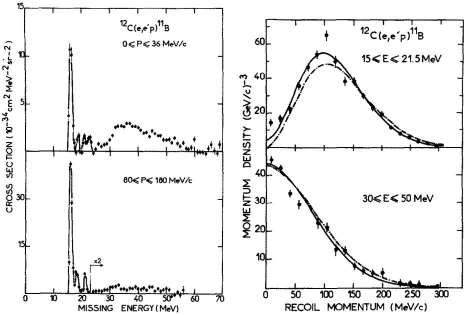
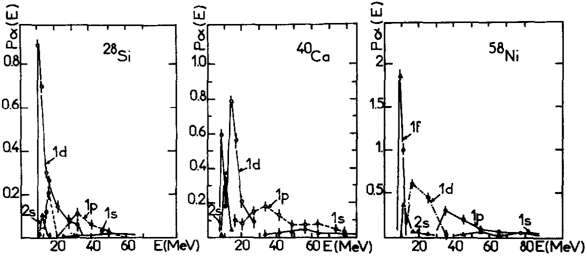
Received 29 August 1975
(Revised 19 January 1976)

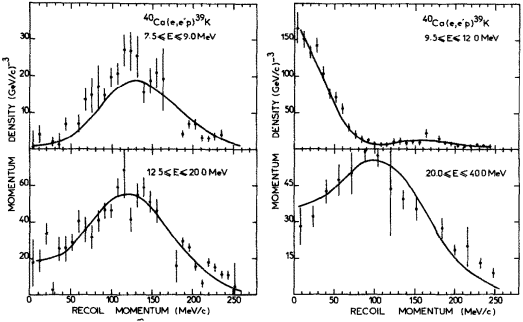
Abstract: The (e, e′p) reaction on ^{12}C, ^{28}Si, ^{40}Ca and ^{58}Ni has been measured at 497 MeV incident electron energy. The experiment covered the region $E \leqq 80$ MeV for the separation energy and $P \leqq 250$ MeV/c for the recoil momentum. Cross sections, calculated in the distorted wave impulse approximation, have been utilized in a shell-model expansion of the spectral function. Average separation and kinetic energies of protons in individual shells are extracted from the data. The validity of Koltun's sum rule is discussed.

- Carbon data



SYSTEMATICS OF ENERGY AND MOMENTUM DISTRIBUTIONS



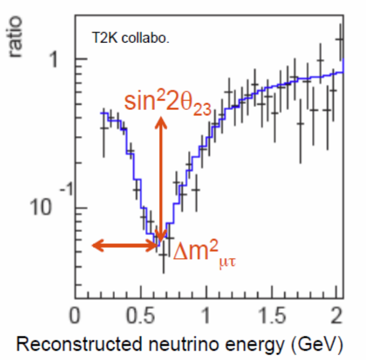


Why Argon and Titanium targets?

The Issue of Neutrino Energy Reconstruction

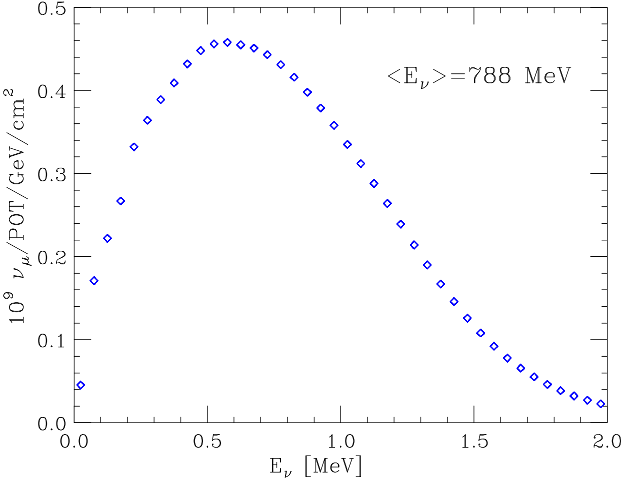
- Oscillation probability after traveling a distance L (two neutrino flavors, for simplicity)

$$P_{\alpha\to\beta} = \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$



- The energy of the incoming neutrino, E_ν is not precisely known, but broadly distributed according to a flux $\Phi(E_\nu)$

MiniBooNE Neutrino Flux



Kinematic Neutrino Energy Reconstruction

- In the charged current quasi elastic (CCQE) channel, assuming single nucleon single knock, the relevant elementary process is

$$\nu_\ell + n \to \ell^- + p$$

- The *reconstructed* neutrino energy is

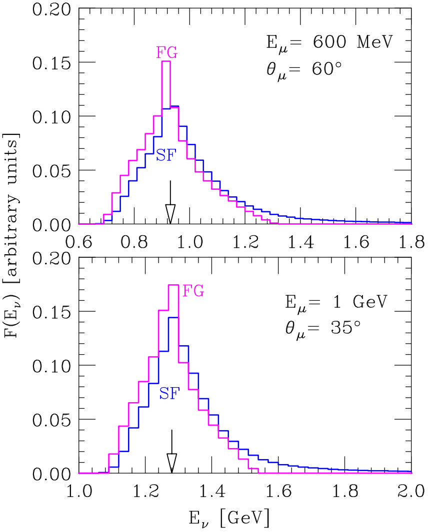
$$E_\nu = \frac{m_p^2 - m_\mu^2 - {E_n}^2 + 2E_\mu E_n - 2\mathbf{k}_\mu \cdot \mathbf{p}_n + |{\mathbf{p}_n}^2|}{2(E_n - E_\mu + |\mathbf{k}_\mu| \cos\theta_\mu - |\mathbf{p}_n| \cos\theta_n)} ,$$

  where $|\mathbf{k}_\mu|$ and θ_μ are measured, while $\mathbf{p}_n$ and E_n are the *unknown* momentum and energy of the interacting neutron

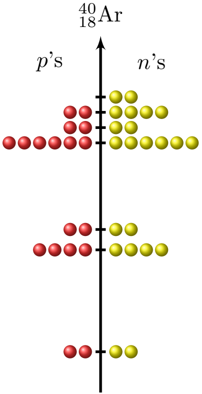
- Existing simulation codes routinely use $|\mathbf{p}_n| = 0$, $E_n = m_n - \epsilon$, with $\epsilon \sim 20$ MeV for carbon and oxygen, or the Fermi gas (FG) model

RECONSTRUCTED NEUTRINO ENERGY IN THE CCQE CHANNEL

- Neutrino energy reconstructed using 2×10^4 pairs of $(|\mathbf{p}|, E)$ values sampled from realistic (SF) and FG oxygen spectral functions
- The average value $\langle E_\nu \rangle$ obtained from the realistic spectral function turns out to be shifted towards larger energy by $\sim$ 70 MeV



- ⋆ The reconstruction of neutrino and antineutrino energy in liquid argon detectors will require the understanding of the spectral functions describing both protons and neutrons
- ⋆ The Ar$(e, e'p)$ cross section only provides information on proton interactions. Arguably, useful information on neutrons can be obtained from the Ti$(e, e'p)$



Jefferson Lab Experiment E12-14-012

- ⋆ Argon & Titanium target
- ⋆ Beam energy $E_e = 2.222$ GeV; parallel kinematics
- ⋆ Five kinematic settings

	E'_e (GeV)	θ_e (deg)	$\|\mathbf{p}'\|$ (MeV)	$\theta_{p'}$ (deg)	$\|\mathbf{q}\|$ (MeV)	p_m (MeV)	E_m (MeV)
kin1	1.777	21.5	915	−50.0	865	50	73
kin2	1.716	20.0	1030	−44.0	846	184	50
kin3	1.799	17.5	915	−47.0	741	174	50
kin4	1.799	15.5	915	−44.5	685	230	50
kin5	1.716	15.5	1030	−39.0	730	300	50

- ⋆ Overall coverage

$$15 \leq p_m \leq 300 \text{ MeV} \quad , \quad 12 \leq E_m \leq 80 \text{ MeV}$$

Determination of the spectral function

⋆ The measured reduced cross-section defined as

$$P_D(p_m, E_m) = \frac{1}{K\sigma_{ep}} \frac{d\sigma_A}{dE_{e'} d\Omega_{e'} dE_p d\Omega_p} ,$$

with $K = |\mathbf{p}'|E_{\mathbf{p}'}$, has been fitted using the model spectral function

$$P_D(p_m, E_m) = \sum_\alpha Z_\alpha |\phi^D_\alpha(\mathbf{p}_m)|^2 f_\alpha(E_m - E_\alpha) + P^D_{\rm corr}(p_m, E_m) ,$$

corrected to take into account the effects of FSI within the DWIA scheme.

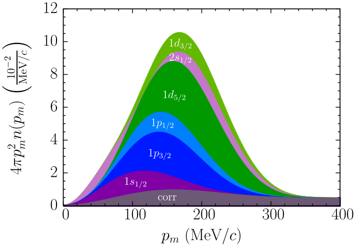
⋆ The unit normalised momentum distributions, $|\phi^D_\alpha(\mathbf{p}_m, \mathbf{p})|^2$ are obtained from Relativistic Mean Field calculations

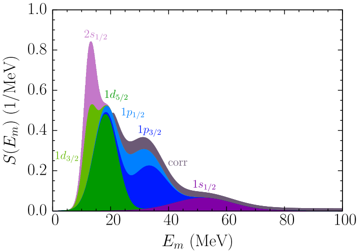
⋆ The energy distributions $f_\alpha(E_m - E_\alpha)$, of width Γ_α, have been assumed to be gaussian

⋆ The fit yields the values of the spectroscopic factors Z_α, the energies of the shell model states E_α, and their widths Γ_α

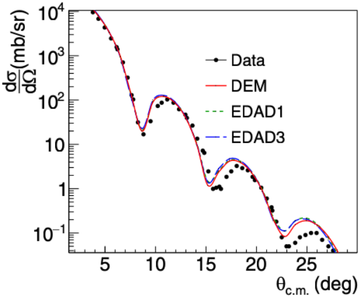
Main Elements of the Analysis

Missing momentum (top) and missing energy (bottom) distributions

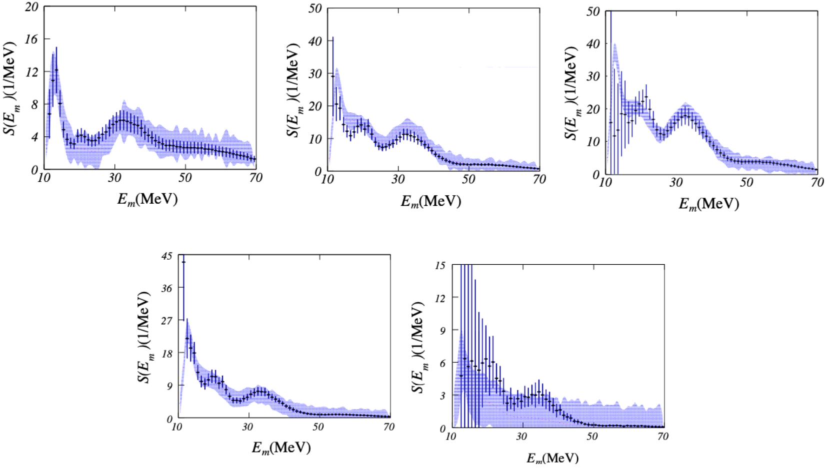




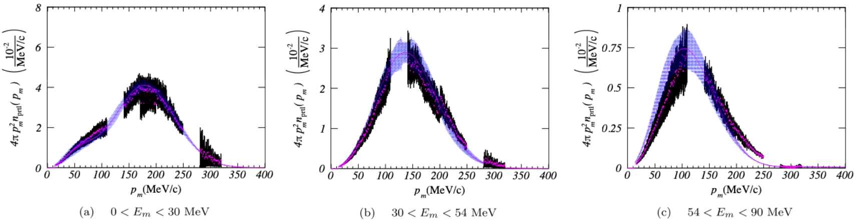
Differential cross section for elastic scattering of 800 MeV protons on Argon. Theoretical results obtained from the optical potential employed in the analysis



Missing Energy Distributions



Missing Momentum Distributions



- ⋆ The agreement—within uncertainty—between distributions corresponding to different kinematical settings supports the validity of the DWIA treatment of FSI, and, more generally, of factorisation

Summary & Prospects

- ⋆ Jlab experiment E12-14-012 is well on track to achieve its primary goal: providing a representation of the data based on a realistic model of the Argon spectral function, suitable for use in simulations of neutrino interactions in liquid argon detectors
- ⋆ The picture emerging from the comparison between data and MC—yielding a $\chi^2/\mathrm{d.o.f} = 1.9$—largely confirms the validity of the theoretical framework underlying the analysis
- ⋆ The development of a more refined model of the spectral function will require the use of advanced theoretical calculations based on microscopic nuclear dynamics
- ⋆ The analysis of Titanium data is in progress. The extent to which a proton in Titanium is a good proxy for a neutron in Argon, as well as the feasibility of a neutron knockout experiment, need to be carefully investigated.

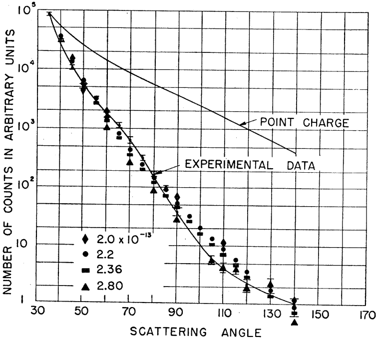
Thank you!

Backup slides

ELASTIC SCATTERING: $e + A \to e' + A$, $\lambda \gg R_A \sim A^{1/3}$

$$\left(\frac{d\sigma}{d\Omega}\right)_{eA} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q})|^2 \,,$$

- The Mott x-section described the electromagnetic interaction of a relativistic electron with a point target



Hofstadter et al, A.D. 1953
Gold target, $E_e = 125$ MeV

KINEMATIC RANGES

TABLE III. Estimate of the spectroscopic strengths probed at each kinematics using the test spectral function. For clarity, only nonvanishing entries are shown. The total strength accessible in the experiment is 14.96.

	$\lvert\mathbf{p}_m\rvert$ range (MeV/c)	E_m range (MeV)	$1d_{3/2}$	$2s_{1/2}$	$1d_{5/2}$	$1p_{1/2}$	$1p_{3/2}$	$1s_{1/2}$	Corr.	Sum
		0–30	0.11	0.48	0.26	0.18	0.18	0.01	0.11	1.34
kin1	15–110	30–54				0.11	0.40	0.44	0.18	1.13
		54–90						0.33	0.05	0.38
		0–30	0.79	0.51	2.37	0.46	0.46	0.01	0.18	4.77
kin2	140–210	30–54				0.27	1.03	0.23	0.31	1.85
		54–90						0.17	0.10	0.28
		0–30	1.04	0.62	3.11	0.64	0.64	0.01	0.26	6.33
kin3	120–220	30–54				0.38	1.44	0.37	0.44	2.64
		54–90						0.28	0.14	0.42
		0–30	0.52	0.56	1.68	0.21	0.21		0.12	3.30
kin4	190–250	30–54				0.12	0.47	0.05	0.21	0.86
		54–90						0.04	0.09	0.13
		0–30	0.12	0.15	0.39	0.03	0.02		0.05	0.76
kin5	260–320	30–54				0.02	0.05		0.11	0.17
		54–90							0.09	0.09

SYSTEMATICS

TABLE IV. Contributions to systematical uncertainties for argon average over all the E_m and p_m bins for each kinematic. All numbers are in %. kin4 and kin5 systematic is the sum in quadrature of the systematic uncertainties on the signal and the background.

	kin1	kin2	kin3	kin4	kin5
1. Total statistical uncertainty	0.53	0.57	0.64	0.54	1.65
2. Total systematic uncertainty	3.14	3.24	3.32	10.23	9.01
a. Beam x & y offset	0.63	0.85	0.69	0.91	1.68
b. Beam energy	0.10	0.10	0.10	0.10	0.10
c. Beam charge	0.30	0.30	0.30	0.30	0.30
d. HRS x & y offset	0.83	1.17	0.78	1.44	1.71
e. Optics (q1, q2, q3)	0.94	0.77	0.55	0.90	1.72
f. Acceptance cut (θ, ϕ, z)	1.16	1.33	1.75	2.19	7.72
g. Target thickness/density/length	0.20	0.20	0.20	0.20	0.20
h. Calorimeter & Čerenkov cut	0.02	0.02	0.02	0.02	0.02
i. Radiative and Coulomb corr.	1.00	1.00	1.00	1.00	1.00
j. β cut	0.47	0.55	0.39	7.74	5.87
k. Boiling effect	0.70	0.70	0.70	0.70	0.70
l. Cross section model	1.00	1.00	1.00	1.00	1.00
m. Trigger and coincidence time cut	0.92	0.52	0.98	5.55	2.58
n. FSI	2.00	2.00	2.00	2.00	2.00