Matching GTMDs onto GPDs at one-loop accuracy

[Based on: arXiv:2207.09526]

Valerio Bertone

IRFU, CEA, Université Paris-Saclay

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A “constructive” introduction

- Let us start with a generic **bi-local** (quark) operator:

$$\mathcal{O} = \overline{\psi}\,(b)\,\Gamma\psi\,(0)$$

- Γ is generic Dirac structure, *i.e.* a linear combination of $\{\mathbb{I}, \gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu}\}$.

$$\Gamma = A\mathbb{I} + B\gamma^5 + C_\mu\gamma^\mu + D_\mu\gamma^5\gamma^\mu + E_{\mu\nu}\sigma^{\mu\nu}$$

- In order to attribute to $\mathcal{O}$ any physical meaning, we need to make it gauge invariant.
  - Introduce the parallel-transport operator W (often called **Wilson line** in this context):

$$W(y, x) = \mathcal{P}\exp\left[-igt^a\int_y^x dz^\mu A^a_\mu(z)\right]$$

- The gauge invariant version of $\mathcal{O}$ is then:

$$\mathcal{O} = \overline{\psi}\,(b)\,\Gamma\,W(b, 0)\psi\,(0)$$

- Now consider the case in which $\mathcal{O}$ is **highly boosted** along $-z$ (as if it was involved in a high-energy collision): this frame is called Breit (or infinite-momentum) frame.
- Working the Breit frame has two main important consequences:
  - $b_z \simeq -cb_t$, therefore in light-cone coordinates $b \simeq (0, b^-, \mathbf{b}_T)$.
  - In addition, the coefficients $\{A, B, C_\mu, D_\mu, E_{\mu\nu}\}$ get enhanced, unchanged, or suppressed:
    - C_+, D_+, E_{+i} enhanced (twist 2), $A, B, C_i, D_i, E_{ij}, E_{+-}$ unchanged (twist 3), C_-, D_-, E_{-i} suppressed (twist 4).

A "constructive" introduction

- Therefore, a particularly interesting operator is the "unpolarised" one:

$$\mathcal{O} = \overline{\psi}(b)\,\gamma^+\,W(b,0)\psi(0)\big|_{b^+=0}$$

(in fact, also the others are interesting but in this seminar I will focus on this one.)

- To connect this operator to an observable we need to take a matrix element.
- We bracket it with two, generally different, hadronic states:

$$\mathcal{M} = \langle H'(p',\lambda')|\overline{\psi}(b)\,\gamma^+\,W(b,0)\psi(0)\,|H(p,\lambda)\rangle\big|_{b^+=0}$$

- Finally, it is usually more phenomenologically relevant to study the **momentum** behaviour of any such matrix element. We thus take its Fourier transform:

$$\Phi = \int db^-d^2\mathbf{b}_T\,e^{ib^-k^+-i\mathbf{b}_T\cdot\mathbf{k}_T}\,\langle H'(p',\lambda')|\overline{\psi}(b)\,\gamma^+\,W(b,0)\psi(0)\,|H(p,\lambda)\rangle\big|_{b^+=0}$$

- This is a (sketchy) definition of **generalised transverse-momentum dependent** (GTMD) correlator.
- GTMDs can be regarded as "**mother distributions**" (*cit.* Meißner, Metz, Schlegel [*JHEP* 08 (2009) 056])
- They encode "the most general one-body information of partons, corresponding to the full one-quark density matrix in momentum space" (*cit.* Lorcé, Parquini, Vanderhaeghen [*JHEP* 05 (2011) 041]).

Further readings: Ji [*Phys.Rev.Lett.* 91 (2003) 062001], Belitsky, Ji, Yuan [*Phys.Rev.D* 69 (2004) 074014], Belitsky, Radyushkin [*Phys.Rept.* 418 (2005) 1-387]

A "constructive" introduction

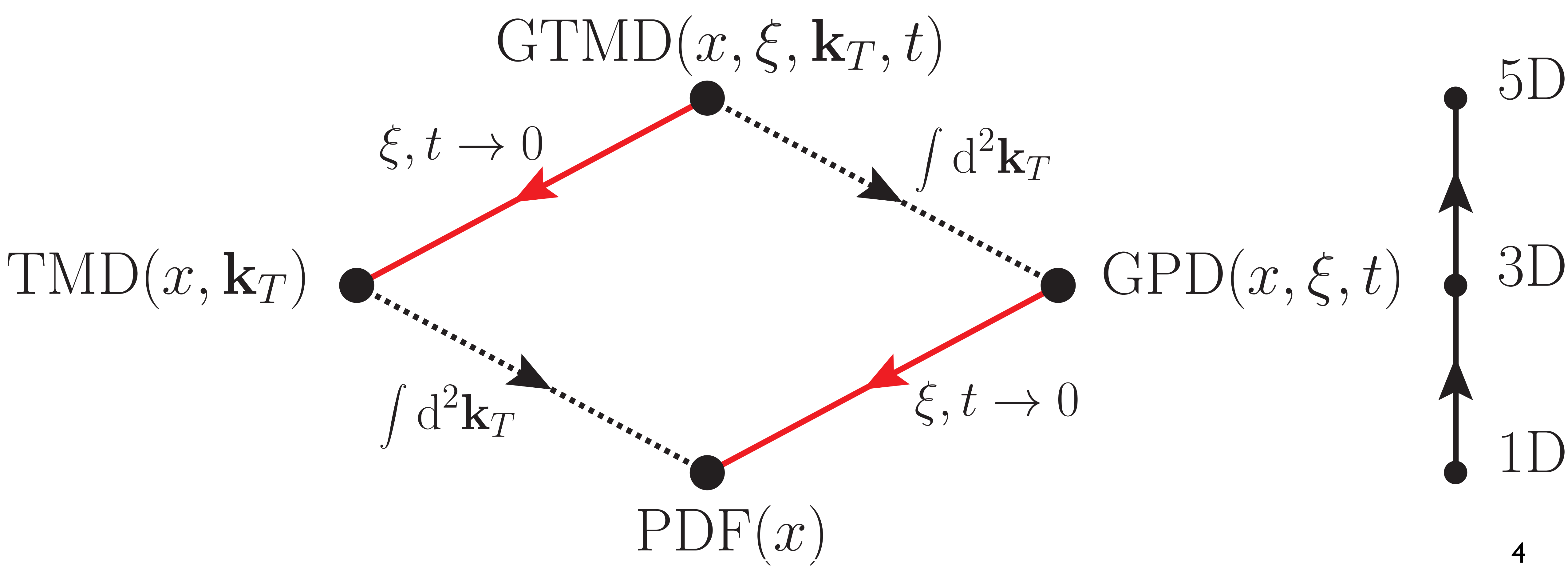
- Pretty much all relevant hadronic distributions in high-energy physics can be made descend from GTMDs. Introducing the definitions:

$$P = \frac{p+p'}{2} \qquad \Delta \equiv p - p'$$

- A common set of kinematic variables used to parameterise GTMDs is:

$$k^+ \equiv x P^+ \qquad \Delta^+ \equiv \xi \frac{P^+}{2} \qquad t = \Delta^2$$

- A (partial) genealogy of GTMDs then looks like this:



A sound GTMD definition

- A proper definition of GTMD distributions requires a combination of a GTMD correlators and soft function. [*Phys.Lett.B* 759 (2016) 336-341]

- Working in $\mathbf{b}_T$ space, *i.e.* the Fourier conjugate of the partonic transverse momentum $\mathbf{k}_T$, is convenient:

$$\hat{\mathcal{F}}_{i/H}(x,\xi,\mathbf{b}_T,t) = \hat{S}_i^{-\frac{1}{2}}(\mathbf{b}_T)\hat{\Phi}_{i/H}(x,\xi,\mathbf{b}_T,t)\,, \quad i = q, g$$

- The *unpolarised* GTMD quark and gluon correlators are defined as:

$$\hat{\Phi}_{q/H}(x,\xi,\mathbf{b}_T,t) = \int \frac{dy}{2\pi} e^{-ix(n\cdot P)y} \left\langle P_{\text{out}} \middle| \left[\overline{\psi}_q W^\dagger_{n,q}\right]\left(\frac{\eta}{2}\right) \frac{\not{n}}{2} \left[W_{n,q}\psi_q\right]\left(-\frac{\eta}{2}\right) \middle| P_{\text{in}} \right\rangle$$

$$\hat{\Phi}_{g/H}(x,\xi,\mathbf{b}_T,t) = \frac{n_\mu n_\nu}{x(n\cdot P)} \int \frac{dy}{2\pi} e^{-ix(n\cdot P)y} \left\langle P_{\text{out}} \middle| \left[F_a^{\mu j} W^\dagger_{n,g}\right]\left(\frac{\eta}{2}\right) \left[W_{n,g}F_a^{\nu j}\right]\left(-\frac{\eta}{2}\right) \middle| P_{\text{in}} \right\rangle$$

- The soft function in non-singular (Feynman) gauge reads:

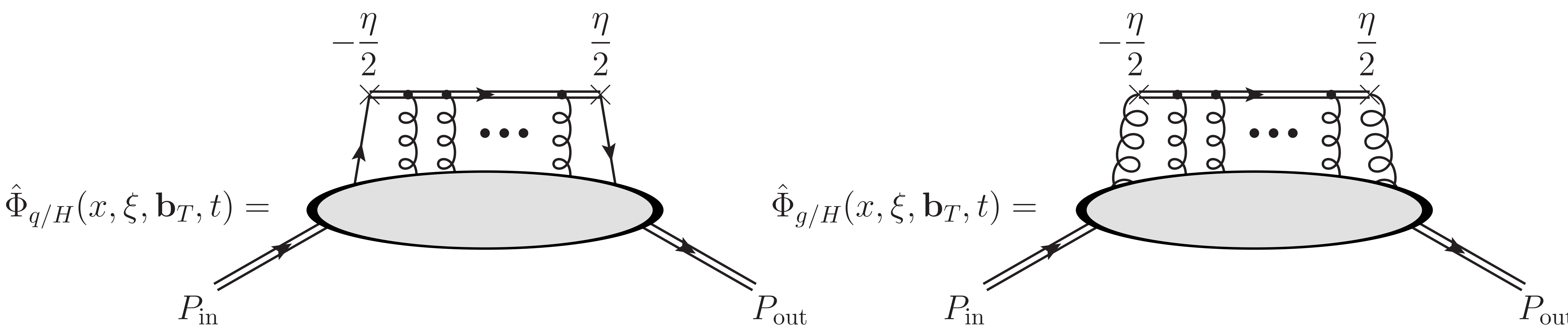
$$\hat{S}_i(\mathbf{b}_T) = \frac{1}{N_i}\text{Tr}_c\langle 0|W_{\overline{n},i}(\mathbf{b}_T)W^\dagger_{n,i}(\mathbf{b}_T)W_{n,i}(\mathbf{0})W^\dagger_{\overline{n},i}(\mathbf{0})|0\rangle\,, \quad i = q, g$$

- where the Wilson line is defined as:

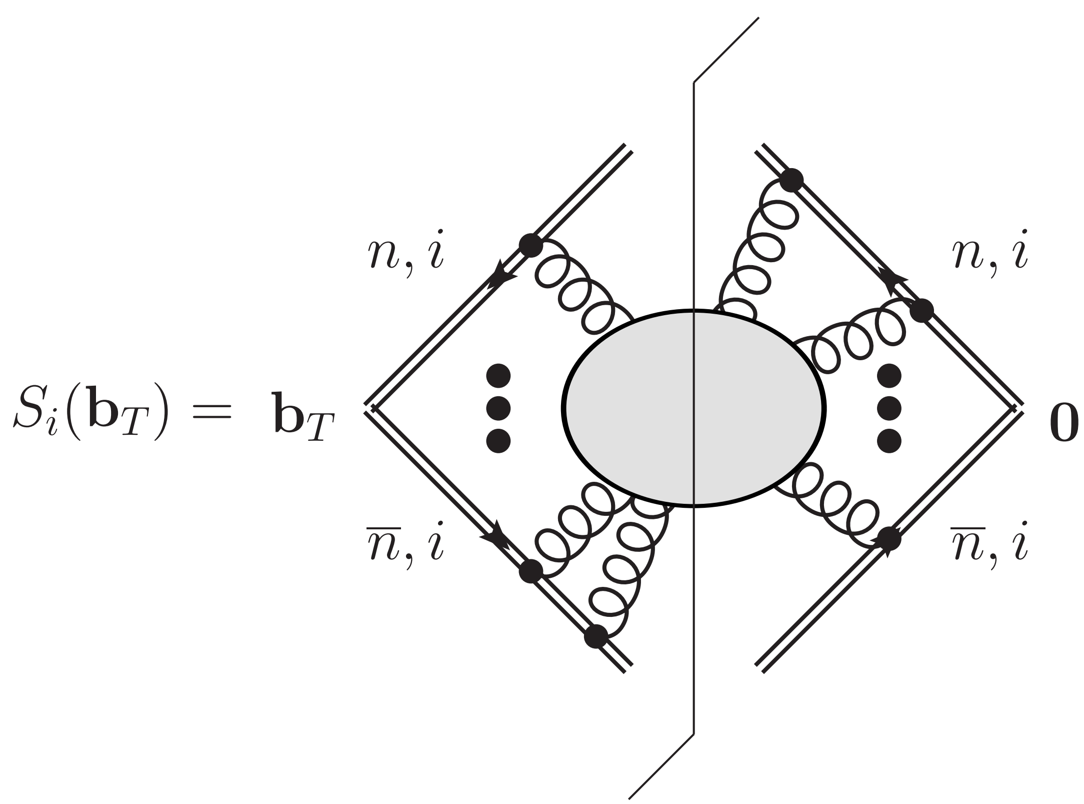
$$W_{v,i}(\mathbf{b}_T) = \mathcal{P}\exp\left[-igt_\alpha^{[i]}v_\mu \int_0^\infty ds\, A_\alpha^\mu(\mathbf{b}_T + sv)\right]$$

A sound GTMD definition

- Graphical representation of the GTMD correlators:



- Graphical representation of the soft function:



$$\eta = yn + \mathbf{b}_T$$

$$P_{\text{in/out}} = P \pm \frac{\Delta}{2}$$

$$t = \Delta^2$$

$$\xi = \frac{2n \cdot \Delta}{n \cdot P}$$

$$n^2 = \overline{n}^2 = 0$$

$$n \cdot \overline{n} = 1$$

Renormalisation of GTMDs

- GTMD correlators and soft function are *separately* affected by **UV**, **IR**, and **rapidity** divergences that need to be regulated in order to perform any calculation:
  - UV and IR divergences are regulated through dim. reg. in $4 - 2\epsilon$ dimensions,
  - rapidity divergences require an *ad hoc* procedure (see below).
- While IR and rapidity divergences cancel out, the UV ones need to be renormalised:

$$S_i(\mathbf{b}_T, \mu, \zeta, \delta) = \lim_{\epsilon\to 0} Z_{S,i}^{-1}(\mathbf{b}_T, Q, \zeta, \mu, \delta, \epsilon)\hat{S}_i(\mathbf{b}_T, Q, \delta, \epsilon)$$

$$\Phi_{i/H}(x, \xi, \mathbf{b}_T, t, \mu, \delta) = \lim_{\epsilon\to 0} Z_{\Phi,i}^{-1}(\xi, \mu, \delta, \epsilon)\hat{\Phi}_{i/H}(x, \xi, \mathbf{b}_T, t, \delta, \epsilon)$$

- Renormalisation of UV divergences leads to the introduction of the **scale** μ, while the renormalisation of rapidity divergences introduces the **scales** ζ **and** Q.
- Renormalisation constants Z in the $\overline{\mathrm{MS}}$ at one-loop accuracy are presented below.
- We can thus obtain the **renormalised** GTMD distributions:

$$\begin{aligned}\mathcal{F}_{i/H}(x, \xi, \mathbf{b}_T, t, \mu, \zeta) &= \lim_{\epsilon,\delta\to 0} Z_{S,i}^{1/2}(\mathbf{b}_T, Q, \zeta, \mu, \delta, \epsilon) Z_{\Phi,i}^{-1}(\xi, \mu, \delta, \epsilon)\hat{\mathcal{F}}_{i/H}(x, \xi, \mathbf{b}_T, t, \delta, \epsilon) \\ &= \lim_{\delta\to 0} S_i^{-1/2}(\mathbf{b}_T, \mu, \zeta, \delta)\Phi_{i/H}(x, \xi, \mathbf{b}_T, t, \mu, \delta)\end{aligned}$$

Renormalisation of GTMDs

- Exploiting the *independence* of the bare quantities from the renormalisation and rapidity scales allows us to derive **evolution equations**:

$$\frac{d\ln \mathcal{F}_{i/H}(x,\xi,\mathbf{b}_T,t,\mu,\zeta)}{d\ln\sqrt{\zeta}} = K_i(\mathbf{b}_T,\mu)$$

$$\frac{d\ln \mathcal{F}_{i/H}(x,\xi,\mathbf{b}_T,t,\mu,\zeta)}{d\ln\mu} = \gamma_i(\mu,\zeta)$$

- The anomalous dimensions K_i and γ_i are naturally related to the **renormalisation constants**:

$$K_i(\mathbf{b}_T,\mu) = \lim_{\epsilon,\delta\to 0}\frac{d\ln Z_{S,i}(\mathbf{b}_T,Q,\zeta,\mu,\delta,\epsilon)}{d\ln\zeta}$$

$$\gamma_i(\mu,\zeta) = \lim_{\epsilon,\delta\to 0}\frac{d\ln[Z_{S,i}^{1/2}(\mathbf{b}_T,Q,\zeta,\mu,\delta,\epsilon)Z_{\Phi,i}^{-1}(\xi,\mu,\delta,\epsilon)]}{d\ln\mu}$$

- Moreover, the requirement that **cross derivatives** are equal leads to introducing a further anomalous dimension:

$$\frac{dK_i(\mathbf{b}_T,\mu)}{d\ln\mu} = \frac{d\gamma_i(\mu,\zeta)}{d\ln\sqrt{\zeta}} \equiv -\gamma_{K,i}(\alpha_s(\mu))$$

Renormalisation of GTMDs

- We can solve the evolution equation obeyed by the anomalous dimensions K_i and γ_i. In this respect it is crucial to choose wisely the **boundary-condition scales**.
  - For the rapidity kernel K_i, the most convenient scale is $\mu = \mu_b = 2e^{-\gamma_E}/|\mathbf{b}_T|$ so that:

$$\frac{dK_i(\mathbf{b}_T,\mu)}{d\ln\mu} = -\gamma_{K,i}(a_s(\mu)) \quad \Rightarrow \quad K_i(\mathbf{b}_T,\mu) = K_i(\mathbf{b}_T,\mu_b) - \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\gamma_{K,i}(\alpha_s(\mu'))$$

  - For the renormalisation kernel γ_i, the most natural choice is $\zeta = \mu^2/(1-\xi^2)$ so that:

$$\frac{d\gamma_i(\mu,\zeta)}{d\ln\sqrt{\zeta}} = -\gamma_{K,i}(a_s(\mu)) \quad \Rightarrow \quad \gamma_i(\mu,\zeta) = \gamma_{F,i}(\alpha_s(\mu)) - \gamma_{K,i}(\alpha_s(\mu))\ln\left(\frac{\sqrt{(1-\xi^2)\zeta}}{\mu}\right)$$

    where we have defined $\gamma_{F,i}(\alpha_s(\mu)) \equiv \gamma_i(\mu,\mu/\sqrt{1-\xi^2})$

- The final form of the GTMD evolution equations reads:

$$\frac{d\ln\mathcal{F}_{i/H}(x,\xi,\mathbf{b}_T,t,\mu,\zeta)}{d\ln\sqrt{\zeta}} = K_i(\mathbf{b}_T,\mu_b) - \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\gamma_{K,i}(a_s(\mu'))$$

$$\frac{d\ln\mathcal{F}_{i/H}(x,\xi,\mathbf{b}_T,t,\mu,\zeta)}{d\ln\mu} = \gamma_{F,i}(a_s(\mu)) - \gamma_{K,i}(a_s(\mu))\ln\left(\frac{\sqrt{(1-\xi^2)\zeta}}{\mu}\right)$$

- All kernels are *purely* perturbative quantities.

Matching on GPDs

- Inspired by TMDs, we define a set of **matching functions** $\mathscr{C}$ that for small values of $|\mathbf{b}_T|$ allows us to express GTMDs in terms of their collinear counterpart: GPDs.

$$\mathcal{F}_{i/H}(x,\xi,\mathbf{b}_T,t,\mu,\zeta) = \int_x^\infty \frac{dy}{y}\mathcal{C}_{i/k}\left(y,\frac{\xi}{x},\mathbf{b}_T,\mu,\zeta\right) F_{k/H}\left(\frac{x}{y},\xi,t,\mu\right)$$

$$\equiv \mathcal{C}_{i/k}(x,\kappa,\mathbf{b}_T,\mu,\zeta) \underset{x}{\otimes} F_{k/H}(x,\xi,t,\mu) \quad \text{GPD}$$

- In order to compute the functions $\mathscr{C}_{i/k}$, we make use of the **parton-in-parton distributions** in which hadronic states are replaced by partonic states:

$$\mathcal{F}_{i/j}(x,\xi,\mathbf{b}_T,\mu,\zeta) = \mathcal{C}_{i/k}(x,\kappa,\mathbf{b}_T,\mu,\zeta) \underset{x}{\otimes} F_{k/j}(x,\xi,\mu)$$

- Since the action of partonic fields on partonic states is computable perturbatively, the following perturbative expansions are meaningful:

$$\mathcal{F}_{i/j}(x,\xi,\mathbf{b}_T,\mu,\zeta) = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n \mathcal{F}_{i/j}^{[n]}(x,\xi,\mathbf{b}_T,\mu,\zeta)\,,$$

$$F_{k/j}(x,\xi,\mu) = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n F_{k/j}^{[n]}(x,\xi,\mu)\,, \qquad \kappa = \frac{\xi}{x}$$

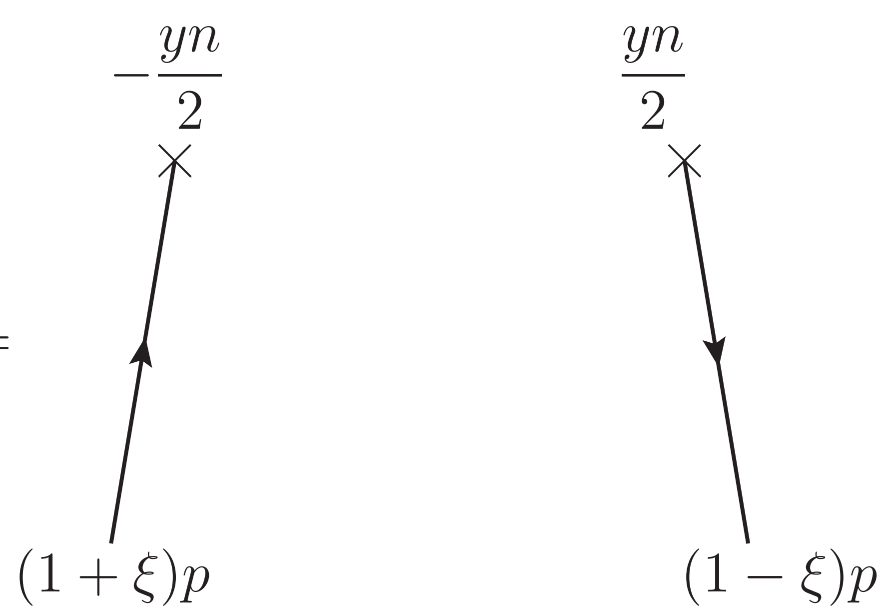
$$\mathcal{C}_{i/k}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = \sum_{n=0}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n \mathcal{C}_{i/k}^{[n]}(x,\kappa,\mathbf{b}_T,\mu,\zeta)\,.$$

- Given the matching formula, the strategy is to compute $\mathscr{F}_{i/j}$ and $F_{k/j}$ in perturbation theory to finally extract $\mathscr{C}_{i/k}$.

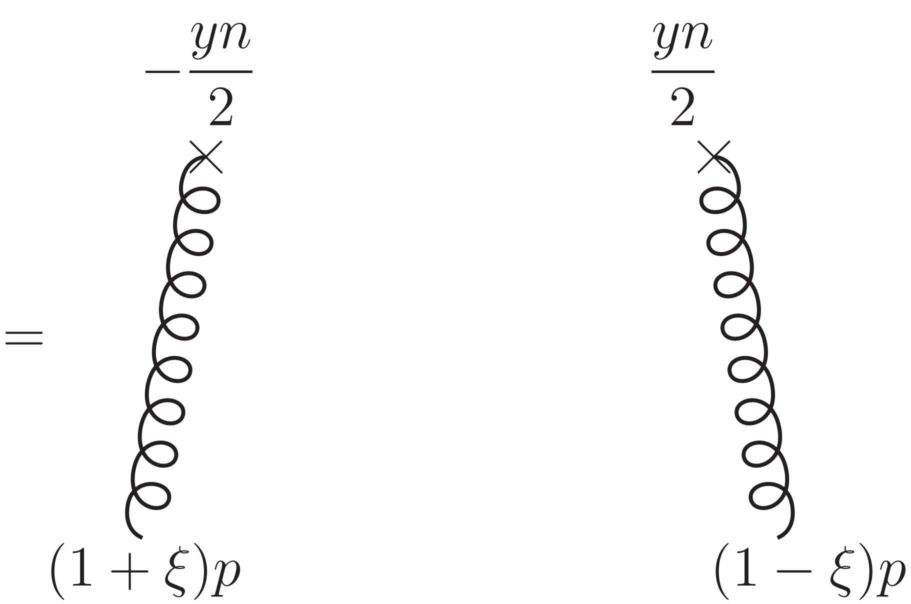
Matching on GPDs

- The leading-order calculation is easily done considering the following diagrams:

$\hat{F}^{[0]}_{q/q}(x,\xi) =$



$\hat{F}^{[0]}_{g/g}(x,\xi) =$



- The result is:

$$\mathcal{F}^{[0]}_{i/j}(x,\xi,\mathbf{b}_T,\mu,\zeta) = F^{[0]}_{i/j}(x,\xi,\mu) = D_j(\xi)\delta_{ij}\delta(1-x)$$

with $D_q(\xi) = \sqrt{1-\xi^2}$ and $D_g(\xi) = 1-\xi^2$.

- It immediately follows that:

$$\mathcal{C}^{[0]}_{i/k}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = \delta_{ik}\delta(1-x)$$

Matching on GPDs

- We now move to NLO where we have:

$$\mathcal{C}^{[1]}_{i/j}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = D_j^{-1}(\xi)\left[\mathcal{F}^{[1]}_{i/j}(x,\xi,\mathbf{b}_T,\mu,\zeta) - F^{[1]}_{i/j}(x,\xi,\mu)\right]$$

- that in terms of **parton-in-parton** GTMD correlators and soft function reads:

$$\mathcal{C}^{[1]}_{i/k}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = D_k^{-1}(\xi)\left[\Phi^{[1]}_{i/k}(x,\xi,\mathbf{b}_T,\mu,\delta) - F^{[1]}_{i/k}(x,\xi,\mu)\right] - \frac{1}{2}\delta_{ik}\delta(1-x)S_i^{[1]}(\mathbf{b}_T,\mu,\zeta,\delta)$$

- The terms in the squared brackets (GTMD correlators and GPDs) are computed diagrammatically and their combination is IR finite.

- The one-loop corrections to the GTMD matching functions can finally be written as:

$$\mathcal{C}^{[1]}_{i/k}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = -\mathcal{P}^{[0],\mathrm{real}}_{i/k}(x,\kappa,\delta)\ln\left(\frac{\mu^2}{\mu_b^2}\right) + \mathcal{R}^{[1]}_{i/k}(x,\kappa) - \frac{1}{2}\delta_{ik}\delta(1-x)S_i^{[1]}(\mathbf{b}_T,\mu,\zeta,\delta)$$

- Notice that both $\mathscr{P}^{[0],\mathrm{real}}_{i/k}$ and $S_i^{[1]}$ are separately affected by a **rapidity divergence**.

Regularising rapidity div.

[Nucl.Phys.B 175 (1980) 27-92]

- To regularise rapidity divergences we resort to the principal-value (PV) prescription:

$$\frac{1}{(n\cdot k)} \to \mathrm{PV}\frac{1}{(n\cdot k)} = \frac{1}{2}\left[\frac{1}{(n\cdot k)+i\delta(n\cdot p)} + \frac{1}{(n\cdot k)-i\delta(n\cdot p)}\right] = \frac{(n\cdot k)}{(n\cdot k)^2+\delta^2(n\cdot p)^2}$$

- Parameterising the + component of the loop momentum k as $k^+ = zP^+$, this translates in:

$$\frac{1}{1-z} \to \left(\frac{1}{1-z}\right)_+ - \delta(1-z)\ln\delta$$

- With this at hand, the "real" part of the splitting functions can be written as:

$$\begin{aligned}\mathcal{P}_{i/k}^{[0],\mathrm{real}}(x,\kappa,\delta) &= \mathcal{P}_{i/k}^{[0]}(x,\kappa) - \mathcal{P}_{i/k}^{[0],\mathrm{virt}}(x,\kappa,\delta)\\ &= \mathcal{P}_{i/k}^{[0]}(x,\kappa) - \delta_{ik}\delta(1-x)2C_i\left[K_i - \ln(1-\xi^2) - 2\int_0^1\frac{dz}{1-z}\right]\\ &= \mathcal{P}_{i/k}^{[0]}(x,\kappa) - \delta_{ik}\delta(1-x)2C_i\left[K_i - \ln(1-\xi^2) + 2\ln\delta\right]\end{aligned}$$

$C_g = C_A = N_c = 3$

$C_q = C_F = \frac{N_c^2-1}{2N_c} = \frac{4}{3}$

$K_q = \frac{3}{2}$

$K_g = \frac{11C_A - 4n_fT_R}{6C_A}$

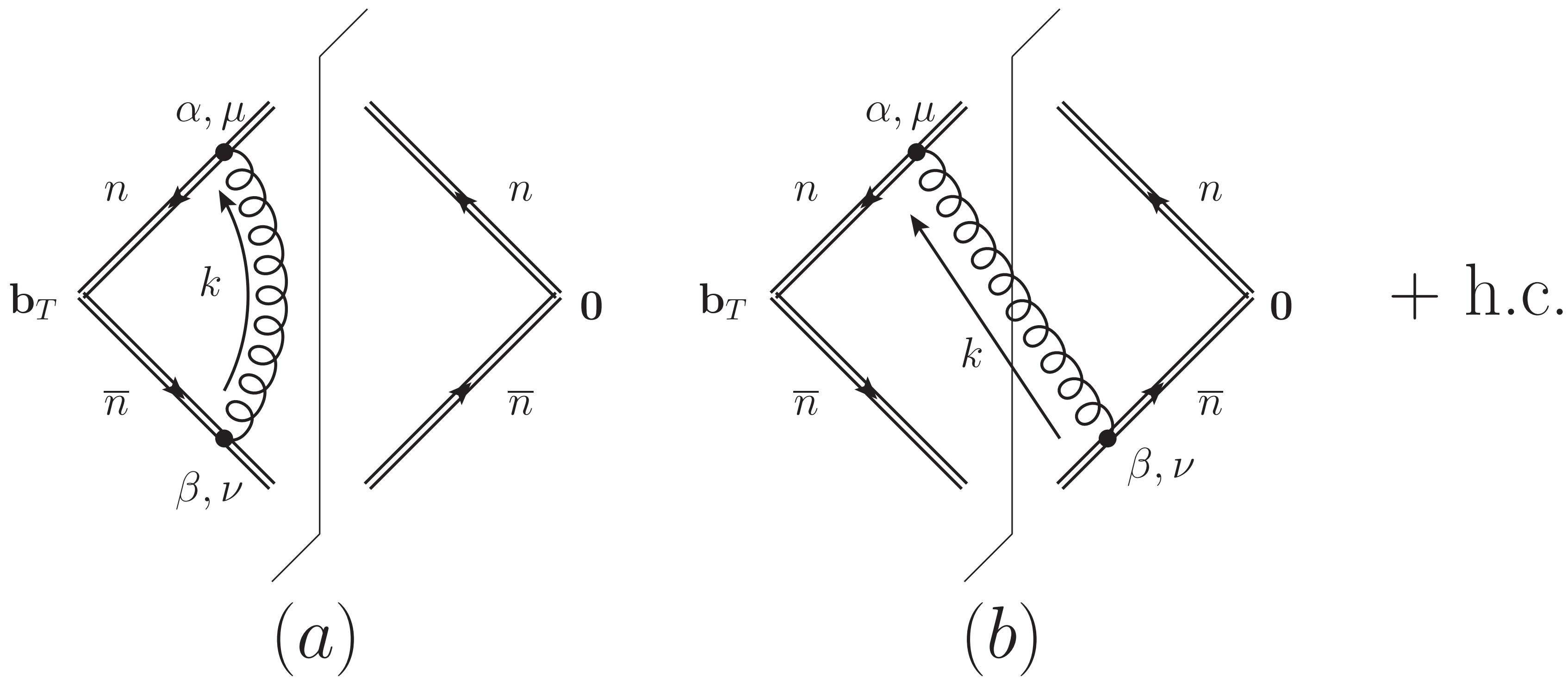
- The rapidity divergence of $\mathscr{P}_{i/k}^{[0],\mathrm{real}}$ is now **explicitly exposed**:

$$\begin{aligned}\mathcal{C}_{i/k}^{[1]}(x,\kappa,\mathbf{b}_T,\mu,\zeta) &= -\mathcal{P}_{i/k}^{[0]}(x,\kappa)\ln\left(\frac{\mu^2}{\mu_b^2}\right) + \mathcal{R}_{i/k}^{[1]}(x,\kappa)\\ &+ \delta_{ik}\delta(1-x)\left[2C_i\left(K_i - \ln(1-\xi^2) + 2\ln\delta\right)\ln\left(\frac{\mu^2}{\mu_b^2}\right) - \frac{1}{2}S_i^{[1]}(\mathbf{b}_T,\mu,\zeta,\delta)\right]\end{aligned}$$

- We need to verify that this rapidity divergence **cancels** against the soft function.

Soft function at one-loop

- The one-loop correction to the soft function arises from the following diagrams:



- The calculation *must* be done using the the same PV regularisation procedure for rapidity divergences used in the GTMD correlators:

$$\hat{S}_i^{[1]}(\mathbf{b}_T, Q, \delta, \epsilon) = -4C_i(4\pi\mu^2)^\epsilon\Gamma(-\epsilon)\left(\frac{b_T^2}{4}\right)^\epsilon\left(\ln\frac{Q^2\delta^2}{\mu_b^2} - \psi(-\epsilon) - \gamma_E\right)$$

$$= 4C_i\left(-\frac{S_\epsilon^2}{\epsilon^2} + \frac{1}{2}\ln^2\left(\frac{\mu^2}{\mu_b^2}\right) - \left(\frac{S_\epsilon}{\epsilon} + \ln\left(\frac{\mu^2}{\mu_b^2}\right)\right)\ln\left(\frac{\mu^2}{Q^2\delta^2}\right) + \frac{\pi^2}{12} + \mathcal{O}(\epsilon)\right)$$

- The appearance of the scale $Q^2 \gg \Lambda_{\rm QCD}^2$ is a consequence of the PV regularisation of the $1/(n\cdot k)$ and $1/(\overline{n}\cdot k)$ eikonal propagators. They introduce the external light-like momenta p and $\overline{p}$ defined such that $(p+\overline{p})^2 = 2p\cdot\overline{p} \equiv Q^2$.

Soft function at one-loop

- The one-loop soft function is affected by a UV double pole that is renormalised in $\overline{\mathrm{MS}}$ by means of the renormalisation constant:

$$Z_{S,i}(\mathbf{b}_T, Q, \zeta, \mu, \delta, \epsilon) = 1 - \frac{\alpha_s}{4\pi} 4C_i \left[\frac{S_\epsilon^2}{\epsilon^2} + \frac{S_\epsilon}{\epsilon} \ln\left(\frac{\mu^2}{Q^2\delta^2}\right) + \ln\left(\frac{\mu^2}{\mu_b^2}\right) \ln\left(\frac{\zeta}{Q^2}\right) \right] + \mathcal{O}(\alpha_s^2)$$

- The *arbitrary* scale ζ is introduced to parameterise the finite part of the renormalisation constant.

- Finally, the one-loop *renormalised* soft function at one loop reads:

$$S_i^{[1]}(\mathbf{b}_T, \mu, \zeta, \delta) = 2C_i \left(4 \ln\left(\frac{\mu^2}{\mu_b^2}\right) \ln\delta + \ln^2\left(\frac{\mu^2}{\mu_b^2}\right) - 2\ln\left(\frac{\mu^2}{\mu_b^2}\right) \ln\left(\frac{\mu^2}{\zeta}\right) + \frac{\pi^2}{6} \right)$$

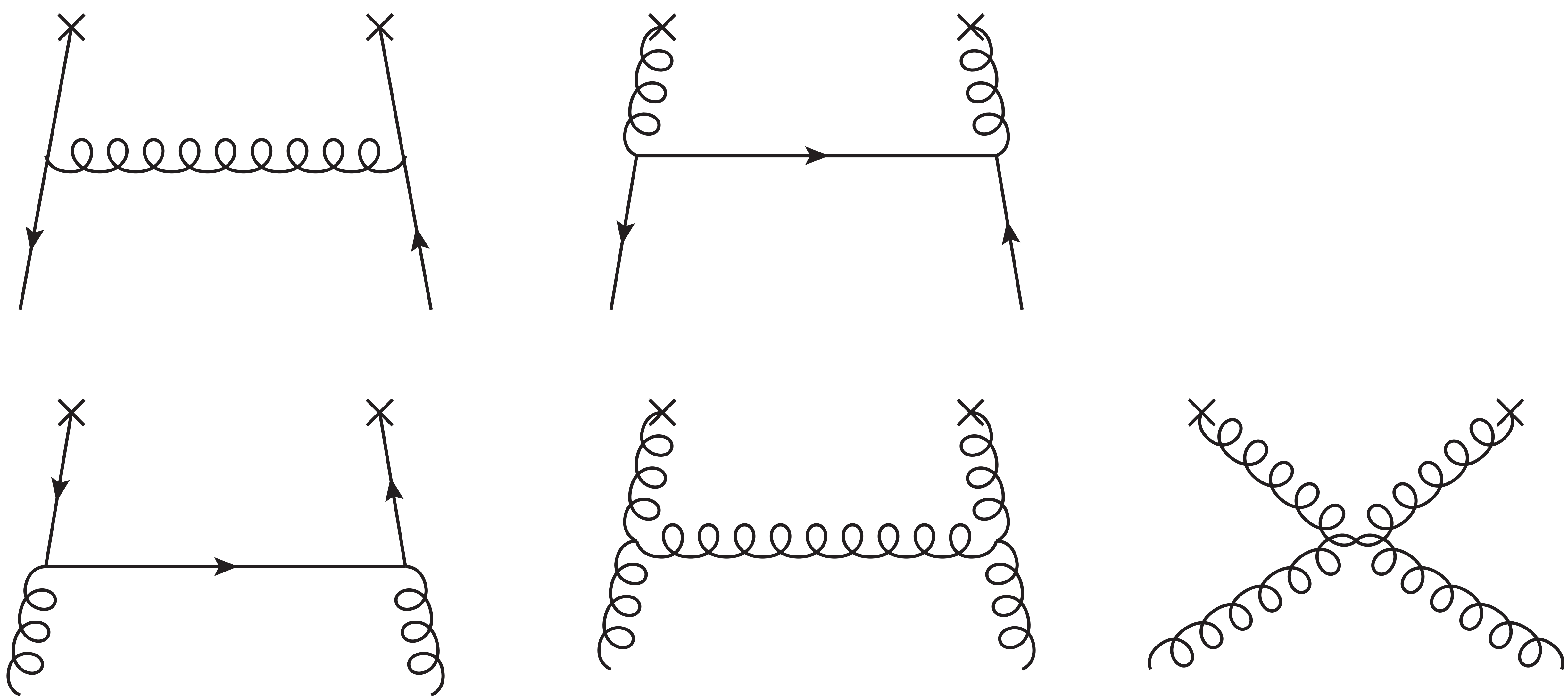
- This result guarantees the **cancellation of the rapidity divergence** in the matching functions that, using this result, become:

$$\begin{aligned} \mathcal{C}_{i/k}^{[1]}(x, \kappa, \mathbf{b}_T, \mu, \zeta) &= -\mathcal{P}_{i/k}^{[0]}(x, \kappa) \ln\left(\frac{\mu^2}{\mu_b^2}\right) + \mathcal{R}_{i/k}^{[1]}(x, \kappa) \\ &- \delta_{ik}\delta(1-x) 2C_i \left[\frac{1}{2} \ln^2\left(\frac{\mu^2}{\mu_b^2}\right) - \left(K_i + \ln\left(\frac{\mu^2}{(1-\xi^2)\zeta}\right)\right) \ln\left(\frac{\mu^2}{\mu_b^2}\right) + \frac{\pi^2}{12} \right] \end{aligned}$$

- This result is *finite* and we are just left with extracting $\mathscr{R}_{i/k}^{[1]}$.

GTMD correlators at one-loop

- $\mathscr{R}^{[1]}_{i/k}$ can be extracted by retaining the $\mathcal{O}(\epsilon/\epsilon)$ order in the computation of the parton-in-parton GPDs:
    - "incidentally", this was done in [arXiv:2206.01412] (just accepted for publication in EPJC).
- At one-loop (and light-cone gauge) the diagrams to be considered are:



- Only "real" diagrams.

GTMD correlators at one-loop

- The the *full* GTMD correlator at one loop takes the form:

$$\hat{\Phi}^{[1]}_{i/k}(x,\xi,\mathbf{b}_T,\mu,\delta) = D_k(\xi)\Bigg[-\frac{S_\epsilon}{\epsilon_{\rm IR}}\mathcal{P}^{[0],\rm real}_{i/k}(x,\kappa,\delta) - \mathcal{P}^{[0],\rm real}_{i/k}(x,\kappa,\delta)\ln\frac{\mu^2}{\mu_b^2} + \mathcal{R}^{[1]}_{i/k}(x,\kappa)$$
$$+ \delta_{ik}\delta(1-x)2C_i\left(K_i - \ln(1-\xi^2) + 2\ln\delta\right)\frac{\mu^{2\epsilon}S_\epsilon}{\epsilon_{\rm UV}} + \mathcal{O}(\epsilon)\Bigg]$$

- Only the "virtual" part of full GTMD correlator is **UV divergent**.

- The UV divergence is renormalised in $\overline{\rm MS}$ by means of the following (flavour diagonal) renormalisation constant:

$$Z_{\Phi,i}(\xi,\mu,\delta,\epsilon) = 1 + \frac{\alpha_s}{4\pi}2C_i\left(K_i - \ln(1-\xi^2) + 2\ln\delta\right)\frac{S_\epsilon}{\epsilon} + \mathcal{O}(\alpha_s^2)$$

- This renormalisation constant, along with that of the soft function, is necessary to compute the one-loop correction to the GTMD anomalous dimensions.

GTMD correlators at one-loop

- Working in the non-singlet/singlet basis:

$$\Phi^- = \sum_q \Phi_{q/k} - \Phi_{\bar{q}/k} \qquad \Phi^+ = \begin{pmatrix} \sum_q \Phi_{q/k} + \Phi_{\bar{q}/k} \\ \Phi_{g/k} \end{pmatrix}$$

- The functions $\mathscr{R}^{[1]}_{i/k}$ take the following general structure:

$$f(x) \underset{x}{\otimes} g(x) \equiv \int_x^\infty \frac{dy}{y} f(y) g\left(\frac{x}{y}\right)$$

$$\mathcal{R}^{\pm,[1]}(y,\kappa) = \theta(1-y)\mathcal{R}_1^{\pm,[1]}(y,\kappa) + \theta(\kappa-1)\mathcal{R}_2^{\pm,[1]}(y,\kappa)$$

"DGLAP" term "ERBL" term

- where:

$$\begin{cases} \mathcal{R}_1^{-,[1]}(y,\kappa) & = & 2C_F \dfrac{1-y}{1-\kappa^2 y^2} \\ \mathcal{R}_2^{-,[1]}(y,\kappa) & = & 2C_F \dfrac{(1-\kappa)y}{1-\kappa^2 y^2} \end{cases}$$

$$\begin{cases} \mathcal{R}_{1,qq}^{+,[1]}(y,\kappa) = \mathcal{R}_1^{-,[1]}(y,\kappa) \\ \mathcal{R}_{2,qq}^{+,[1]}(y,\kappa) = 2C_F \dfrac{1-\kappa}{\kappa(1-\kappa^2 y^2)} \end{cases} \qquad \begin{cases} \mathcal{R}_{1,qg}^{+,[1]}(y,\kappa) = 4n_f T_R \dfrac{y(1-y)}{(1-\kappa^2 y^2)^2} \\ \mathcal{R}_{2,qg}^{+,[1]}(y,\kappa) = 4n_f T_R \dfrac{(1-\kappa)y^2}{(1-\kappa^2 y^2)^2} \end{cases}$$

$$\begin{cases} \mathcal{R}_{1,gq}^{+,[1]}(y,\kappa) = 2C_F \dfrac{(1-\kappa^2)y}{1-\kappa^2 y^2} \\ \mathcal{R}_{2,gq}^{+,[1]}(y,\kappa) = -2C_F \dfrac{1-\kappa^2}{\kappa(1-\kappa^2 y^2)} \end{cases} \qquad \begin{cases} \mathcal{R}_{1,gg}^{+,[1]}(y,\kappa) = 8C_A \dfrac{\kappa^2 y(1-y)}{(1-\kappa^2 y^2)^2} \\ \mathcal{R}_{2,gg}^{+,[1]}(y,\kappa) = C_A \dfrac{(1-\kappa)(1+\kappa-(1-7\kappa)\kappa^2 y^2)}{\kappa(1-\kappa^2 y^2)^2} \end{cases}$$

Anomalous dimensions

- We can finally compute the one-loop correction to the GTMD anomalous dimensions:

$$K_i(\mathbf{b}_T,\mu) = \lim_{\epsilon,\delta\to 0}\frac{d\ln Z_{S,i}(\mathbf{b}_T,Q,\zeta,\mu,\delta,\epsilon)}{d\ln\zeta}$$

$$\gamma_i(\mu,\zeta) = \lim_{\epsilon,\delta\to 0}\frac{d\ln[Z_{S,i}^{1/2}(\mathbf{b}_T,Q,\zeta,\mu,\delta,\epsilon)Z_{\Phi,i}^{-1}(\xi,\mu,\delta,\epsilon)]}{d\ln\mu}$$

- Given the renormalisation constants presented above, and their combination:

$$Z_{S,i}^{1/2}Z_{\Phi,i}^{-1} = 1-\frac{\alpha_s}{4\pi}2C_i\left[\frac{S_\epsilon^2}{\epsilon^2}+\frac{S_\epsilon}{\epsilon}\left(K_i+\ln\left(\frac{\mu^2}{(1-\xi^2)Q^2}\right)\right)+\ln\left(\frac{\mu^2}{\mu_b^2}\right)\ln\left(\frac{\zeta}{Q^2}\right)\right]+\mathcal{O}(\alpha_s^2)$$

- one readily finds:

$$K_i(\mathbf{b}_T,\mu) = -\frac{\alpha_s}{4\pi}4C_i\ln\left(\frac{\mu^2}{\mu_b^2}\right)+\mathcal{O}(a_s^2)$$

$$\gamma_i(\mu,\zeta) = \frac{\alpha_s}{4\pi}4C_i\left(K_i+\ln\left(\frac{\mu^2}{(1-\xi^2)\zeta}\right)\right)+\mathcal{O}(\alpha_s^2)$$

- The first coefficient of the expansion of the anomalous dimensions is:

$$K_i^{[0]}=0 \qquad \gamma_{F,i}^{[0]}=4C_iK_i \qquad \gamma_{K,i}^{[0]}=8C_i$$

- Unsurprisingly, these results coincide with those obtained in the TMD framework.

[Collins, *Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol.* 32 (2011) 1-624]

Forward limit

- Given the general result:

$$\mathcal{C}^{[1]}_{i/k}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = -\mathcal{P}^{[0]}_{i/k}(x,\kappa)\ln\left(\frac{\mu^2}{\mu_b^2}\right) + \mathcal{R}^{[1]}_{i/k}(x,\kappa) - \delta_{ik}\delta(1-x)2C_i\left[\frac{1}{2}\ln^2\left(\frac{\mu^2}{\mu_b^2}\right) - \left(K_i + \ln\left(\frac{\mu^2}{(1-\xi^2)\zeta}\right)\right)\ln\left(\frac{\mu^2}{\mu_b^2}\right) + \frac{\pi^2}{12}\right]$$

- setting $\mu = \mu_b$ eliminates all logarithmic terms:

$$\mathcal{C}^{[1]}_{i/k}(x,\kappa,\mathbf{b}_T,\mu_b,\zeta) = \mathcal{R}^{[1]}_{i/k}(x,\kappa) - \delta_{ik}\delta(1-x)C_i\frac{\pi^2}{6}$$

- We can now take the *forward limit* $\kappa \to 0$ that is equivalent to taking $\xi \to 0$:

$$\lim_{\kappa\to 0}\mathcal{C}^{[1],-}(y,\kappa,\mathbf{b}_T,\mu_b,\zeta) = \lim_{\kappa\to 0}\mathcal{C}^{[1],+}_{qq}(y,\kappa,\mathbf{b}_T,\mu_b,\zeta) = 2C_F(1-y) - C_F\frac{\pi^2}{6}\delta(1-y)$$

$$\lim_{\kappa\to 0}\mathcal{C}^{[1],+}_{qg}(y,\kappa,\mathbf{b}_T,\mu_b,\zeta) = 4n_f T_R y(1-y)$$

$$\lim_{\kappa\to 0}\mathcal{C}^{[1],+}_{gq}(y,\kappa,\mathbf{b}_T,\mu_b,\zeta) = 2C_F y$$

$$\lim_{\kappa\to 0}\mathcal{C}^{[1],+}_{gg}(y,\kappa,\mathbf{b}_T,\mu_b,\zeta) = -C_A\frac{\pi^2}{6}\delta(1-y)$$

- which reproduces the well-known TMD results [Collins, *Camb.Monogr.Part.Phys.Nucl.Phys.Cosmol.* 32 (2011) 1-624].

Reconstructing GTMDs

- We can now use the matching functions to reconstruct GTMDs.
- The unpolarised GTMD distributions can be decomposed as: Meißner, Metz, Schlegel [*JHEP* 08 (2009) 056]

$$\mathcal{F}_{i/H} = \frac{1}{2M}\overline{u}(P_{\text{out}})\left[F^i_{1,1} + \frac{i\sigma^{\mathbf{k}_T n}}{n\cdot P}F^i_{1,2} + \frac{i\sigma^{\mathbf{\Delta}_T n}}{n\cdot P}F^i_{1,3} + \frac{i\sigma^{\mathbf{k}_T\mathbf{\Delta}_T}}{M^2}F^i_{1,4}\right]u(P_{\text{in}})$$

- Each function $F^i_{1,l}$ is generally complex and can thus be decomposed into a real and an imaginary part:

$$F^i_{1,l} = F^{i,e}_{1,l} + iF^{i,o}_{1,l} \qquad\qquad F^{i,e}_{1,l}, F^{i,o}_{1,l} \in \mathbb{R}$$

- The real part of $F^i_{1,1}$ ($F^{i,e}_{1,1}$) in $\mathbf{b}_T$ space for small $|\mathbf{b}_T|$ and for $\mu^2 \simeq \zeta \simeq \mu_b^2$ is related to the GPDs H_j and E_j precisely by means of the matching functions:

$$F^{i,e}_{1,1}(x,\xi,b_T,t,\mu,\zeta) \underset{b_T\simeq 0}{=} \mathcal{C}_{i/j}(x,\kappa,b_T,\mu,\zeta) \underset{x}{\otimes} \left[(1-\xi^2)H_j(x,\xi,t,\mu) - \xi^2 E_j(x,\xi,t,\mu)\right]$$

- Moreover, the forward limit of $F^{i,e}_{1,1}$ is the unpolarised TMD $f_{1,i}$:

$$\lim_{\xi,t\to 0} F^{i,e}_{1,1}(x,\xi,b_T,t,\mu,\zeta) = f_{1,i}(x,b_T,\mu,\zeta)$$

Reconstructing GTMDs

- We can evolve $F_{1,1}^{i,e}$ to *any* scale by solving the evolution equations:
  - $\mathcal{O}(\alpha_s)$ matching functions allow us to reach **NNLL accuracy**. Anomalous dimensions (that coincide with the TMD ones) need to be evaluated accordingly.
- Extrapolation to large $|\mathbf{b}_T|$ is obtained *a la* CSS, *i.e.* by means of a b_* prescription:

$$b_*(b_T) = \frac{b_0}{Q}\left(\frac{1-\exp\left(-\frac{b_T^4 Q^4}{b_0^4}\right)}{1-\exp\left(-\frac{b_T^4}{b_0^4}\right)}\right)^{\frac{1}{4}}$$

- and introducing an appropriate non-perturbative function $f_{\rm NP}$. The final result is:

$$\begin{aligned} F_{1,1}^{i,e}(x,\xi,b_T,t,\mu,\zeta) &= \mathcal{C}_{i/j}(x,\kappa,b_*,\mu_{b_*},\mu_{b_*}^2)\underset{x}{\otimes}\left[(1-\xi^2)H_j(x,\xi,t,\mu_{b_*})-\xi^2 E_j(x,\xi,t,\mu_{b_*})\right] \\ &\times R_i\left[(\mu,\zeta)\leftarrow(\mu_{b_*},\mu_{b_*}^2)\right] \\ &\times f_{\rm NP}(x,b_T,(1-\xi^2)\zeta) \end{aligned}$$

- The evolution operator (or Sudakov form factor) is given by:

$$R_i = \exp\left\{K_i(b_*,\mu_{b_*})\ln\frac{\sqrt{(1-\xi^2)\zeta}}{\mu_{b_*}}+\int_{\mu_{b_*}}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_{F,i}(\alpha_s(\mu'))-\gamma_{K,i}(\alpha_s(\mu'))\ln\frac{\sqrt{(1-\xi^2)\zeta}}{\mu'}\right]\right\}$$

- Finally the GTMDs in $\mathbf{k}_T$ space are obtained by inverse Fourier transform:

$$F_{1,1}^{i,e}(x,\xi,k_T,t,\mu,\zeta) = \frac{1}{2\pi}\int_0^\infty db_T\, b_T J_0(k_T b_T) F_{1,1}^{i,e}(x,\xi,b_T,t,\mu,\zeta)$$

Numerical setup

- The numerical code used to compute $F_{1,1}^{i,e}$ is public:

  https://github.com/vbertone/GTMDMatching

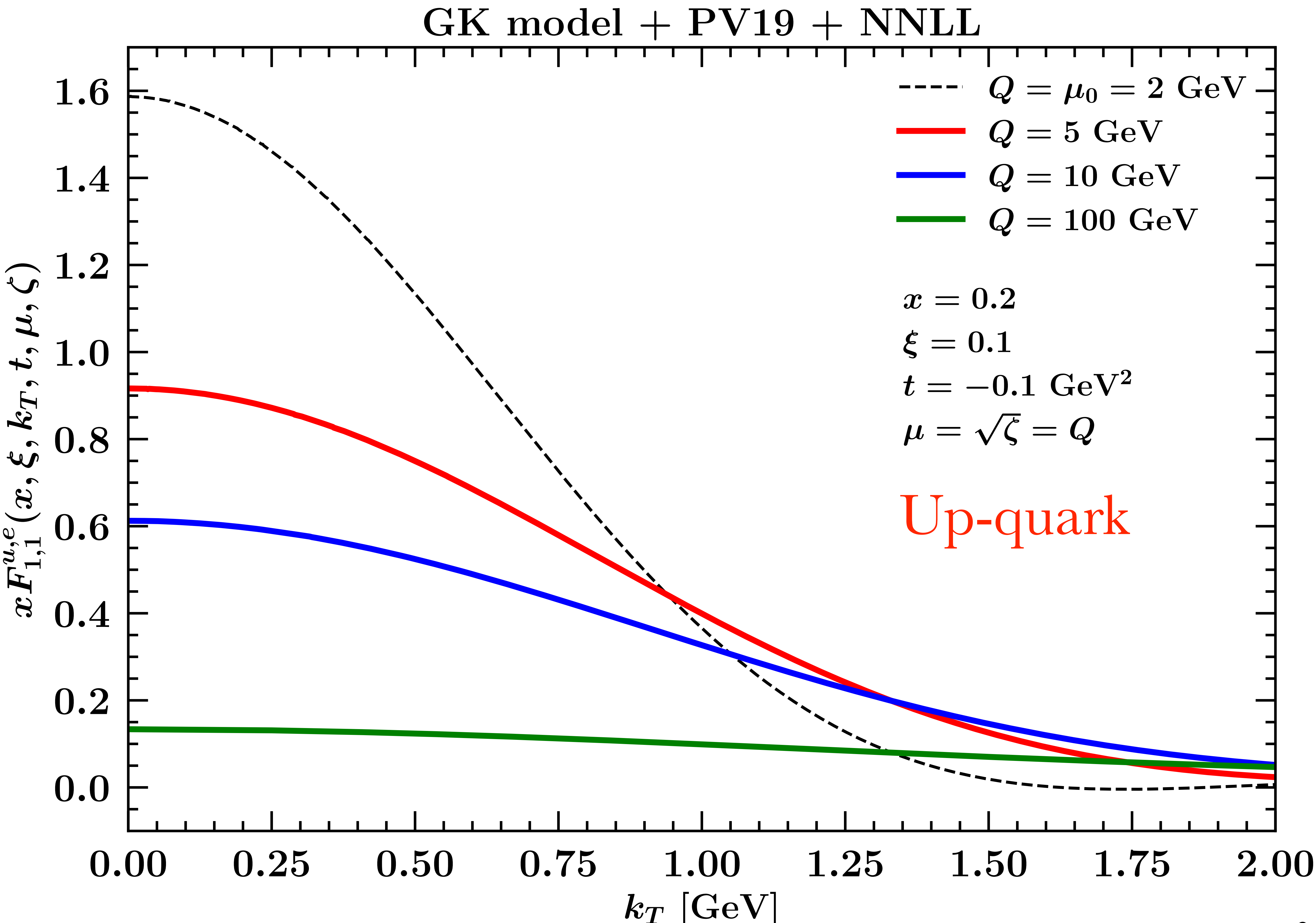
- and is based on a combination different public codes:

- **PARTONS** [https://partons.cea.fr/partons/doc/html/index.html] for the handling of GPDs:
  - the Goloskokov-Kroll (GK) model for the GPDs H_j and E_j has been used.

- **NangaParbat** [https://github.com/MapCollaboration/NangaParbat] for the handling of TMDs:
  - the PV19 [*JHEP* 07 (2020) 117] determination of $f_{\rm NP}$ along with the b_* function.

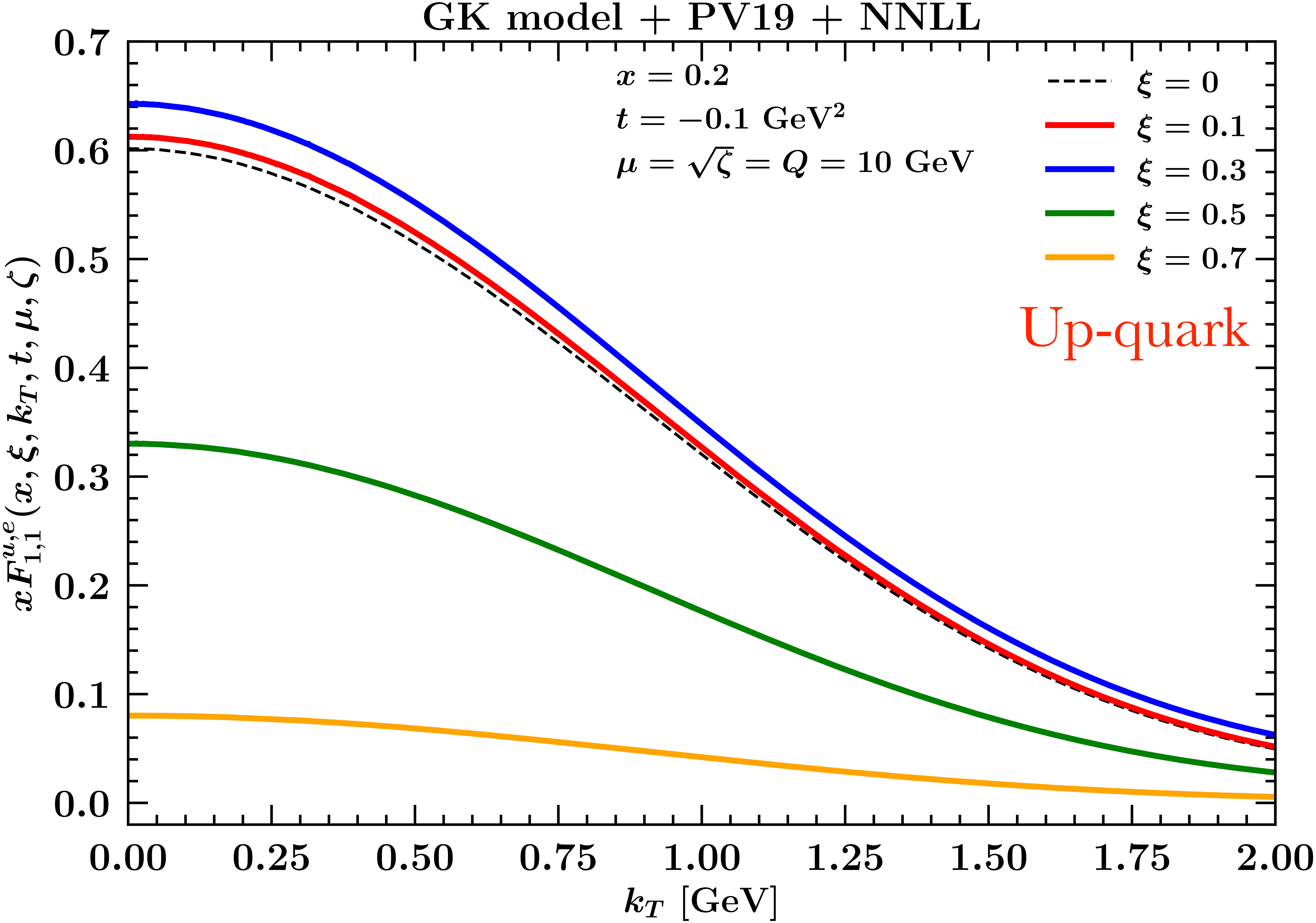
- **APFEL++** [https://github.com/vbertone/apfelxx] is used for:
  - the numerical computation of the **convolutions**,
  - the **collinear evolution of GPDs**,
  - the computation of the **Sudakov form factor**,
  - the **inverse Fourier transform**.

Results



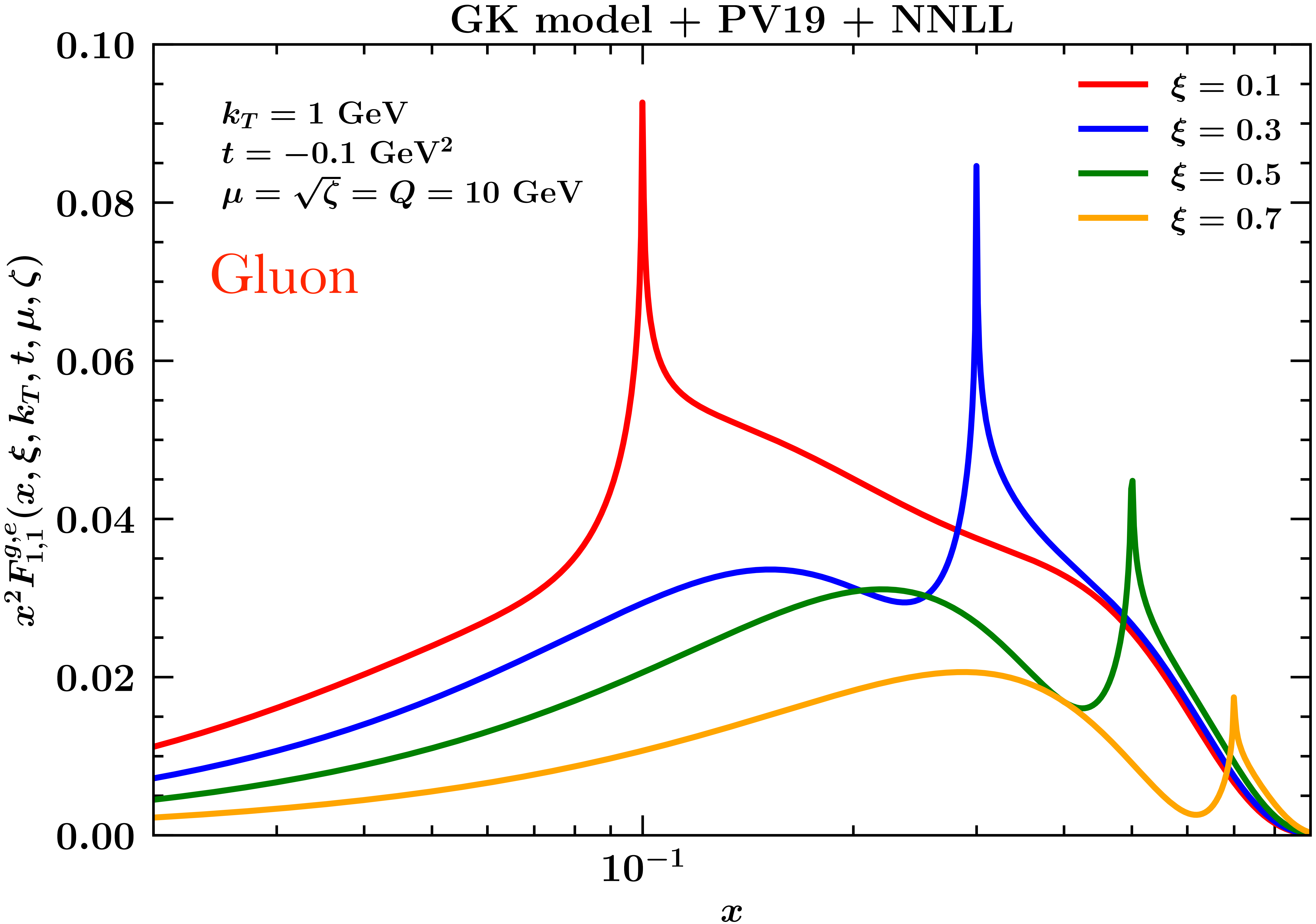
GK model + PV19 + NNLL
$xF^{u,e}_{1,1}(x,\xi,k_T,t,\mu,\zeta)$
$Q = \mu_0 = 2$ GeV
$Q = 5$ GeV
$Q = 10$ GeV
$Q = 100$ GeV
$x = 0.2$
$\xi = 0.1$
$t = -0.1$ GeV^2
$\mu = \sqrt{\zeta} = Q$
Up-quark
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
k_T [GeV]

Results



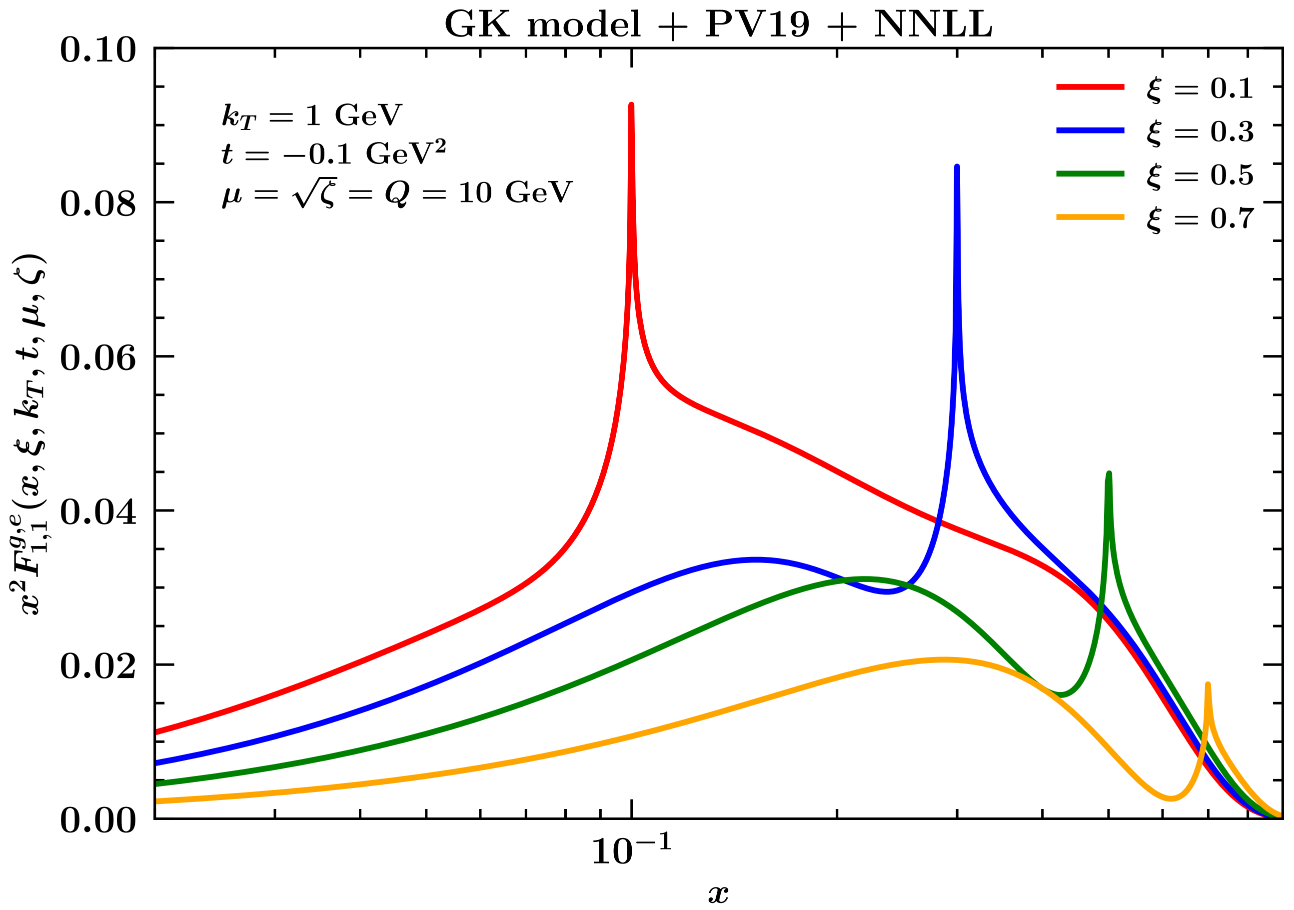
GK model + PV19 + NNLL
$x = 0.2$
$t = -0.1$ GeV^2
$\mu = \sqrt{\zeta} = Q = 10$ GeV
$\xi = 0$
$\xi = 0.1$
$\xi = 0.3$
$\xi = 0.5$
$\xi = 0.7$
Up-quark
$xF_{1,1}^{u,e}(x, \xi, k_T, t, \mu, \zeta)$
k_T [GeV]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Results


GK model + PV19 + NNLL
0.10
0.08
0.06
0.04
0.02
0.00
10^{-1}
x
$x^2 F_{1,1}^{g,e}(x, \xi, k_T, t, \mu, \zeta)$
$k_T = 1$ GeV
$t = -0.1$ GeV^2
$\mu = \sqrt{\zeta} = Q = 10$ GeV
Gluon
$\xi = 0.1$
$\xi = 0.3$
$\xi = 0.5$
$\xi = 0.7$

Results

- The x behaviour of $F_{1,1}^{i,e}$ presents a divergence at $x = \xi$.
- This enhancements is probably signalling the need for some sort of **resummation**.
- Further investigations are necessary.



- Logarithmic enhancements caused by large **threshold logarithms** of the kind $\ln(1 - x/\xi)$ have been studied in the past:
  - Altinoluk *et al.* [JHEP 10 (2012) 049] have studied the resummation of these logarithms in deeply-virtual Compton scattering (DVCS) obtaining a Sudakov-like resummation by means of the hyperbolic cosine.
  - More recently Braun *et al.* [JHEP 09 (2020) 117], within the context of a fixed-order NNLO calculation of DVCS, have challenged the result.
  - A few days ago Schoenleber [arXiv:2209.09015] has derived a resummation formula that closely resembles that obtained for threshold logs in inclusive DIS. This result agrees with [JHEP 09 (2020) 117] but disagrees with [JHEP 10 (2012) 049].
  - Musatov and Radyushkin [Phys.Rev.D 56 (1997) 2713-2735] have analysed $\gamma^* \gamma \to \pi^0$ arguing that the Sudakov resummation of thresholds logarithms takes place in an "unconventional" form, leading to an enhancement rather than to a suppression in the Sudakov region.

Conclusions

- At present, the study of the hadronic structure is a very alive branch of particle physics.

- While it started with the need to to describe high-energy processes, it has proven to be a very prolific research field that gives us access to a mine of information.

- As the study of PDFs has reached impressive precision, the determination of TMDs is quickly catching up, and phenomenological extractions of GPDs are taking their first steps, **GTMDs** represent the **next and ultimate frontier**.

- GTMDs can be regarded as "mother" distributions, directly connected with the long-sought Wigner distributions, from which all others descend.

- Tools to attack GTMDs are currently being developed:
  - the computation of the matching functions is only one of them,
  - other examples are: factorisation theorems, model developments, lattice calculations, etc.

- Much work still needs to be done but a lot of effort is being put into this field both on the theoretical and experimental side. The EIC is on the horizon and is expected to be a real breakthrough for the study of the hadronic structure.

Backup

Matching on GPDs

- We now move to NLO where we have:

$$\mathcal{C}^{[1]}_{i/j}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = D_j^{-1}(\xi)\left[\mathcal{F}^{[1]}_{i/j}(x,\xi,\mathbf{b}_T,\mu,\zeta) - F^{[1]}_{i/j}(x,\xi,\mu)\right]$$

- Considering the definition of the GTMD distributions and the perturbative expansions of **parton-in-parton** GTMD correlators and soft function:

$$S_i(\mathbf{b}_T,\mu,\zeta,\delta) \quad = \quad 1+\sum_{n=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n S_i^{[n]}(\mathbf{b}_T,\mu,\zeta,\delta)$$

$$\Phi_{i/j}(x,\xi,\mathbf{b}_T,\mu,\delta) \quad = \quad D_j(\xi)\delta_{ij}\delta(1-x)+\sum_{n=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n \Phi^{[n]}_{i/j}(x,\xi,\mathbf{b}_T,\mu,\delta)$$

- such that:

$$\mathcal{F}^{[1]}_{i/j}(x,\xi,\mathbf{b}_T,\mu,\zeta) = \Phi^{[1]}_{i/j}(x,\xi,\mathbf{b}_T,\mu,\delta) - \frac{1}{2}D_j(\xi)\delta_{ij}\delta(1-x)S_i^{[1]}(\mathbf{b}_T,\mu,\zeta,\delta)$$

- the one-loop correction to the matching functions are thus computed as:

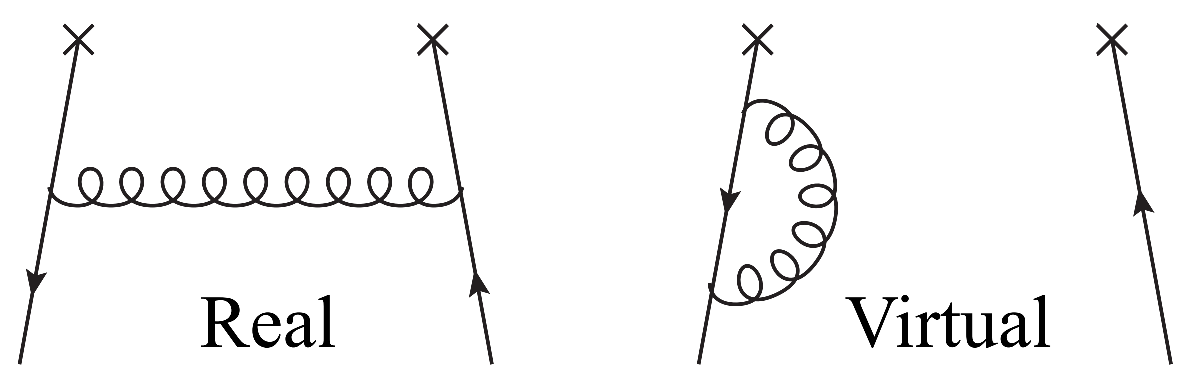
$$\mathcal{C}^{[1]}_{i/k}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = D_k^{-1}(\xi)\left[\Phi^{[1]}_{i/k}(x,\xi,\mathbf{b}_T,\mu,\delta) - F^{[1]}_{i/k}(x,\xi,\mu)\right] - \frac{1}{2}\delta_{ik}\delta(1-x)S_i^{[1]}(\mathbf{b}_T,\mu,\zeta,\delta)$$

Matching on GPDs

- The computation of $\mathscr{C}_{i/k}^{[1]}$ can be simplified by observing that:

$$\Phi_{i/k}^{[1]} = \Phi_{i/k}^{[1],\mathrm{real}} + \Phi_{i/k}^{[1],\mathrm{virt}} \qquad F_{i/k}^{[1]} = F_{i/k}^{[1],\mathrm{real}} + F_{i/k}^{[1],\mathrm{virt}}$$

- where by "real" we denote those diagrams that have attachments between the $-\eta/2$ and the $\eta/2$ legs, while "virtual" those that have not, *e.g.*:



- It is easy to see that in $\mathbf{b}_T$ space:

$$\Phi_{i/k}^{[1],\mathrm{virt}} = F_{i/k}^{[1],\mathrm{virt}}$$

- so that:

$$\Phi_{i/k}^{[1]} - F_{i/k}^{[1]} = \Phi_{i/k}^{[1],\mathrm{real}} - F_{i/k}^{[1],\mathrm{real}}$$

- Therefore, we only need to consider "real" diagrams ("virtuals" cancel out).

Matching on GPDs

- Moreover, the "real" part of the *renormalised* one-loop GPDs has the form:

$$F_{i/k}^{[1],\text{real}}(x,\xi,\mu) = D_k(\xi) S_\epsilon \left[\mathcal{P}_{i/k}^{[0],\text{real}}(x,\kappa,\delta)\left(\ln\mu^2 - \frac{\mu^{2\epsilon}}{\epsilon_{\text{IR}}}\right) - \epsilon\mathcal{R}_{i/k}^{[1]}(x,\kappa)\left(\frac{\mu^{2\epsilon}}{\epsilon_{\text{UV}}} - \frac{\mu^{2\epsilon}}{\epsilon_{\text{IR}}}\right)\right]$$

- while "real" part of the (UV convergent) GTMD correlator is:

$$\Phi_{i/k}^{[1],\text{real}}(x,\xi,\mathbf{b}_T,\mu,\delta) = -D_k(\xi)\left[\mathcal{P}_{i/k}^{[0],\text{real}}(x,\kappa,\delta) - \epsilon\mathcal{R}_{i/k}^{[1]}(x,\kappa)\right]\mu^{2\epsilon}\frac{\pi^\epsilon b_T^{2\epsilon}(1+\gamma_{\text{E}}\epsilon)}{\epsilon_{\text{IR}}} + \mathcal{O}(\epsilon)$$

- Their combination is finite in four dimension, we can then take the $\epsilon \to 0$ limit:

$$\Phi_{i/k}^{[1]}(x,\xi,\mathbf{b}_T,\mu,\delta) - F_{i/k}^{[1]}(x,\xi,\mu) = D_k(\xi)\left[-\mathcal{P}_{i/k}^{[0],\text{real}}(x,\kappa,\delta)\ln\left(\frac{\mu^2}{\mu_b^2}\right) + \mathcal{R}_{i/k}^{[1]}(x,\kappa)\right]$$

- The one-loop corrections to the GTMD matching functions can thus be written as:

$$\mathcal{C}_{i/k}^{[1]}(x,\kappa,\mathbf{b}_T,\mu,\zeta) = -\mathcal{P}_{i/k}^{[0],\text{real}}(x,\kappa,\delta)\ln\left(\frac{\mu^2}{\mu_b^2}\right) + \mathcal{R}_{i/k}^{[1]}(x,\kappa) - \frac{1}{2}\delta_{ik}\delta(1-x)S_i^{[1]}(\mathbf{b}_T,\mu,\zeta,\delta)$$

- Notice that both $\mathscr{P}_{i/k}^{[0]}$ and $S_i^{[1]}$ are separately affected by a **rapidity divergence**.

GTMD correlators at one-loop

- Although we showed that for the computation of the matching functions we only need the "real" contribution to the GTMD correlators (the "virtual" one cancels against GPDs), we have computed the *full* GTMD correlator ("real" + "virtual'):

$$\begin{aligned}\hat{\Phi}^{[1]}_{i/k}(x,\xi,\mathbf{b}_T,\mu,\delta) &= D_k(\xi)\bigg[-\frac{S_\epsilon}{\epsilon_{\rm IR}}\mathcal{P}^{[0],{\rm real}}_{i/k}(x,\kappa,\delta)-\mathcal{P}^{[0],{\rm real}}_{i/k}(x,\kappa,\delta)\ln\frac{\mu^2}{\mu_b^2}+\mathcal{R}^{[1]}_{i/k}(x,\kappa)\\ &+ \delta_{ik}\delta(1-x)2C_i\left(K_i-\ln(1-\xi^2)+2\ln\delta\right)\frac{\mu^{2\epsilon}S_\epsilon}{\epsilon_{\rm UV}}+\mathcal{O}(\epsilon)\bigg]\end{aligned}$$

- Because of the "virtual" part, the full GTMD correlator is **UV divergent**.

- The UV divergence is renormalised in $\overline{\rm MS}$ by means of the following (flavour diagonal) renormalisation constant:

$$Z_{\Phi,i}(\xi,\mu,\delta,\epsilon)=1+\frac{\alpha_s}{4\pi}2C_i\left(K_i-\ln(1-\xi^2)+2\ln\delta\right)\frac{S_\epsilon}{\epsilon}+\mathcal{O}(\alpha_s^2)$$

- As shown below, this renormalisation constant is necessary to compute the one-loop correction to the GTMD anomalous dimensions.