Resolving PDFs & GPDs of the Nucleon from Lattice QCD

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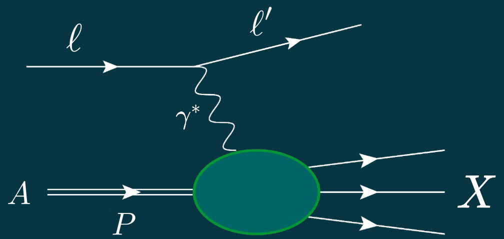
On Behalf of the HadStruc Collaboration

Jefferson Lab

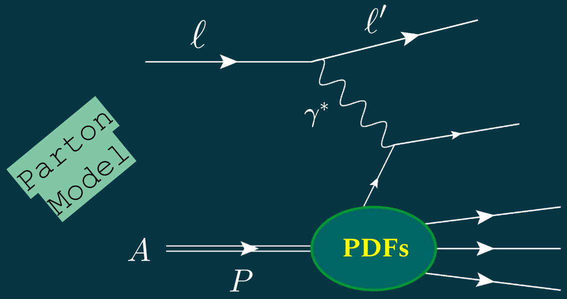
Deep Inelastic Scattering (DIS) & PDFs

$$\ell + A(P) \rightarrow \ell' + X$$



Invariants

$$\begin{cases} q^\mu = \ell^\mu - \ell'^\mu \\ s = (P+\ell)^2 \\ W^2 = (P+q)^2 \end{cases} \qquad Q^2 \equiv -q^2 \geq 0 \qquad x = \frac{Q^2}{2P\cdot q}$$



Inclusive cross section in terms of leptonic/hadronic tensors

$$\frac{d\sigma_{\rm DIS}(x,Q^2,s)}{d^3\ell'} \propto L_{\mu\nu}W^{\mu\nu}$$

$L_{\mu\nu}$: Perturbative

$W^{\mu\nu}$: Non-perturbative

$$W^{\mu\nu}(q,P) = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\langle P,S|\mathcal{J}^\mu(z)\,\mathcal{J}^\nu(0)|P,S\rangle$$

Lorentz decomposition into structure functions (SFs)

$$\hat{P}^\mu = P^\mu - \frac{P\cdot q}{q^2}q^\mu$$

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2) + \frac{\hat{P}^\mu\hat{P}^\nu}{P\cdot q}F_2(x,Q^2)$$

$$+\, i\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha S_\beta}{P\cdot q}g_1(x,Q^2) + i\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha\left(S_\beta - P_\beta\frac{S\cdot q}{P\cdot q}\right)}{P\cdot q}g_2(x,Q^2) + \text{P.V.}$$

QCD factorization theorems relate cross sections (SFs) to PDFs

Eg. J. Collins, D. Soper, G. Sterman, Adv. Ser. Direct. High Energy Phys. 5, 1 (1989)

$$F_i(x,Q^2) = \sum_{a=q,\bar{q},g} f_{a/h}(x,\mu^2)\otimes H_i^a\left(x,\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right) + h.t.$$

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle h(p)|\,\overline{\psi}\left(\tfrac{z}{2}\right)\gamma^+\Phi^{(f)}_{\hat{z}^-}\left(\left\{\tfrac{z}{2},-\tfrac{z}{2}\right\}\right)\psi\left(-\tfrac{z}{2}\right)|h(p)\rangle$$

➢ number densities & probabilistic interpretation

First-Principles Lattice QCD

Feynman Path Integral representation

- infinite trajectories of QM system

R. Feynman, Rev. Mod. Phys. 20, 367 (1948)

$$\langle\Omega|\, T\left\{\phi\left(x_1\right)\phi\left(x_2\right)\right\}|\Omega\rangle = \mathcal{Z}^{-1}\int\mathcal{D}\left[\phi\right]\phi\left(x_1\right)\phi\left(x_2\right)e^{iS[\phi]}$$

$$\langle\Omega|\,\hat{\mathcal{O}}\,|\Omega\rangle = \mathcal{Z}^{-1}\int\mathcal{D}\left[\phi\right]\mathcal{O}\left[\phi\right]e^{iS[\phi]}$$

Strict UV/IR cutoffs

- fermions restricted to lattice sites
- oriented fields $\quad U_\mu\left(x\right)\in \mathrm{SU}\left(3\right)$
  - *gauge links* $\quad U_\mu\left(x\right)\equiv e^{iaA_\mu(x)}$
- Dirac operator & propagators

Oscillatory action - *Sign Problem*

$$\mathcal{Z} = \int\mathcal{D}\left[\psi_i,\overline{\psi}_i,A_\mu\right]e^{iS_{\mathrm{QCD}}\left[\psi_i,\overline{\psi}_i,A_\mu\right]} \quad\Rightarrow\quad \mathcal{Z}_E = \int\mathcal{D}\left[\psi_i,\overline{\psi}_i,A_\mu\right]e^{-S_E\left[\psi_i,\overline{\psi}_i,A_\mu\right]}$$

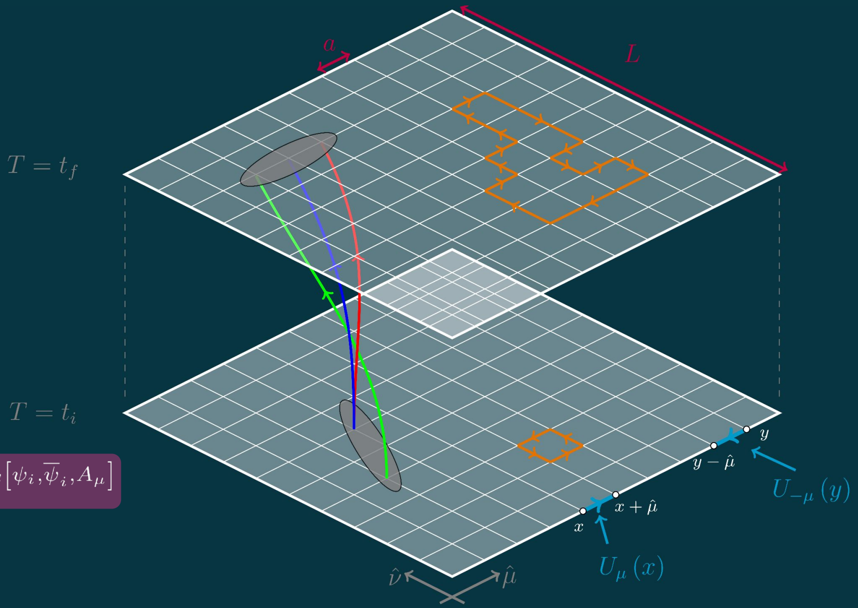
Integrate out fermionic fields

$$\left\langle T\left\{\prod_i\mathcal{O}_i\right\}\right\rangle \equiv \int\mathcal{D}\left[U\right]\langle\prod_i\mathcal{O}_i\rangle_{\mathrm{F}}\;P\left[U\right] \qquad \left\langle T\left\{\prod_i\mathcal{O}_i\right\}\right\rangle \approx \frac{1}{N}\sum_{k=1}^{N}\prod_i\mathcal{O}_i\left[U_k\right]$$

First-principles scheme to numerically compute quantities directly from QCD Lagrangian

$$\Lambda = \left\{x\in\mathbb{R}^d \mid x = na, n\in\mathbb{Z}^d\right\}$$

K. G. Wilson, Phys. Rev. D 10, 2445 (1974)



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Strict UV/IR cutoffs

- fermions restricted to lattice sites
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$$U_\mu(x) \in \mathrm{SU}(3)$$

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$$\mathcal{Z} = \int \mathcal{D}\left[\psi_i, \overline{\psi}_i, A_\mu\right] e^{iS_{\mathrm{QCD}}\left[\psi_i,\overline{\psi}_i,A_\mu\right]} \quad \Rightarrow \quad \mathcal{Z}_E = \int \mathcal{D}\left[\psi_i, \overline{\psi}_i, A_\mu\right] e^{-S_E[}$$

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$$\left\langle T\left\{\prod_i \mathcal{O}_i\right\}\right\rangle \equiv \int \mathcal{D}[U]\,\langle\prod_i \mathcal{O}_i\rangle_{\mathrm{F}}\; P[U] \qquad \left\langle T\left\{\prod_i \mathcal{O}_i\right\}\right\rangle \approx \frac{1}{N}\sum_{k=1}^{N}$$

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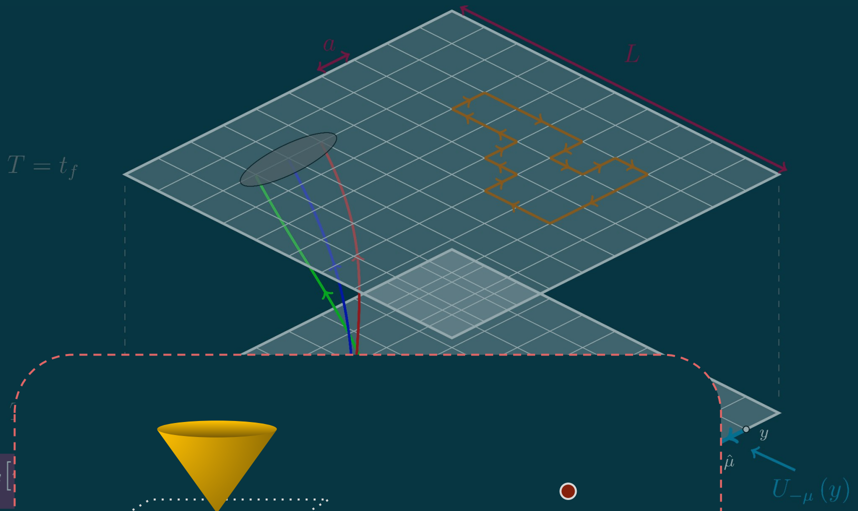
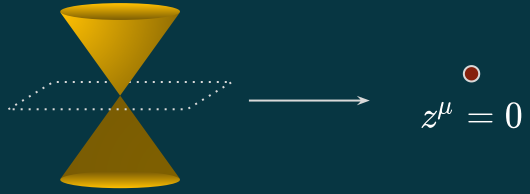




Collapse of light-cone in Euclidean metric of LQCD

From Matrix Elements in Lattice QCD to PDFs

Two popular, and related, methods to obtain PDFs
from matrix elements of space-like quantities in Lattice QCD

$$M^{[\Gamma]}(p,z) = \langle h(p)|\overline{\psi}(z)\,\Gamma\Phi_{\hat{z}}^{(f)}(\{z,0\})\,\psi(0)\,|h(p)\rangle$$

LaMET

Large Momentum Effective Theory

Quasi-PDF: Fourier transform - distribution of parton longitudinal space-like momenta

Factorizes into PDF with power corrections in $1/p_z^2$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002

SDF

Short Distance Factorization

Short-distance OPE applied to matrix element

Factorizes into PDF with power corrections in z^2

V. Braun and D. Mueller, Eur.Phys.J.C 55 (2008) 349-361
A. Radyushkin, Phys.Rev.D 96 (2017) 3, 034025
Y. Q. Ma and J. W. Qiu, Phys. Rev. Lett. 120 (2018) 2, 022003

Several other methods exist to extract SFs from suitable Euclidean correlations

- Hadronic tensor

  J. Liang et al., Phys.Rev.D 101 (2020) 11, 114503

- “OPE without OPE”

  K.U. Can et al., Phys.Rev.D 102 (2020) 114505
  A.J. Chambers et al., Phys.Rev.Lett. 118 (2017) 24, 242001

- Auxiliary quark methods (Pion DAs & moments from OPE)

  HOPE Collab., Phys.Rev.D 105 (2022) 3, 034506
  W. Detmold et al., PoS LATTICE2018 (2018) 106
  G. Bali et al., Eur.Phys.J.C 78 (2018) 3, 217
  G. Bali et al., Phys.Rev.D 98 (2018) 9, 094507

- Current-current correlators

  R.S. Sufian, J. Karpie, CE et al., Phys.Rev.D 99 (2019) 7, 074507
  R.S. Sufian, CE, J. Karpie et al., Phys.Rev.D 102 (2020) 5, 054508

Comments on Space-like Parton Bilinears

A non-trivial light-cone limit:

$$M^{[\Gamma]}(p,z) = \langle h(p)|\,\overline{\psi}(z)\,\Gamma\Phi^{(f)}_{\hat{z}}(\{z,0\})\,\psi(0)\,|h(p)\rangle$$

Standard log. singularities → perturbative evolution

Additional UV singularities for space-like Wilson line

$$Z_{\text{link}}(z_3,a) \simeq e^{-A|z_3|/a}$$

Add. UV divergences must be regulated and removed prior to cont. limit

Bare Matrix Element

Non-Pert. Renormalization

M. Constantinou and H. Panagopoulos, Phys. Rev. D96, 054506 (2017)
C. Alexandrou et al., Nucl. Phys. B923, 394 (2017)

Map momentum dependence

- truncated FT.
- Backus-Gilbert, etc.

Matching (LaMET)

$$\tilde{q}(\tilde{x},\mu,p_z) = \int_{-1}^{1}\frac{dx}{|x|}C\left(\frac{\tilde{x}}{x},\frac{\mu}{p_z}\right)q(x,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\text{QCD}}}{p_z^2},\frac{m^2}{p_z^2}\right)$$

X. Ji, Sci. China Phys.Mech.Astron. 57, 1407 (2014)

Inverse Problems...

Reduced Distribution

K. Orginos, et al., Phys. Rev. D96, 094503 (2017)

- Ioffe-time independent
- Multiplicatively renormalizable

T. Ishikawa et al., Phys.Rev.D 96 (2017) 9, 094019
X. Ji et al., Phys.Rev.Lett. 120 (2018) 11, 112001
J. Green et al., Phys.Rev.Lett. 121 (2018) 2, 022004

$$\mathfrak{M}(\nu,z^2) = \frac{M_4(p,z)/M_4(p,0)}{M_4(0,z)/M_4(0,0)}$$

Matching (SDF) / PDF Extraction

$$\mathcal{M}(\nu,z^2) = C\left(z^2\mu^2,\alpha_s(\mu^2)\right)\otimes Q(\nu,\mu^2) + \mathcal{O}\left(z^2\Lambda^2_{\text{QCD}}\right)$$

T. Izubuchi, et al., Phys.Rev. D98 (2018) no.5, 056004
A. Radyushkin, Phys.Lett. B781 (2018) 433-442
A. Radyushkin, Phys. Rev. D 98 (2018) no.1, 014019
J.-H. Zhang, et al., Phys.Rev. D97 (2018) no.7, 074508

Towards the Unpolarized PDF from Pseudo-Distributions

$$\nu \equiv p \cdot z$$

A matrix element of a distinct character

$$M^\alpha(p,z) = \langle h(p)|\overline{\psi}(z)\gamma^\alpha\Phi_{\hat{z}}^{(f)}(\{z,0\})\psi(0)|h(p)\rangle = 2p^\alpha\mathcal{M}(\nu,z^2) + 2z^\alpha\mathcal{N}(\nu,z^2)$$

Unpolarized Light-cone PDF

Unpolarized leading-twist PDF defined in terms of $k^-, \mathbf{k}_\perp$ integrated parton correlator

$$p^\alpha = \left(p^+, \frac{m_h^2}{2p^+}, \mathbf{0}_\perp\right)$$

$$z^\alpha = (0, z^-, \mathbf{0}_\perp) \qquad \alpha = +$$

[Unpolarized] Ioffe-time Distribution (ITD)

$$\mathcal{M}(p^+z^-, 0)_{\mu^2} \equiv Q(\nu,\mu^2) = \int_{-1}^{1} dx\ e^{i\nu x} f_{q/h}(x,\mu^2)$$

V. Braun et al., Phys.Rev.D 51 (1995) 6036-6051

Unpolarized Pseudo-PDF

Generalization of unpolarized light-cone PDF onto space-like intervals; Lorentz covariant parton momentum fraction

Frame amenable to calculation in Lattice QCD

$$\alpha = 4 \qquad p^\alpha = (\mathbf{0}_\perp, p_z, E) \qquad z^\alpha = (\mathbf{0}_\perp, z_3, 0)$$

[Unpolarized] Ioffe-time Pseudo-distribution (pseudo-ITD)

$$\mathcal{M}(p_z z_3, z_3^2) = \int_{-1}^{1} dx\ e^{i\nu x}\mathcal{P}(x, z_3^2)$$

A. Radyushkin, Phys.Rev.D 96 (2017) 3, 034025

Obtaining the Pseudo-Distribution

Needed correlation functions:

$$C_2(p_z,T) = \langle \mathcal{N}(-p_z,T_f)\,\overline{\mathcal{N}}(p_z,T_0)\rangle = \sum_n |\mathcal{A}_n|^2\, e^{-E_nT}$$

$$C_3(p_z,T,\tau;z_3) = V_3\,\langle \mathcal{N}(-p_z,T_f)\,\mathring{\mathcal{O}}^{[\gamma_4]}_{\mathrm{WL}}(z_3,\tau)\,\overline{\mathcal{N}}(p_z,T_0)\rangle$$
$$= V_3 \sum_{n,n'} \langle \mathcal{N}|n'\rangle\langle n|\overline{\mathcal{N}}\rangle\langle n'|\,\mathring{\mathcal{O}}^{[\gamma_4]}_{\mathrm{WL}}(z_3,\tau)\,|n\rangle\, e^{-E_{n'}(T-\tau)}e^{-E_nT}$$

$$C_{mn}(t) = \sum_{\vec{x},\vec{y}} \langle 0|\,\mathcal{O}_m(t,\vec{x})\,\mathcal{O}_n^{\dagger}(0,\vec{y})\,|0\rangle$$
$$\equiv \mathrm{Tr}\left[\Phi_m(t)\otimes\tau(t,0)\,\tau(t,0)\,\tau(t,0)\otimes\Phi_n(0)\right]$$

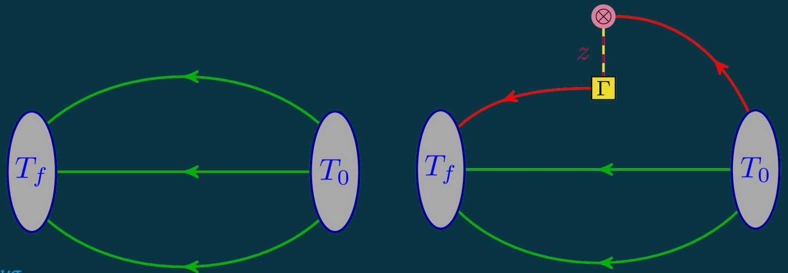
Contamination from unwanted states & $O(3)\mapsto O_h^{[D]}$

- interpolators that best reflect properties of desired state $\langle 0|\,\hat{\mathcal{O}}(\vec{p})\,|h(\vec{p})\rangle \gg \langle 0|\,\hat{\mathcal{O}}(\vec{p})\,|h'(\vec{p})\rangle$

Distillation: Low-rank and *non-iterative* approximation of a gauge-covariant smearing kernel

M. Peardon et al., Phys. Rev. D80, 054506 (2009)

$$J_{\sigma,n_\sigma} = e^{\sigma\nabla^2} = \sum_\lambda e^{-\sigma\lambda}\,|\lambda\rangle\langle\lambda| \qquad \square(\vec{x},\vec{y};t)_{ab} = \sum_{k=1}^{R_{\mathcal{D}}} \xi_a^{(k)}(\vec{x},t)\,\xi_b^{(k)\dagger}(\vec{y},t)$$



Wick contractions factorize distillation space

"Perambulators" $\tau^{kl}_{\alpha\beta}(t_f,t_0) = \xi^{(k)\dagger}(t_f)\,M^{-1}_{\alpha\beta}(t_f,t_0)\,\xi^{(l)}(t_0)$

"Elementals" $\Phi^{(i,j,k)}_{\mu\nu\sigma}(t) = \epsilon^{abc}\left(\mathcal{D}_1\xi^{(i)}\right)^a\left(\mathcal{D}_2\xi^{(j)}\right)^b\left(\mathcal{D}_3\xi^{(k)}\right)^c(t)\,S_{\mu\nu\sigma}$

$$\Xi^{(l,k)}_{\alpha\beta}(T_f,T_0;\tau,z_3) = \sum_{\vec{y}} \xi^{(l)\dagger}(T_f)\,D^{-1}_{\alpha\sigma}(T_f,\tau;\vec{y}+z_3\hat{z})\,\left[\gamma^4\right]_{\sigma\rho}\,\Phi^{(f)}_{\hat{z}}(\{\vec{y}+z_3\hat{z},\vec{y}\})\,D^{-1}_{\rho\beta}(\tau,T_0;\vec{y})\,\xi^{(k)}(T_0)$$

Unpolarized PDFs

Space-like Wilson line

Nucleon Interpolators with Distillation

Excited-state contamination

- ➢ optimize operator/state overlaps - saturate correlation functions at early temporal separations

Generic light-quark nucleon interpolator smeared with distillation

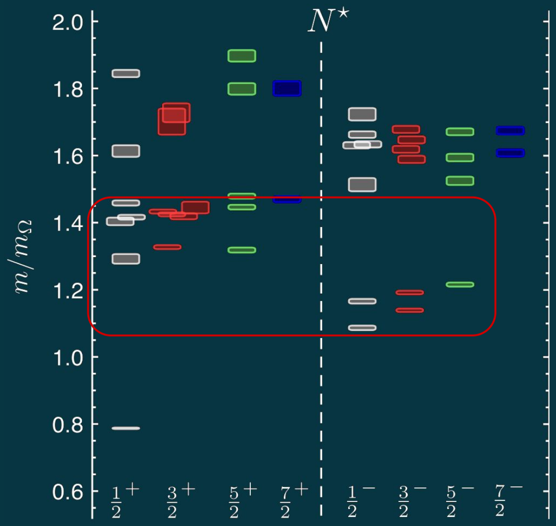
$$\mathcal{O}_i(t) = \epsilon^{abc}\left(\mathcal{D}_1\Box u\right)_a^\alpha\left(\mathcal{D}_2\Box d\right)_b^\beta\left(\mathcal{D}_3\Box u\right)_c^\gamma(t)\, S_i^{\alpha\beta\gamma}$$

Dirac structure/covariant derivatives

Discretized continuum-like interpolators of definite permutational symmetries

$$\mathcal{O}_B = \left(\mathcal{F}_{\mathcal{P}(\mathrm{F})}\otimes\mathcal{S}_{\mathcal{P}(\mathrm{S})}\otimes\mathcal{D}_{\mathcal{P}(\mathrm{D})}\right)\{q_1q_2q_3\}$$

$$\left(N_M\otimes\left(\tfrac{1}{2}^+\right)^1_M\otimes D^{[2]}_{L=1,A}\right)^{J^P=\frac{1}{2}^+}\equiv N^2P_A\tfrac{1}{2}^+$$



R. Edwards, et. al., Phys. Rev. D84, 074508 (2011)

(Generally) Continuum spins reducible under octahedral group

Canonical subductions

- ➢ spinors/derivatives combined into object of definite J^P

$$\mathcal{O}^{\{J\}}_{^n\Lambda,r} = \sum_m S^{J,m}_{^n\Lambda,r}\;\mathcal{O}^{\{J,m\}}$$

R. Edwards, et. al., Phys. Rev. D84, 074508 (2011)
J. Dudek and R. Edwards, Phys. Rev. D85, 054016 (2012)

Helicity subductions

C. Thomas, et al., Phys. Rev. D85, 014507 (2012)
C. Thomas, private communication

- ➢ boost breaks O_h^D symmetry to little groups

$$\left[\mathbb{O}^{J^P,\lambda}(\vec{p})\right]^\dagger = \sum_m \mathcal{D}^{(J)}_{m,\lambda}(R)\left[O^{J^P,m}(\vec{p})\right]^\dagger$$

- ➢ subduce into little groups

$$\left[\mathbb{O}^{J^P,|\lambda|}_{\Lambda,\mu}(\vec{p})\right]^\dagger = \sum_{\hat{\lambda}=\pm|\lambda|} S^{\tilde{\eta},\hat{\lambda}}_{\Lambda,\mu}\left[\mathbb{O}^{J^P,\hat{\lambda}}(\vec{p})\right]^\dagger$$

Unpolarized Pseudo-ITD: Lattice Implementation

JLab/WM/LANL 2+1 Flavor Isotropic Lattices

ID	a (fm)	m_π (MeV)	β	c_{SW}	$L^3 \times T$	N_{cfg}
E1	0.094(1)	358(3)	6.3	1.205	$32^3 \times 64$	349

➢ isovector combination only

Parameters/Statistics

ID	N_{vec}	N_{srcs}	T/a	$p_z \times \left(\frac{2\pi}{L}\right)$	z/a
E1	64	4	$4, 6, \cdots, 14$ $0.38, \cdots, 1.32$ fm	$0, \pm 1, \cdots, \pm 6$ $0, 0.41, \cdots, 2.47$ GeV	$0, \pm 1, \cdots, \pm 12, \cdots$ $0, 0.094, \cdots, 1.13$ fm

[Unpolarized] Short-distance factorization

$$B(u) = \left(\frac{1+u^2}{1-u}\right)_+ \qquad L(u) = \left[4\frac{\ln(1-u)}{1-u} - 2(1-u)\right]_+$$

$$\mathfrak{M}(\nu, z^2) = \left\{\delta(1-u) - \frac{\alpha_s C_F}{2\pi}\int_0^1 du \left[\ln\left(\frac{e^{2\gamma_E+1}z^2\mu^2}{4}\right) B(u) + L(u)\right]\right\} \mathcal{Q}(u\nu, \mu^2) + \mathcal{O}(z^2\Lambda^2_{QCD})$$

High-momenta essential

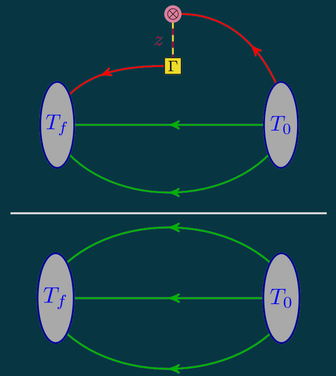
G. S. Bali et al. Phys. Rev. D93, 094515 (2016)
CE, R. Edwards, K. Orginos, D. Richards, PRD 103 (2021) 3, 034502

$$\xi^{(k)}_\pm(\vec{z}, t) \equiv e^{i\vec{\zeta}_\pm \cdot \vec{z}}\xi^{(k)}(\vec{z}, t), \qquad \vec{\zeta}_\pm = \pm 2 \cdot \frac{2\pi}{L}\hat{z}$$

Summation method - further excited-state suppression

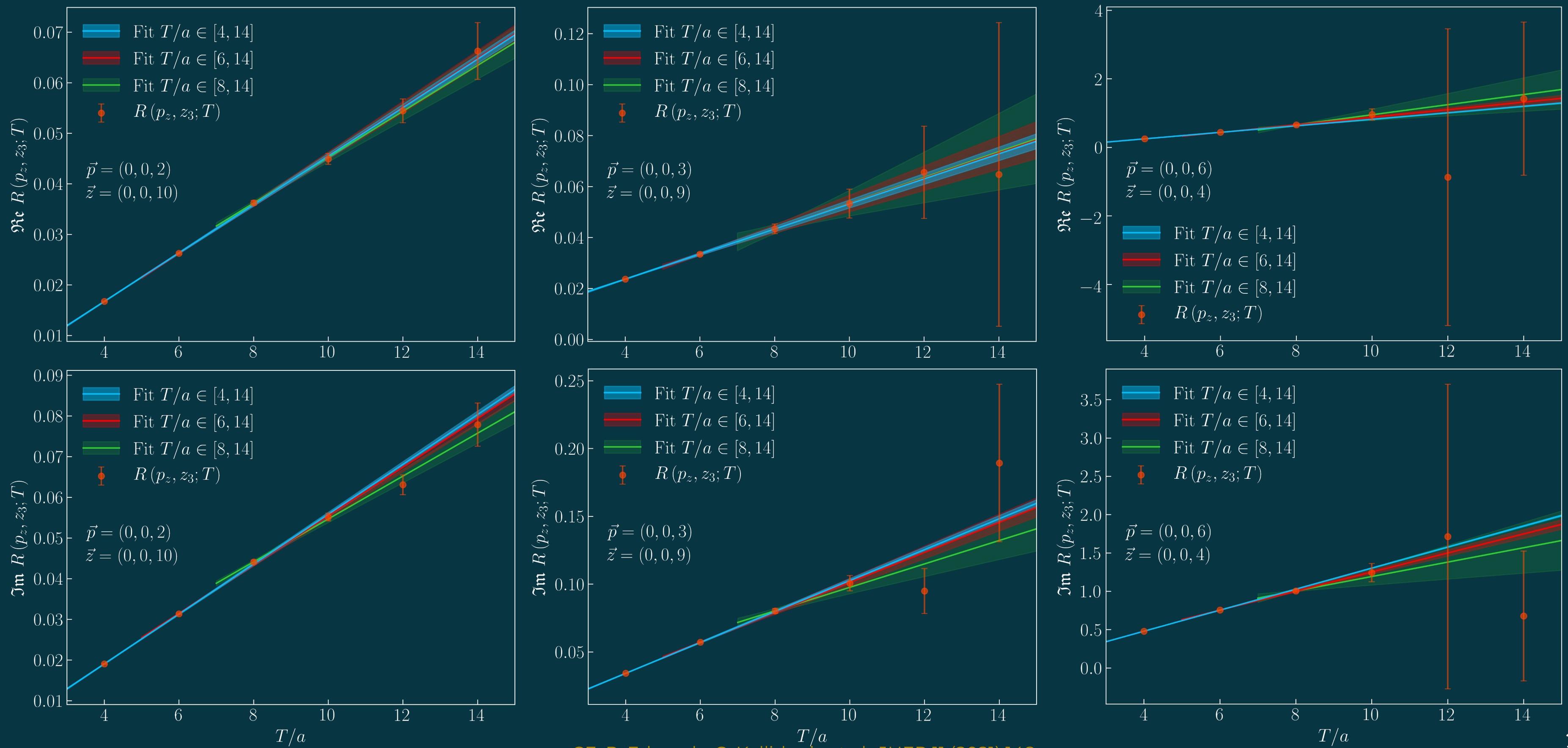
L. Maiani et al., Nucl. Phys. B293 (1987)
C. Bouchard et al., Phys. Rev. D 96, no. 1, 014504 (2017)

$$R(p_z, z_3; T) = \sum_{\tau/a=1}^{T-1} \frac{C_3(p_z, T, \tau; z_3)}{C_2(p_z, T)}$$



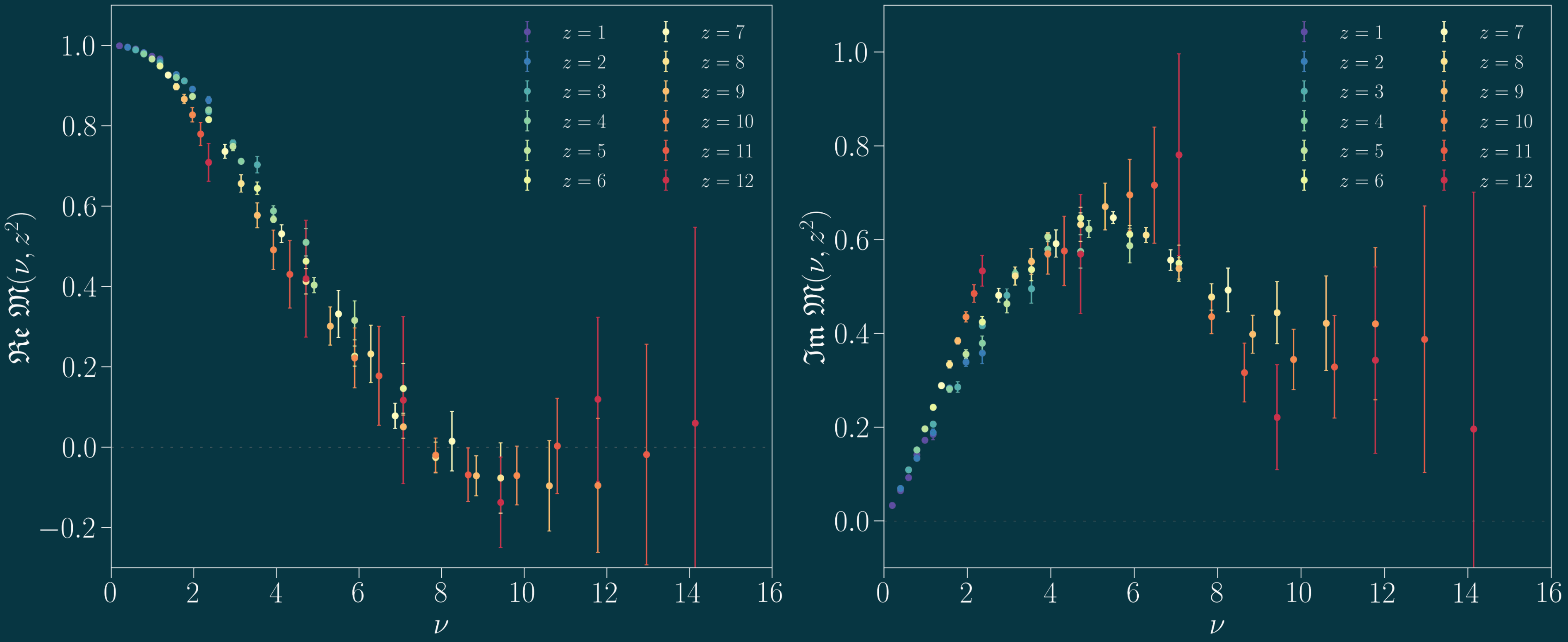
$$R_{fit}(p_z, z_3; T) = \mathcal{A} + M_4(p_z, z_3)T + \mathcal{O}\left(e^{-\Delta E T}\right)$$

Selected Unpolarized Matrix Elements



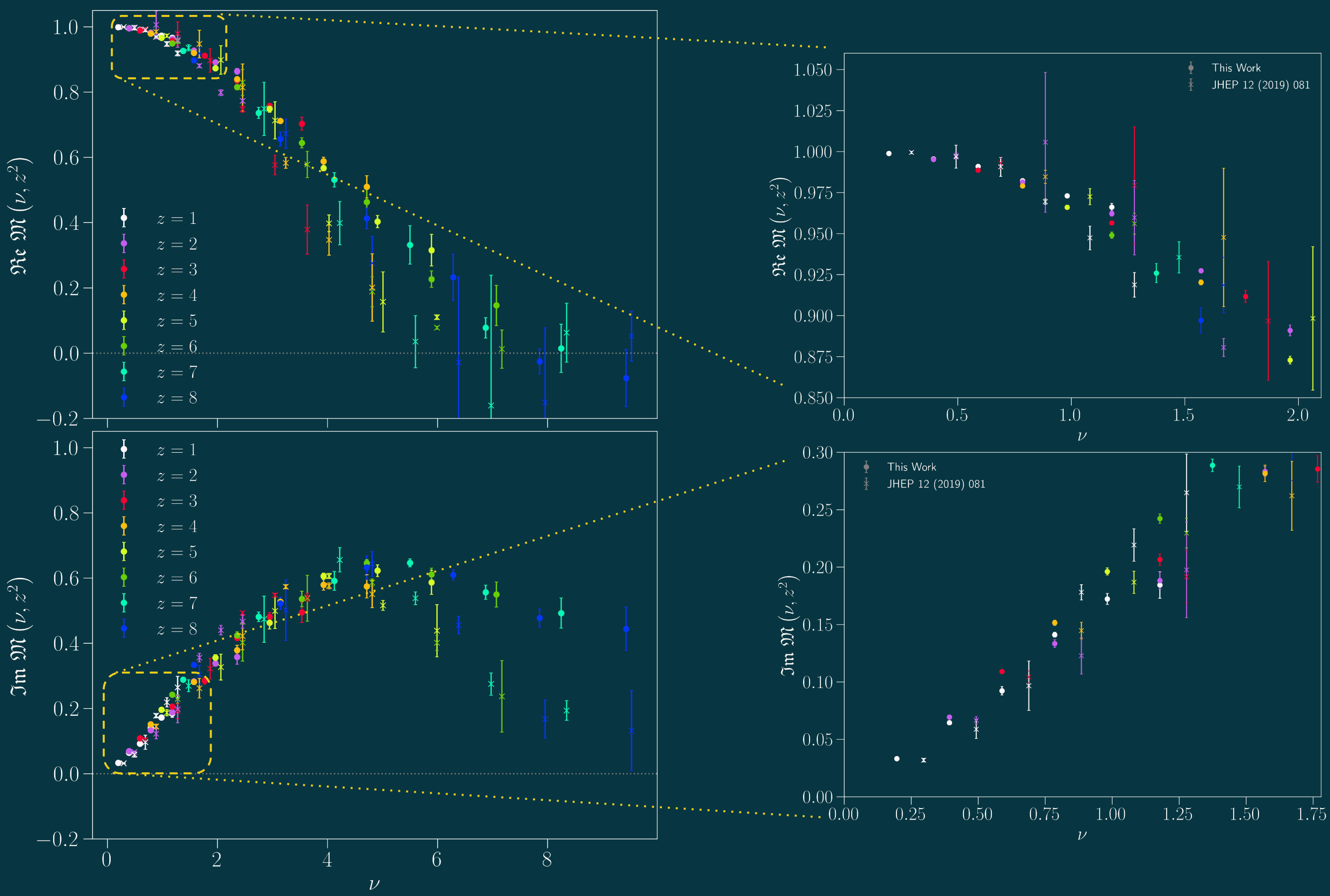
CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Unpolarized Ioffe-time Pseudo-Distribution



CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Efficacy of Distillation



B. Joó et al., JHEP 12 (2019) 081
[Gaussian smearing]

$N_{\text{cfg}} = 417 \qquad N_{\text{src}} = 8 \qquad N_\zeta = 5$

$N_{\text{inv}}/\text{cfg} \simeq 8.6\text{k}$

CE, R. Edwards, C. Kallidonis et al.,
JHEP 11 (2021) 148
[Distillation]

$N_{\text{cfg}} = 349 \qquad N_{\text{src}} = 4 \qquad N_\zeta = 3$

$N_{\text{inv}}/\text{cfg} \simeq 16\text{k}$

Parameterizing the Unknown PDF

Jacobi (hypergeometric) polynomials

$$P_n^{(\alpha,\beta)}(z) = \frac{\Gamma(\alpha+n+1)}{n!\Gamma(\alpha+\beta+n+1)} \sum_{j=0}^{n} \binom{n}{j} \frac{\Gamma(\alpha+\beta+n+j+1)}{\Gamma(\alpha+j+1)} \left(\frac{z-1}{2}\right)^j$$

- $z \in [-1, 1]$ — Interval
- $(1-z)^\alpha (1+z)^\beta$ — Metric
- $\alpha, \beta > -1$ — Validity

$$\int_{-1}^{1} dz\,(1-z)^\alpha (1+z)^\beta P_n^{(\alpha,\beta)}(z) P_m^{(\alpha,\beta)}(z) = \delta_{n,m} h_n(\alpha,\beta)$$

Flexibility of PDF functional form captured without bias via $\{\Omega_n^{(\alpha,\beta)}\}$

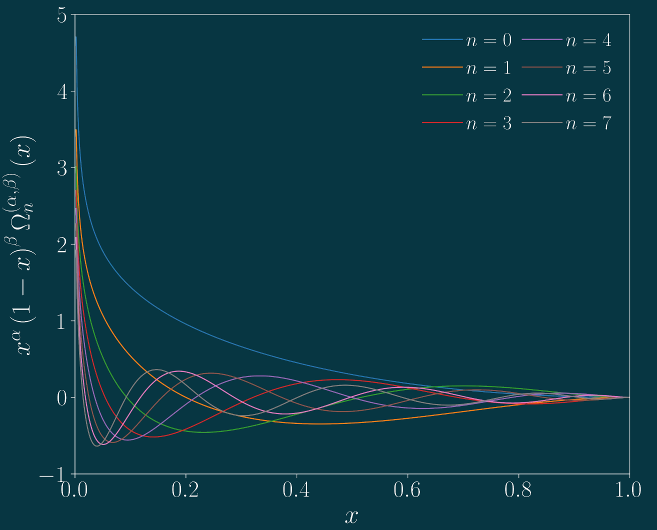
$$f_{q/h}(x) = x^\alpha (1-x)^\beta \sum_{n=0}^{\infty} C_{q,n}^{(\alpha,\beta)} \Omega_n^{(\alpha,\beta)}(x)$$

J. Karpie, K. Orginos, A. Radyushkin et al., JHEP 11 (2021) 024

A convenient change of variables: $z \mapsto 1 - 2x$

$$\Omega_n^{(\alpha,\beta)}(x) = \sum_{j=0}^{n} \underbrace{\frac{\Gamma(\alpha+n+1)}{n!\Gamma(\alpha+\beta+n+1)} \binom{n}{j} \frac{(-1)^j \Gamma(\alpha+\beta+n+j+1)}{\Gamma(\alpha+j+1)}}_{\omega_{n,j}^{(\alpha,\beta)}} x^j$$

- $x \in [0, 1]$ — Interval
- $x^\alpha (1-x)^\beta$ — Metric
- $\alpha, \beta > -1$ — Validity



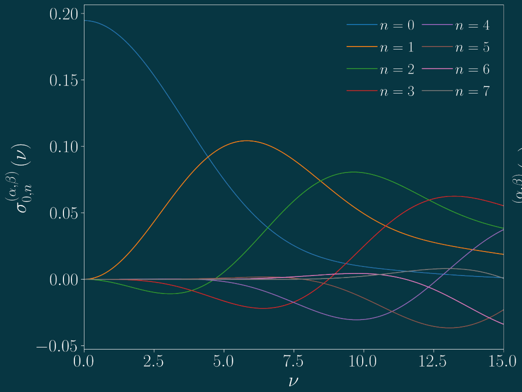
Regularization via Orthogonal Polynomials

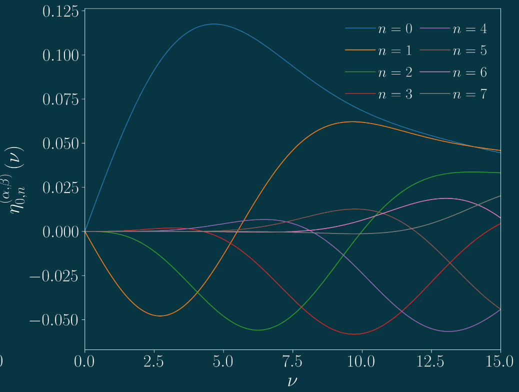
Ill-posed (pseudo-)ITD/PDF matching relation:
(α,β) lose meaning when inf. # terms included

$$\mathfrak{M}\left(\nu, z^2\right) = \int_{-1}^{1} dx\; \mathcal{K}\left(x\nu, z^2\mu^2\right) \boxed{f_{q/h}\left(x, \mu^2\right)} + \sum_{k=1}^{\infty} \mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$$

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \int_0^1 dx\; \mathcal{K}_{\rm v}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta \Omega_n^{(\alpha,\beta)}\left(x\right)}$$
$$= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{\left(-1\right)^k}{\left(2k\right)!} c_{2k}\left(z^2\mu^2\right) \omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+1, \beta+1\right)\nu^{2k}$$

$$\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \int_0^1 dx\; \mathcal{K}_{+}\left(x\nu, z^2\mu^2\right) \boxed{x^\alpha\left(1-x\right)^\beta \Omega_n^{(\alpha,\beta)}\left(x\right)}$$
$$= \sum_{j=0}^{n}\sum_{k=0}^{\infty} \frac{\left(-1\right)^k}{\left(2k+1\right)!} c_{2k+1}\left(z^2\mu^2\right) \omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+2, \beta+1\right)\nu^{2k+1}$$





Regularization via Orthogonal Polynomials

Ill-posed (pseudo-)ITD/PDF matching relation:
(α,β) lose meaning when inf. # terms included

$$\mathfrak{M}\left(\nu,z^2\right)=\int_{-1}^{1}dx\,\mathcal{K}\left(x\nu,z^2\mu^2\right)\boxed{f_{q/h}\left(x,\mu^2\right)}+\sum_{k=1}^{\infty}\mathcal{B}_k\left(\nu\right)\left(z^2\right)^k$$

$$\sigma_n^{(\alpha,\beta)}\left(\nu,z^2\mu^2\right)=\int_0^1 dx\,\mathcal{K}_{\rm v}\left(x\nu,z^2\mu^2\right)\boxed{x^\alpha\left(1-x\right)^\beta\Omega_n^{(\alpha,\beta)}\left(x\right)}$$
$$=\sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}c_{2k}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)}B\left(\alpha+2k+j+1,\beta+1\right)\nu^{2k}$$
$$\eta_n^{(\alpha,\beta)}\left(\nu,z^2\mu^2\right)=\int_0^1 dx\,\mathcal{K}_{+}\left(x\nu,z^2\mu^2\right)\boxed{x^\alpha\left(1-x\right)^\beta\Omega_n^{(\alpha,\beta)}\left(x\right)}$$
$$=\sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}c_{2k+1}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)}B\left(\alpha+2k+j+2,\beta+1\right)\nu^{2k+1}$$

$$\mathfrak{Re}\,\mathfrak{M}_{\rm fit}\left(\nu,z^2\right)=\sum_{n=0}^{\infty}\sigma_n^{(\alpha,\beta)}\left(\nu,z^2\mu^2\right)C_{{\rm v},n}^{lt\,(\alpha,\beta)}+\frac{a}{|z|}\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{{\rm v},n}^{az\,(\alpha,\beta)}$$
$$+z^2\Lambda_{\rm QCD}^2\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{{\rm v},n}^{t4\,(\alpha,\beta)}+z^4\Lambda_{\rm QCD}^4\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{{\rm v},n}^{t6\,(\alpha,\beta)}$$

$$\mathfrak{Im}\,\mathfrak{M}_{\rm fit}\left(\nu,z^2\right)=\sum_{n=0}^{\infty}\eta_n^{(\alpha,\beta)}\left(\nu,z^2\mu^2\right)C_{+,n}^{lt\,(\alpha,\beta)}+\frac{a}{|z|}\sum_{n=0}^{\infty}\eta_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{+,n}^{az\,(\alpha,\beta)}$$
$$+z^2\Lambda_{\rm QCD}^2\sum_{n=0}^{\infty}\eta_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{+,n}^{t4\,(\alpha,\beta)}+z^4\Lambda_{\rm QCD}^4\sum_{n=0}^{\infty}\eta_{0,n}^{(\alpha,\beta)}\left(\nu\right)C_{+,n}^{t6\,(\alpha,\beta)}$$

$$\Downarrow$$

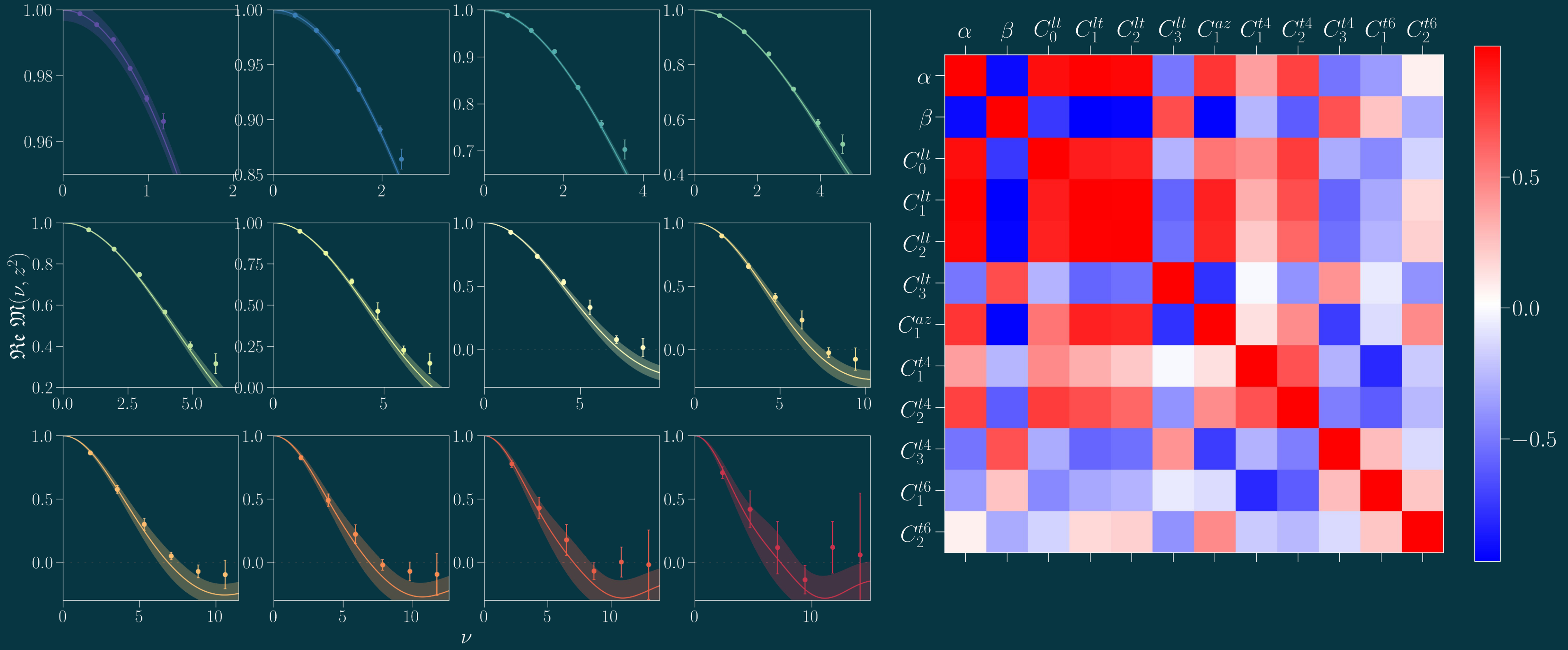
Jacobi polynomial basis are only non-linear terms
Separable non-linear optimization → variable projection

G. Golub and V. Pereyra, SIAM Journal on Numerical Analysis 10, 413 (1973)

Strategy of parametric fits with Jacobi polynomials

1. scan over truncation orders
    a. search for optimal expansion coefficients for each
2. establish polynomial hierarchy
    a. preference given to low-order polynomials
    b. restrict x-space contaminating distributions to be sub-leading to leading-twist PDF
    c. Bayesian priors (gaussian)
3. separability of non-linear optimization

Optimal Fit for Unpolarized Valence Quark PDF

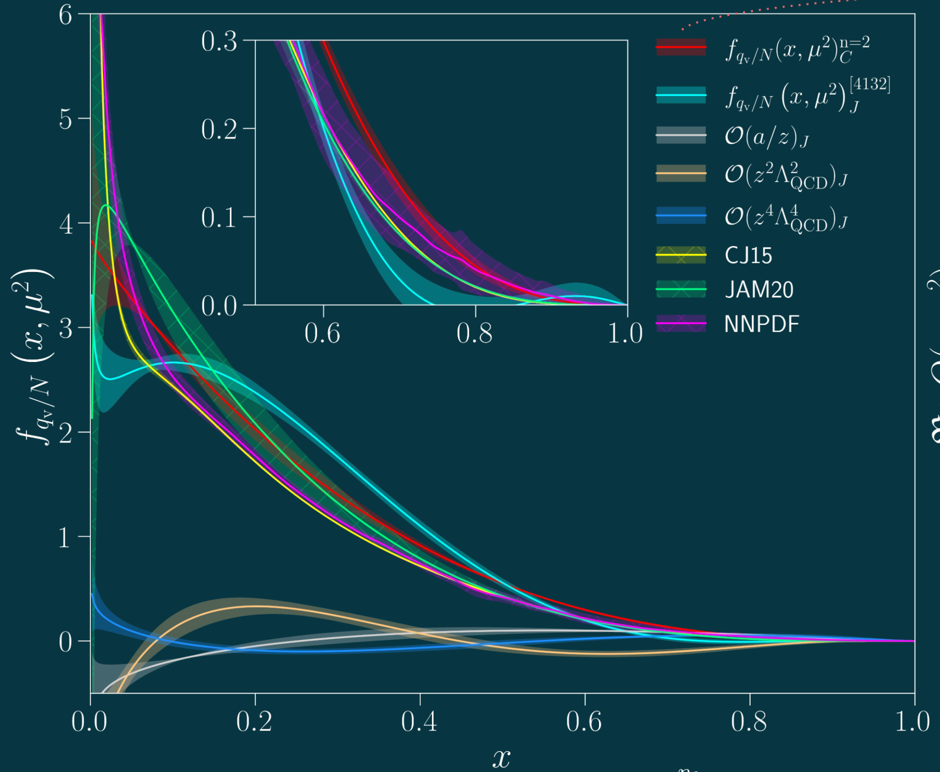


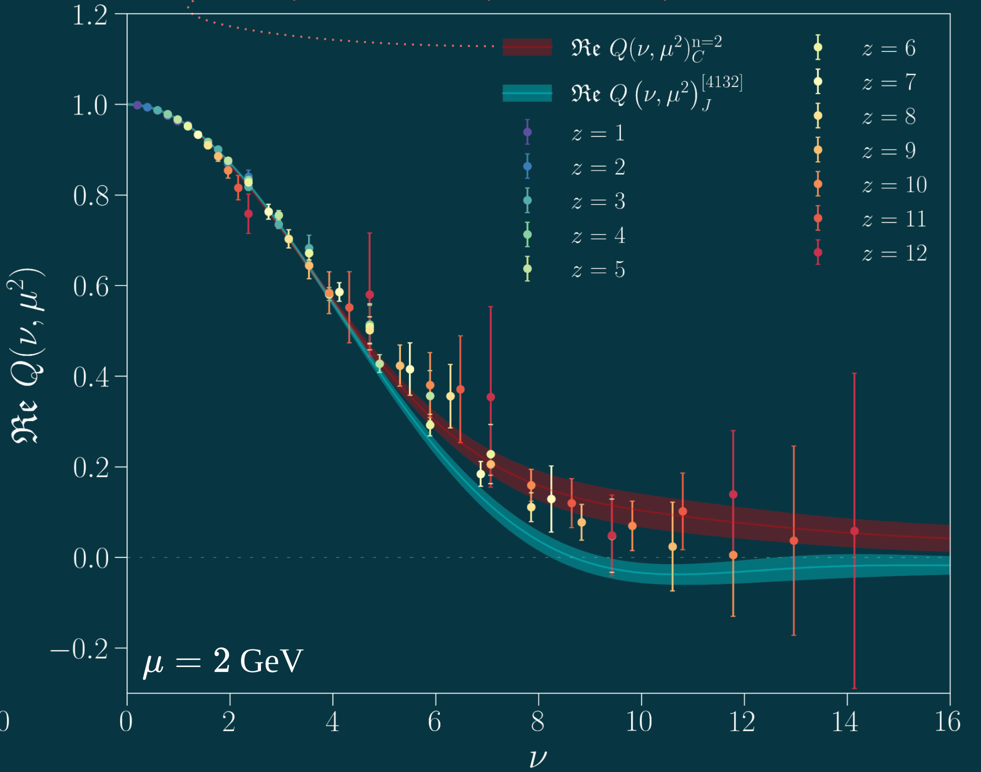
CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Unpolarized Valence Quark PDF and Leading-Twist ITD

CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

2-param Ansatz (uncorrelated)

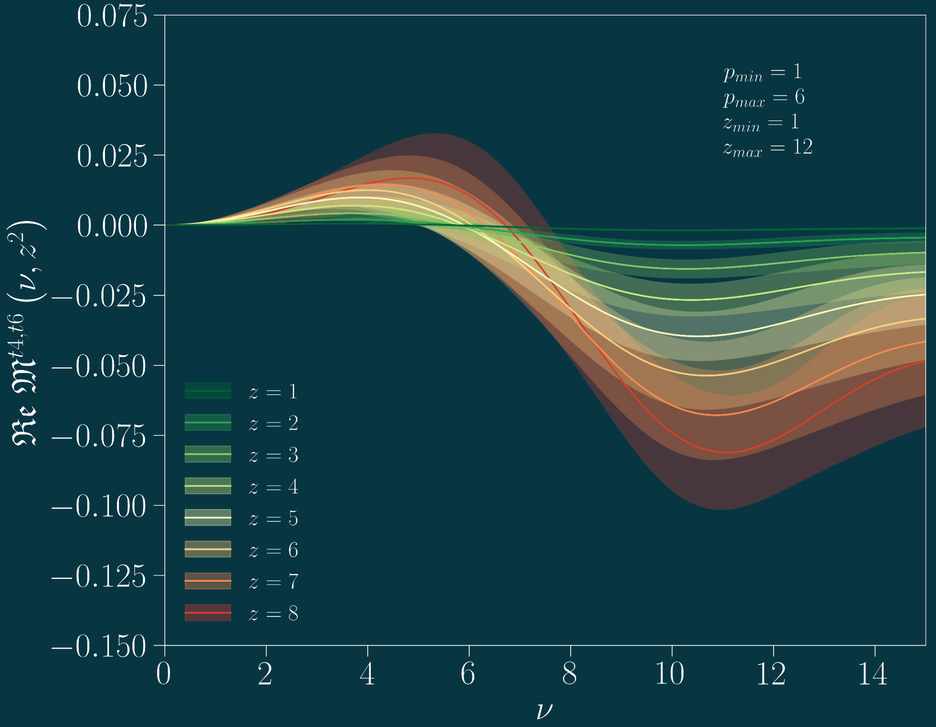


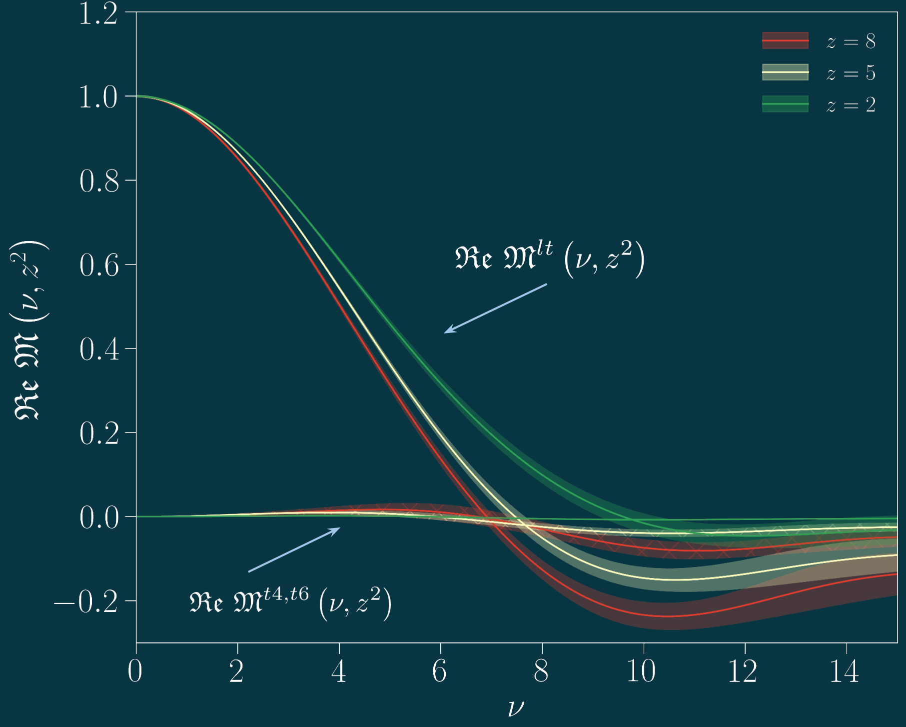


$$f_{q_v/N}\left(x,\mu^2\right)_J = x^\alpha\left(1-x\right)^\beta \sum_{n=0}^{n_{lt}} C_{v,n}^{(\alpha,\beta)}\Omega_n^{(\alpha,\beta)}(x)$$

$$\mathcal{O}\left(corr\right)_J = x^\alpha\left(1-x\right)^\beta \sum_{n=1}^{n_{corr}} C_{v,n}^{corr(\alpha,\beta)}\Omega_n^{(\alpha,\beta)}(x)$$

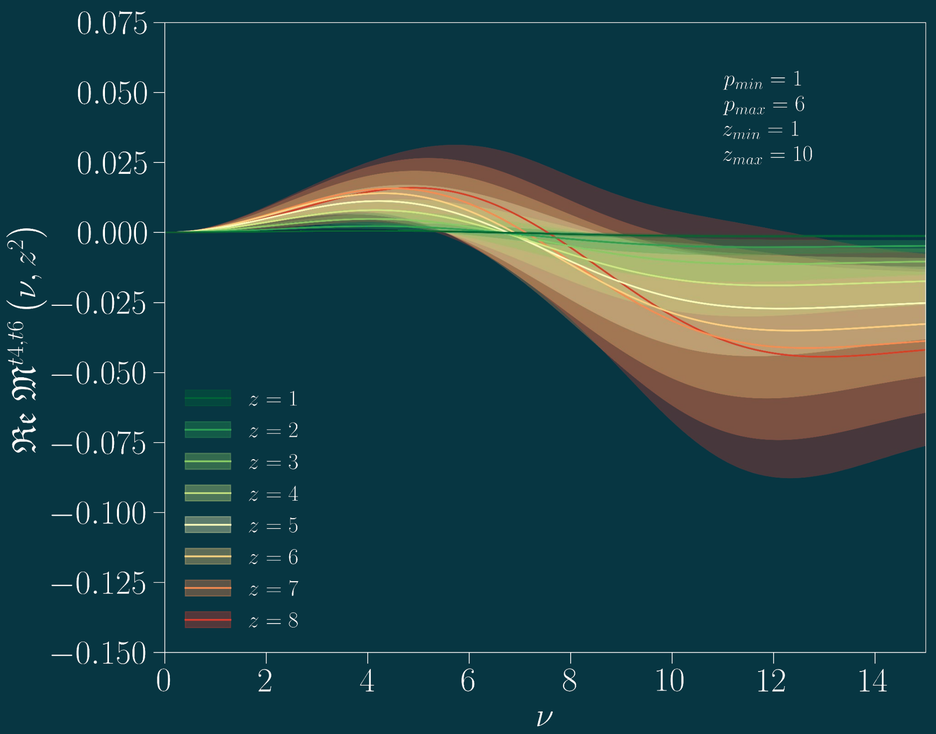
Parameterized Higher-Twist Contamination (Unpol.)

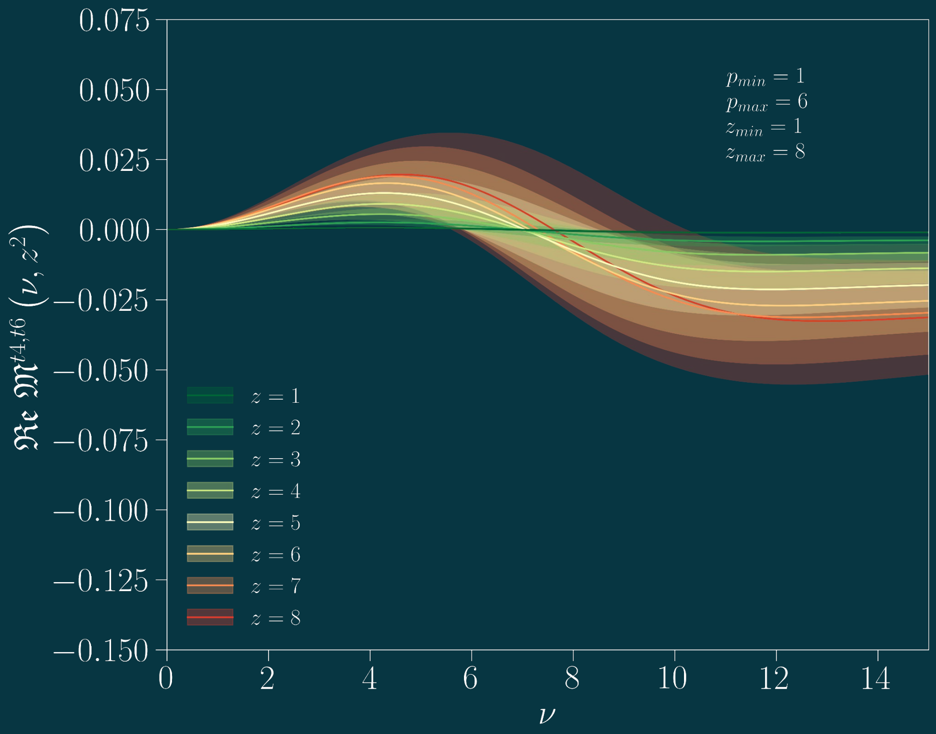




$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}(\nu, z^2) \equiv z^2\Lambda^2_{\text{QCD}} \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\text{v},n}^{t4\ (\alpha,\beta)} + z^4\Lambda^4_{\text{QCD}} \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\text{v},n}^{t6\ (\alpha,\beta)}$$

Wilson Line Cuts & Higher-Twist Variability (Unpol.)





$$\mathfrak{Re}\ \mathfrak{M}^{t4,t6}\left(\nu, z^2\right) \equiv z^2 \Lambda_{\text{QCD}}^2 \sum_{n=1}^{n_{t4}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\text{v},n}^{t4\,(\alpha,\beta)} + z^4 \Lambda_{\text{QCD}}^4 \sum_{n=1}^{n_{t6}} \sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\text{v},n}^{t6\,(\alpha,\beta)}$$

Short-Distance Tension

Dramatic effect of a discretization correction

$\{n_{lt}, n_{az}, n_{t4}, n_{t6}\}_{\mathrm{v}/+}$	α	β	$C^{lt}_{\mathrm{v},0}$	$C^{lt}_{\mathrm{v},1}$	$C^{lt}_{\mathrm{v},2}$	$C^{lt}_{\mathrm{v},3}$
$\{4,1,3,2\}_{\mathrm{v}}$	$-0.209(147)$	$1.330(415)$	$1.606(257)$	$0.427(752)$	$-0.880(409)$	$-0.675(122)$
$\{4,0,3,2\}_{\mathrm{v}}$	$-0.376(37)$	$2.032(496)$	$1.340(165)$	$0.335(261)$	$-0.125(100)$	$-0.651(140)$

$C^{az}_{\mathrm{v},1}$	$C^{t4}_{\mathrm{v},1}$	$C^{t4}_{\mathrm{v},2}$	$C^{t4}_{\mathrm{v},3}$	$C^{t6}_{\mathrm{v},1}$	$C^{t6}_{\mathrm{v},2}$	χ^2_r
$-0.279(48)$	$0.052(53)$	$-0.371(106)$	$-0.407(122)$	$-0.045(37)$	$0.228(52)$	$2.620(345)$
–	$-0.090(52)$	$-0.112(77)$	$0.274(99)$	$0.011(39)$	$0.397(84)$	$45.68(1.72)$

Visualize scale dependence in reduced pseudo-ITD via mock pseudo-PDF fit

$$\frac{\Gamma(5+\alpha)}{\Gamma(1+\alpha)\Gamma(4)} x^{\alpha}(1-x)^3$$

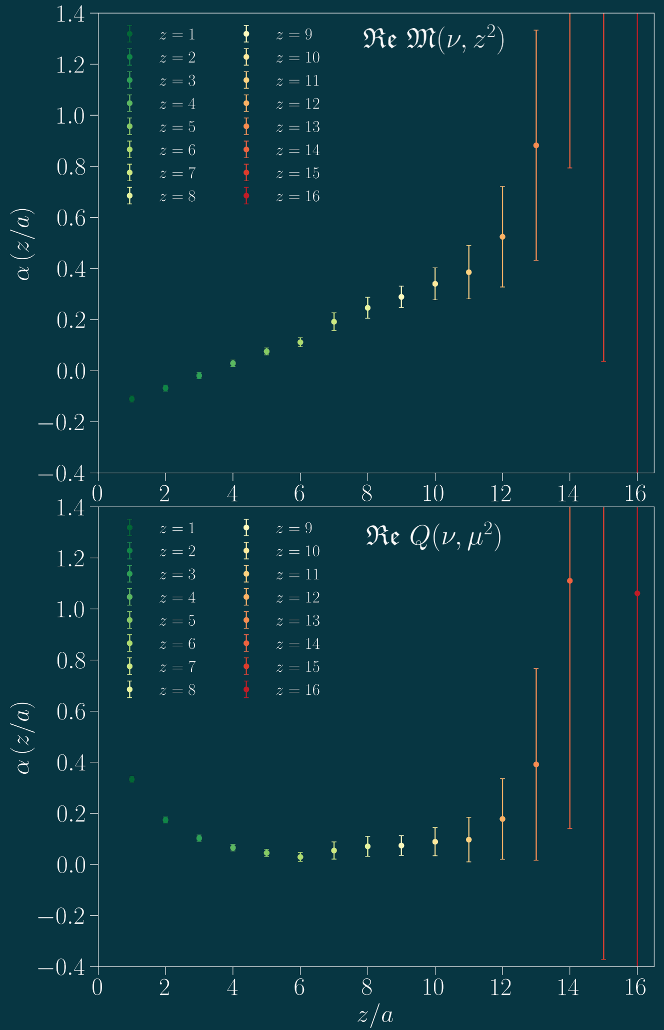
$$\mathfrak{Re}\ \mathfrak{M}_{\mathrm{fit}}\left(\nu, z^2\right) = \int_0^1 dx \cos(x\nu)\, \mathfrak{Re}\ \mathcal{P}\left(x, z^2; \alpha, 3\right)$$

Evolution/matching with pseudo-PDF fit

$$\mathfrak{Re}\ \mathcal{Q}\left(\nu, \mu^2\right) = \mathfrak{Re}\ \mathfrak{M}\left(\nu, z^2\right) + \frac{\alpha_s C_F}{2\pi} \int_0^1 du\, \mathfrak{P}\left(u\nu, z^2; \alpha, \beta=3\right) \left[\ln\left(\frac{z^2\mu^2 e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right]$$

$$\mathfrak{P}\left(u\nu, z^2; \alpha, \beta\right) = {}_2F_3\left(\frac{1+\alpha}{2}, \frac{2+\alpha}{2}; \frac{1}{2}, \frac{5+\alpha}{2}, \frac{6+\alpha}{2}; -\frac{\nu^2}{4}\right)$$

- ➢ redo two-parameter fits to matched ITD



Short-Distance Tension

Dramatic effect of a discretization correction

$\{n_{lt}, n_{az}, n_{t4}, n_{t6}\}_{\mathrm{v}/+}$	α	β	$C^{lt}_{\mathrm{v},0}$	$C^{lt}_{\mathrm{v},1}$	$C^{lt}_{\mathrm{v},2}$	$C^{lt}_{\mathrm{v},3}$
$\{4,1,3,2\}_{\mathrm{v}}$	−0.209(147)	1.330(415)	1.606(257)	0.427(752)	−0.880(409)	−0.675(122)
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$C^{az}_{\mathrm{v},1}$	$C^{t4}_{\mathrm{v},1}$	$C^{t4}_{\mathrm{v},2}$	$C^{t4}_{\mathrm{v},3}$	$C^{t6}_{\mathrm{v},1}$	$C^{t6}_{\mathrm{v},2}$	χ^2_r
−0.279(48)	0.052(53)	−0.371(106)	−0.407(122)	−0.045(37)	0.228(52)	2.620(345)
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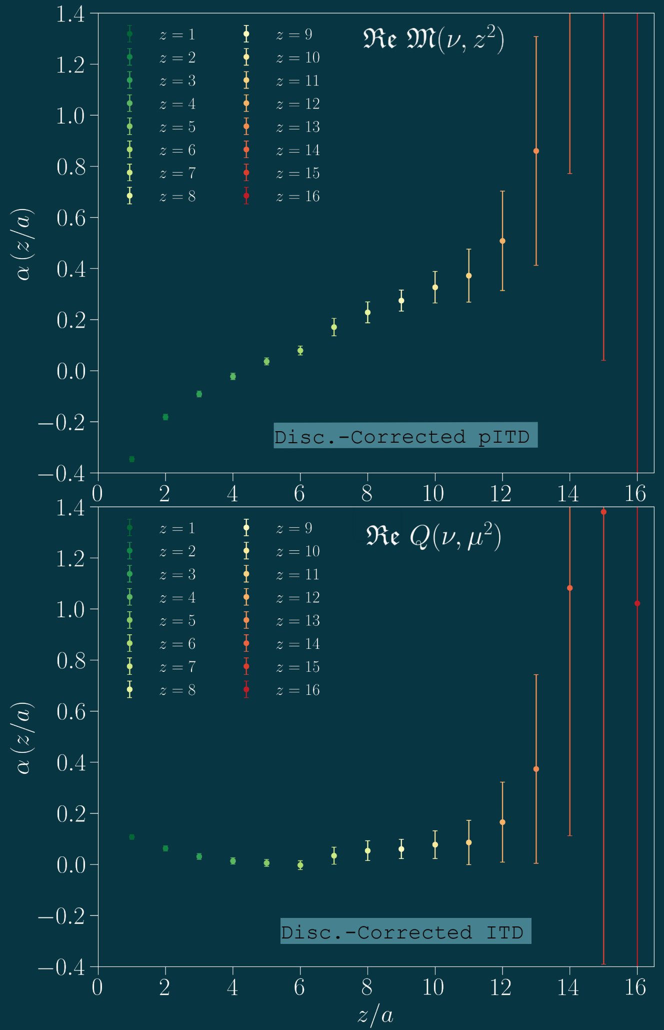
Evolution/matching with pseudo-PDF fit

$$\mathfrak{Re}\ \mathcal{Q}(\nu,\mu^2) = \mathfrak{Re}\ \mathfrak{M}(\nu, z^2) + \frac{\alpha_s C_F}{2\pi}\int_0^1 du\, \mathfrak{P}(u\nu, z^2; \alpha, \beta=3)\left[\ln\left(\frac{z^2\mu^2 e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right]$$

$$\mathfrak{P}(u\nu, z^2; \alpha, \beta) = {}_2F_3\left(\frac{1+\alpha}{2}, \frac{2+\alpha}{2}; \frac{1}{2}, \frac{5+\alpha}{2}, \frac{6+\alpha}{2}; -\frac{\nu^2}{4}\right)$$

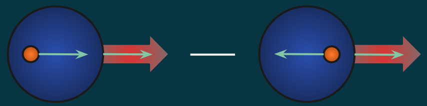
➢ redo two-parameter fits to matched ITD



Spin-Dependent PDFs from Pseudo-Distributions

Quark Helicity Distribution

Helicity asymmetry of partons within hadronic state of definite helicity

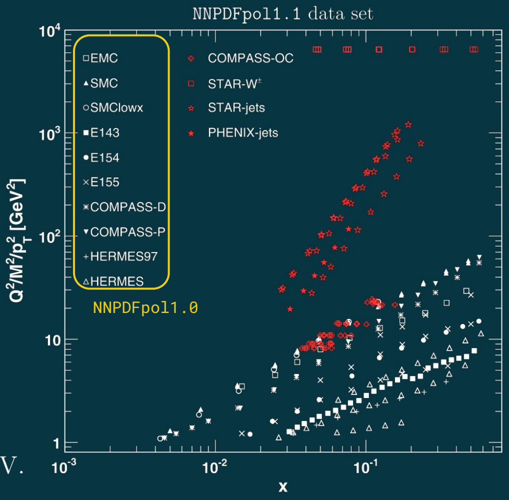


Known primarily from polarized inclusive DIS (ex. JLAB - CLAS)

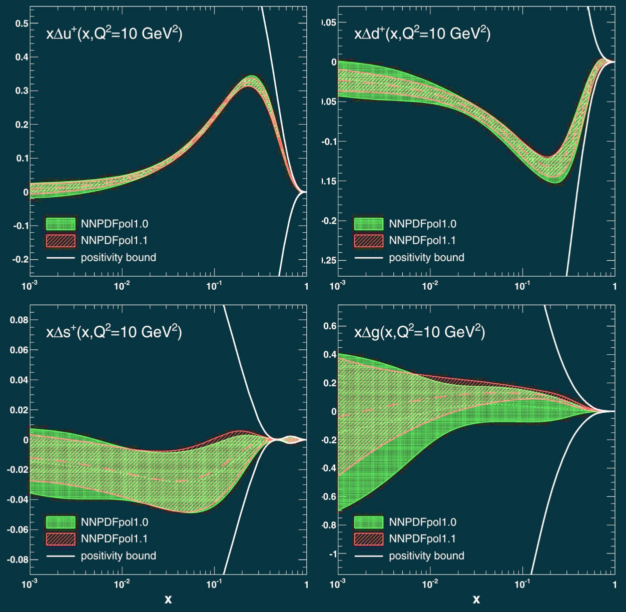
$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1\left(x, Q^2\right) + \frac{\hat{P}^\mu \hat{P}^\nu}{P \cdot q} F_2\left(x, Q^2\right)$$

$$+ i\epsilon^{\mu\nu\alpha\beta} \frac{q_\alpha S_\beta}{P \cdot q} g_1\left(x, Q^2\right) + i\epsilon^{\mu\nu\alpha\beta} \frac{q_\alpha \left(S_\beta - P_\beta \frac{S \cdot q}{P \cdot q}\right)}{P \cdot q} g_2\left(x, Q^2\right) + \text{P.V.}$$

$$g_1\left(x, Q^2\right) = \frac{1}{2} \sum_q e_q^2 \left[\Delta q\left(x, Q^2\right) + \Delta \bar{q}\left(x, Q^2\right)\right]$$



E. Nocera et al., Nuc.Phys.B 887 (2014) 276-308



Afford insight into proton spin puzzle [EMC] Phys.Lett.B 206 (1988) 364

$$S_q\left(Q^2\right) = \frac{1}{2} \int_0^1 dx \sum_{u,d,s} \left[\Delta q\left(x, Q^2\right) + \Delta \bar{q}\left(x, Q^2\right)\right]$$

Polarized semi-inclusive DIS (SIDIS) processes offer additional measures of helicity PDFs (e.g. COMPASS/HERMES)

Fragmentation Functions → can be constrained simultaneous analysis of single inclusive annihilation (SIA) reactions

$$g_1^h\left(x, Q^2, z\right) = \frac{1}{2} \sum_q e_q^2 \left[\Delta q\left(x, Q^2\right) D_q^h\left(z, Q^2\right) + \Delta \bar{q}\left(x, Q^2\right) D_{\bar{q}}^h\left(z, Q^2\right)\right]$$

Towards the Helicity PDF from Pseudo-Distributions

$$\nu \equiv p \cdot z$$

A matrix element of a distinct character

$$M^{\alpha 5}(p,z) = \langle h(p)|\,\overline{\psi}(z)\,\gamma^\alpha\gamma^5\Phi_{\hat{z}}^{(f)}(\{z,0\})\,\psi(0)\,|h(p)\rangle = -2m_N S^\alpha \mathcal{M}(\nu,z^2) - 2im_N p^\alpha (z\cdot S)\,\mathcal{N}(\nu,z^2) + 2m_N^3 z^\alpha (z\cdot S)\,\mathcal{R}(\nu,z^2)$$

Light-cone Helicity PDF — **Helicity Pseudo-PDF**

Leading-twist helicity PDF defined in terms of $k^-, \mathbf{k}_\perp$ integrated parton correlator

$$p^\alpha = \left(p^+, \frac{m_h^2}{2p^+}, \mathbf{0}_\perp\right)$$

$$z^\alpha = \left(0, z^-, \mathbf{0}_\perp\right) \qquad \alpha = +$$

[Helicity] Ioffe-time Distribution (ITD)

$$\left\{\mathcal{M}(p^+z^-,0) + ip^+z^-\mathcal{N}(p^+z^-,0)\right\}_{\mu^2} \equiv \Delta Q(\nu,\mu^2) = \int_{-1}^{1} dx\; e^{i\nu x} g_{q/h}(x,\mu^2)$$

$$\mathcal{Y}(p^+z^-,0)_{\mu^2}$$

Generalization of light-cone helicity PDF onto space-like intervals; Lorentz covariant parton momentum fraction

$$p^\alpha = (\mathbf{0}_\perp, p_z, E) \qquad S^\alpha = (\mathbf{0}_\perp, S_z, S_4)$$

$$z^\alpha = (\mathbf{0}_\perp, z_3, 0) \qquad \alpha = 3$$

[Helicity] Ioffe-time Pseudo-distribution (pseudo-ITD)

$$\mathcal{Y}(p_z z_3, z_3^2) = \int_{-1}^{1} dx\; e^{i\nu x}\Delta\mathcal{P}(x, z_3^2)$$

Helicity Pseudo-ITD: Lattice Implementation

A slightly altered version of the reduced distribution:

Additional source of polynomial corrections - isolation of it beyond that which in factorization exceeds accuracy of data

$$\mathfrak{M}\left(\nu,z^2\right)=\frac{M_3\left(p,z\right)/M_3\left(p,0\right)}{M_3\left(0,z\right)/M_3\left(0,0\right)}=\frac{\mathcal{Y}\left(\nu,z^2\right)\mathcal{Y}\left(0,0\right)|_{p=z=0}+m_N^2z^2\mathcal{R}\left(\nu,z^2\right)\mathcal{Y}\left(0,0\right)|_{p=z=0}}{\mathcal{Y}\left(\nu,0\right)|_{z=0}\,\mathcal{Y}\left(0,z^2\right)|_{p=0}+m_N^2z^2\mathcal{R}\left(0,z^2\right)|_{p=0}\,\mathcal{Y}\left(\nu,0\right)|_{z=0}}$$

[Helicity] Short-distance factorization

$$B\left(u\right)=\left(\frac{1+u^2}{1-u}\right)_+ \qquad L\left(u\right)=\left[4\frac{\ln\left(1-u\right)}{1-u}-4\left(1-u\right)\right]_+$$

$$\mathfrak{M}\left(\nu,z^2\right)=\left\{\delta\left(1-u\right)-\frac{\alpha_sC_F}{2\pi}\int_0^1du\left[\ln\left(\frac{e^{2\gamma_E+1}z^2\mu^2}{4}\right)B\left(u\right)+L\left(u\right)\right]\right\}\Delta Q\left(u\nu,\mu^2\right)+\mathcal{O}\left(z^2\Lambda_{\rm QCD}^2\right)$$

High-momenta (remains) essential

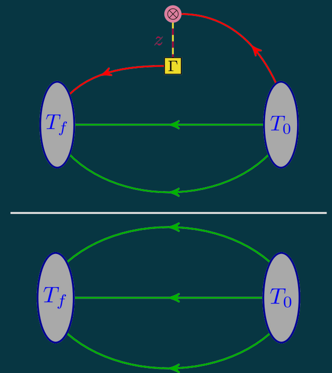
G. S. Bali et al. Phys. Rev. D93, 094515 (2016)
CE, R. Edwards, K. Orginos, D. Richards, PRD 103 (2021) 3, 034502

$$\xi_\pm^{(k)}\left(\vec{z},t\right)\equiv e^{i\vec{\zeta}_\pm\cdot\vec{z}}\xi^{(k)}\left(\vec{z},t\right) \qquad \vec{\zeta}_\pm = \pm2\cdot\frac{2\pi}{L}\hat{z}$$

Summation method - further excited-state suppression

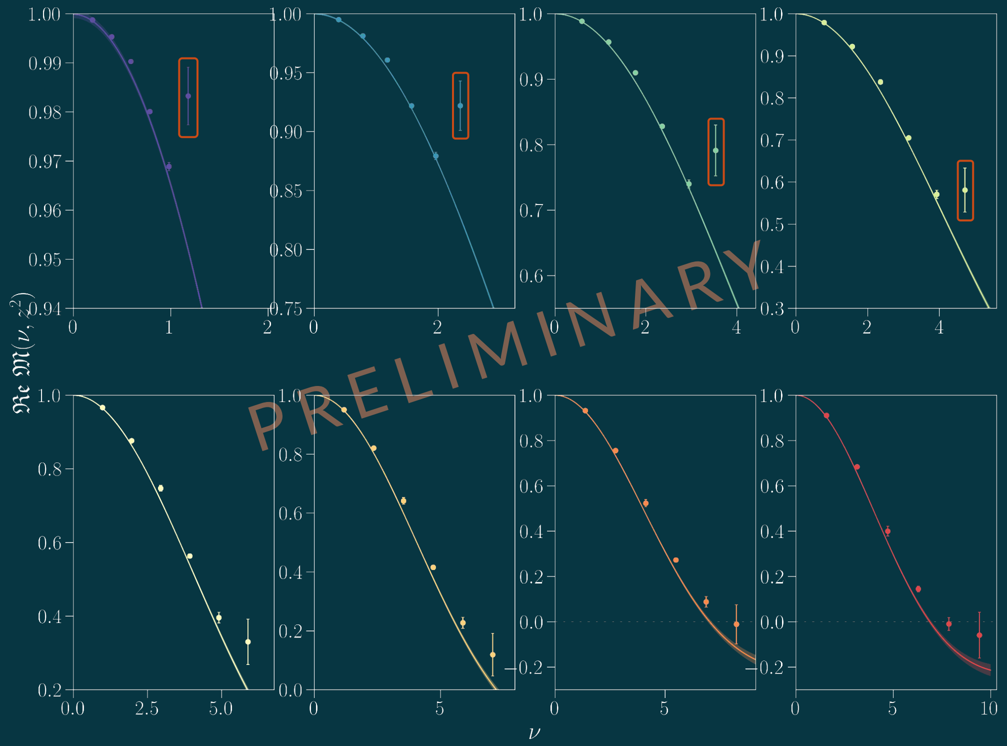
L. Maiani et al., Nucl. Phys. B293 (1987)
C. Bouchard et al., Phys. Rev. D 96, no. 1, 014504 (2017)

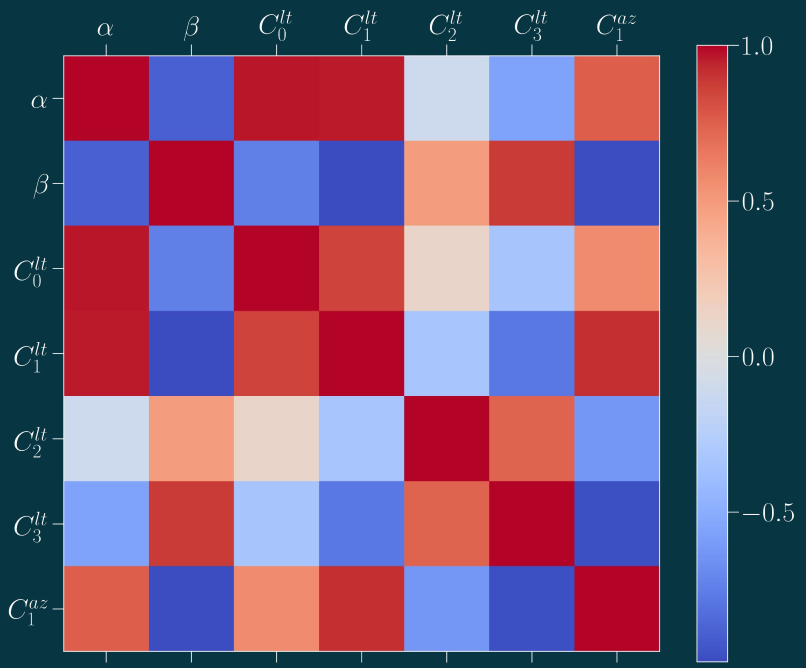
$$R\left(p_z,z_3;T\right)=\sum_{\tau/a=1}^{T-1}\frac{C_3\left(p_z,T,\tau;z_3\right)}{C_2\left(p_z,T\right)}$$



$$R_{\rm fit}\left(p_z,z_3;T\right)=\mathcal{A}+M_4\left(p_z,z_3\right)T+\mathcal{O}\left(e^{-\Delta ET}\right)$$

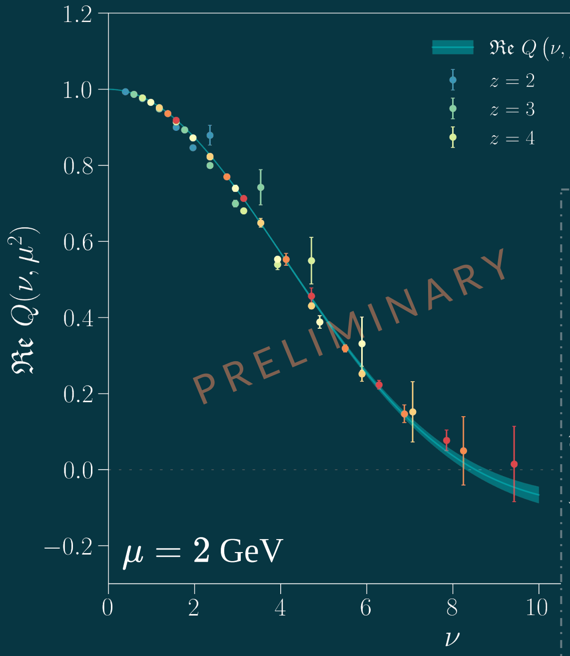
A Preliminary Fit for Helicity Valence Quark PDF

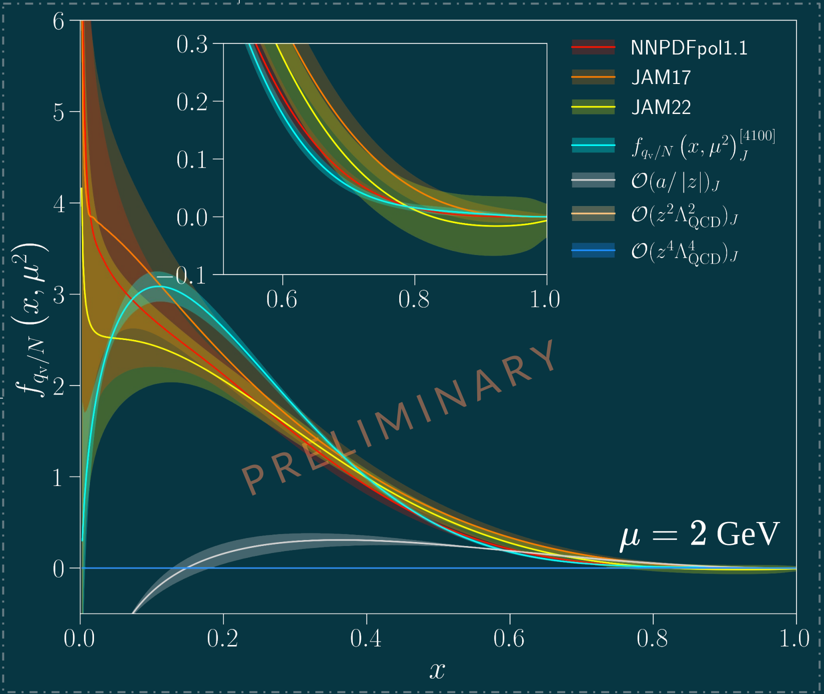




➢ Highest momentum data require further investigation → Cut on $p_{\max}^{\text{latt}} = 5$

Helicity Valence Quark PDF and Leading-Twist ITD





Matched helicity ITD well-described by FT of leading-twist helicity PDF (highest momentum data in tension)

[JAM17]: 1st global analysis of spin-dep. PDFs & FFs (NLO)

- ➢ polarized inclusive/semi-inclusive DIS & single-inclusive e+e- annihilation (PDFs/FFs fit simultaneously)

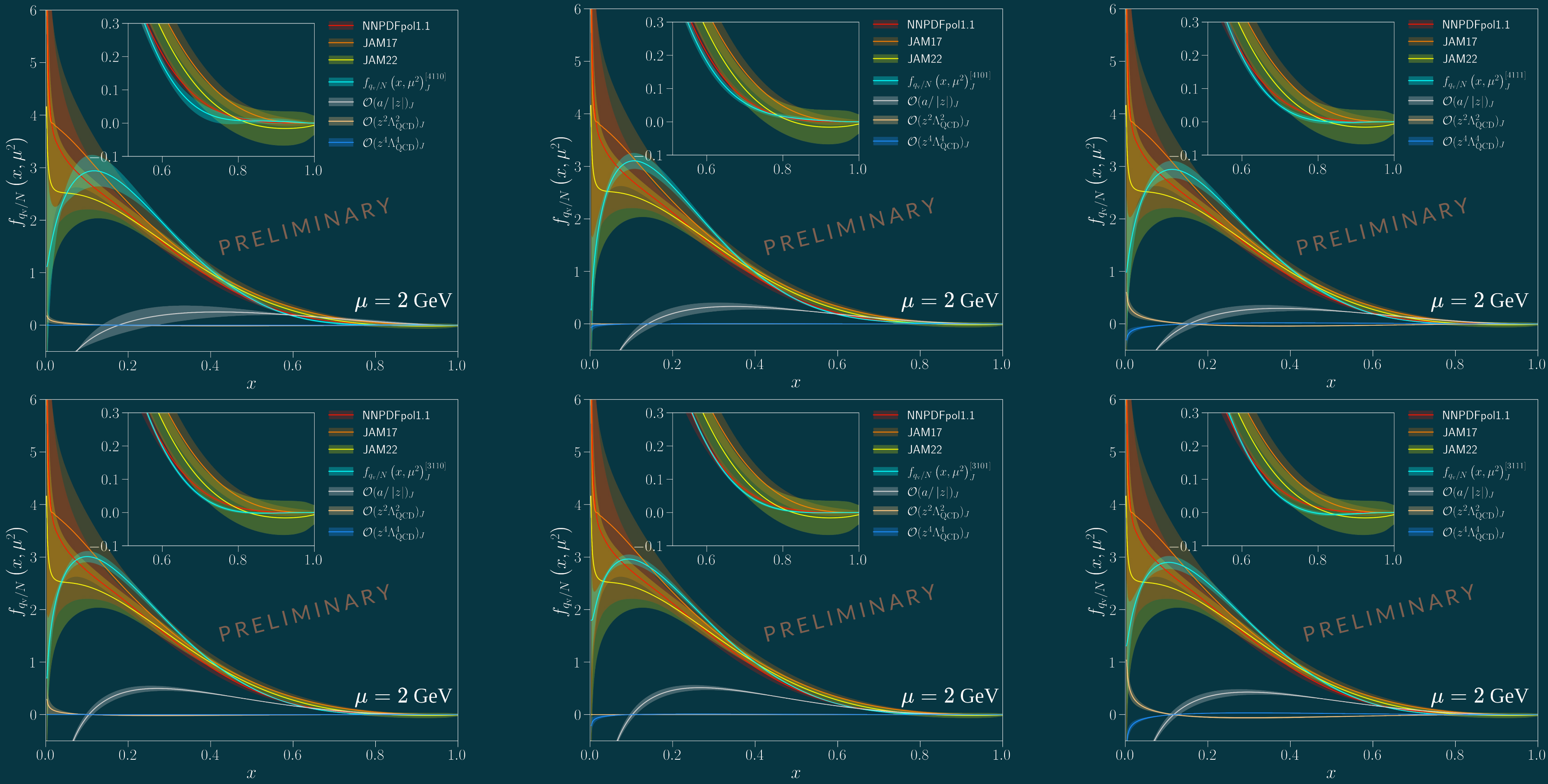
JAM Collab. N. Sato et al., Phys.Rev.D 93 (2016) 7, 074005

[JAM22]: global analysis of spin-dep. PDFs & FFs (NLO)

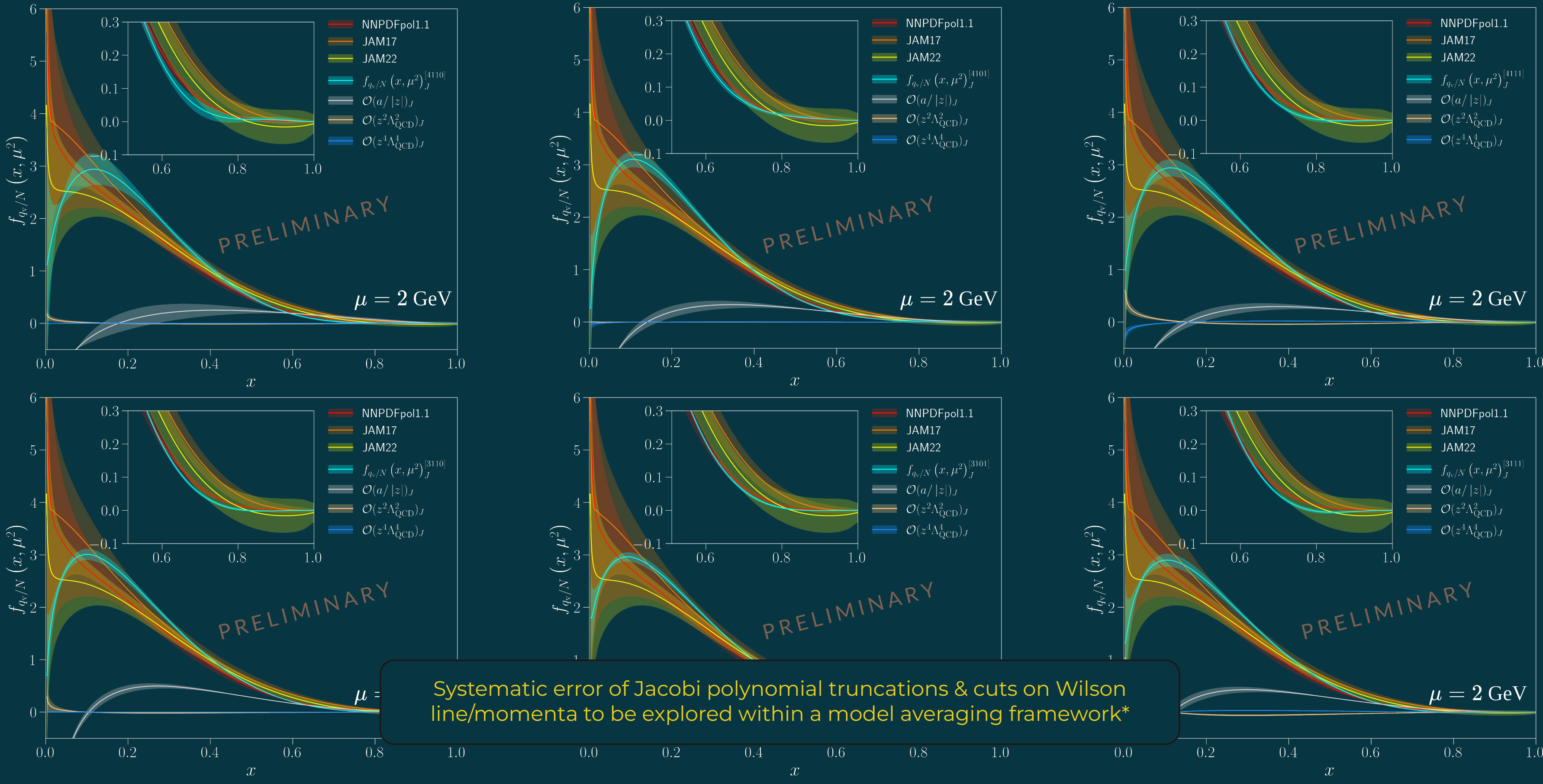
- ➢ simultaneous global analysis of unpol./pol. PDFs/FFs
- ➢ pp jet production (STAR/PHENIX) (esp. SSAs)
- ➢ PDF positivity assumption relaxed

JAM Collab. C. Cocuzza et al., arXiv:2202.03372 [hep-ph]

Other Preliminary Helicity Valence Quark Fits



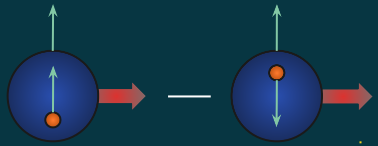
Other Preliminary Helicity Valence Quark Fits



Systematic error of Jacobi polynomial truncations & cuts on Wilson line/momenta to be explored within a model averaging framework*

Quark Transversity Distribution

Distribution of transversely polarized quarks
w/in hadron polarized transverse to (inf.) momentum



- ➢ only chiral-odd twist-2 collinear PDF (decouples from inclusive DIS)
  - ○ additional process needed to accommodate parton helicity flip (i.e. another chiral-odd fn. to access from exp.)
  - ○ challenging to accomplish experimentally - limited info. on dist.
- ➢ numerous candidate 2-hadron processes

$p^{\uparrow}p^{\uparrow} \rightarrow \ell\ell' X$ (Ralston-Soper '79)

$ep^{\uparrow} \rightarrow e'\pi X$ (Collins '93)

$pp^{\uparrow} \rightarrow \Lambda^{\uparrow} X$ (De Florian et al., '98)

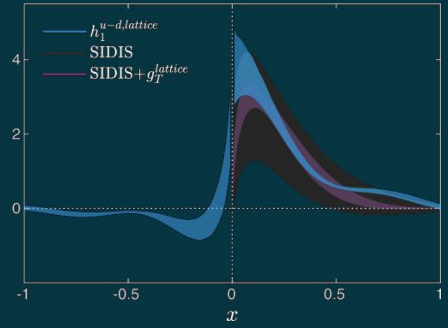
$ep^{\uparrow} \rightarrow e'\left(\pi^{+}\pi^{-}\right) X$

Dearth of data sensitive to transversity PDF
(and non-conservation of tensor change)

$$g_T \equiv \int_0^1 dx [h_1^q(x) - h_1^{\bar{q}}(x)]$$

- ➢ ideal for study from LQCD

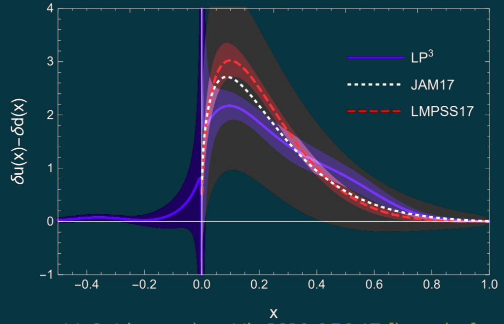
TMCs accounted for



C. Alexandrou et al., Phys.Rev.D 98 (2018) 9, 091503

C. Alexandrou et al., Phys.Rev.D 99 (2019) 11, 114504

Thorough investigation of systematic effects (eg. excited-states, non-pert. renormalization, FT truncation & matching schemes)



Y.-S. Liu et al., arXiv:1810.05043 [hep-lat]

Transversity PDF at phys. pion mass using quasi-distributions & LaMET

Towards the Transversity PDF from Pseudo-Distributions

A matrix element of a distinct character

$$\nu \equiv p \cdot z$$

$$M^{\alpha\beta}(p,z) = \langle h(p)| \overline{\psi}(z) i\sigma^{\alpha\beta}\gamma^5 \Phi_{\hat{z}}^{(f)}(\{z,0\})\psi(0)|h(p)\rangle =$$
$$2(p^\alpha S_\perp^\beta - p^\beta S_\perp^\alpha)\mathcal{M}(\nu,z^2) + 2im_N^2(z^\alpha S_\perp^\beta - z^\beta S_\perp^\alpha)\mathcal{N}(\nu,z^2) + 2m_N^2(z^\alpha p^\beta - z^\beta p^\alpha)(z\cdot S_\perp)\mathcal{R}(\nu,z^2)$$

Light-cone Transversity PDF — **Transversity Pseudo-PDF**

Leading-twist transversity PDF defined in terms of $k^-, \mathbf{k}_\perp$ integrated parton correlator

$$p^\alpha = \left(p^+, \frac{m_h^2}{2p^+}, \mathbf{0}_\perp\right)$$

$$z^\alpha = (0, z^-, \mathbf{0}_\perp) \qquad \alpha = + \quad \beta = i$$

[Transversity] Ioffe-time Distribution (ITD)

$$\mathcal{M}(p^+z^-,0)_{\mu^2} \equiv \delta Q(\nu,\mu^2) = \int_{-1}^{1} dx\; e^{i\nu x} h_{q/h}(x,\mu^2)$$

Generalization of light-cone transversity PDF onto space-like intervals; Lorentz covariant parton momentum fraction

$$p^\beta = (\mathbf{0}_\perp, p_z, E) \qquad S^\beta = (S_\perp, 0, S_4)$$

$$\alpha = 4 \qquad \beta = i \qquad z^\beta = (\mathbf{0}_\perp, z_3, 0)$$

[Transversity] Ioffe-time Pseudo-distribution (pseudo-ITD)

$$\mathcal{M}(p_z z_3, z_3^2) = \int_{-1}^{1} dx\; e^{i\nu x}\delta\mathcal{P}(x, z_3^2)$$

Transversity Pseudo-ITD: Lattice Implementation

Standard reduced distribution, but rotational symmetry allows for further matelem sampling...

$$\mathfrak{M}\left(\nu, z^2\right) = \frac{M_{4i}\left(p,z\right)/M_{4i}\left(p,0\right)}{M_{4i}\left(0,z\right)/M_{4i}\left(0,0\right)}$$

[Transversity] Short-distance factorization

$$B\left(u\right) = \left(\frac{2u}{1-u}\right)_+ \qquad L\left(u\right) = 4\left[\frac{\ln\left(1-u\right)}{1-u}\right]_+$$

$$\mathfrak{M}\left(\nu, z^2\right) = \left\{\delta\left(1-u\right) - \frac{\alpha_s C_F}{2\pi}\int_0^1 du\left[\ln\left(\frac{e^{2\gamma_E+1}z^2\mu^2}{4}\right)B\left(u\right) + L\left(u\right)\right]\right\}\delta Q\left(u\nu,\mu^2\right) + \mathcal{O}\left(z^2\Lambda^2_{\mathrm{QCD}}\right)$$

CE, J. Karpie, N. Karthik et al., Phys.Rev.D 105 (2022) 034507 & V. Braun, Y. Ji, A. Vladimirov, JHEP 10 087 (2021)

High-momenta (still) essential

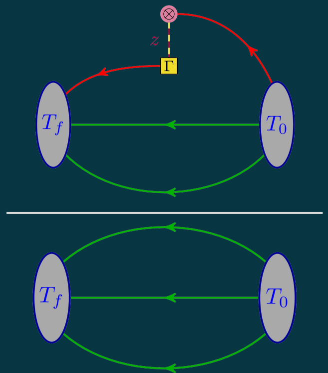
G. S. Bali et al. Phys. Rev. D93, 094515 (2016)
CE, R. Edwards, K. Orginos, D. Richards, PRD 103 (2021) 3, 034502

$$\xi^{(k)}_{\pm}\left(\vec{z},t\right) \equiv e^{i\vec{\zeta}_{\pm}\cdot\vec{z}}\xi^{(k)}\left(\vec{z},t\right) \qquad \vec{\zeta}_{\pm} = \pm 2\cdot\frac{2\pi}{L}\hat{z}$$

2-state fits to two- and three-point correlation functions favored over summation method

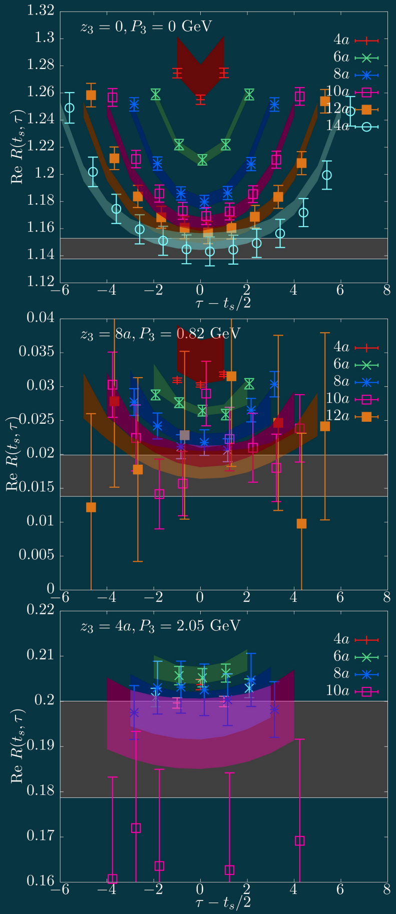
- ➢ ratio still exposes matrix element

$$R\left(p_z, z_3; T\right) = \sum_{\tau/a=1}^{T-1}\frac{C_3\left(p_z,T,\tau;z_3\right)}{C_2\left(p_z,T\right)}$$



- ➢ n.b. dedicated calculation of renormalized tensor charge for future

Bare Transversity Matrix Elements

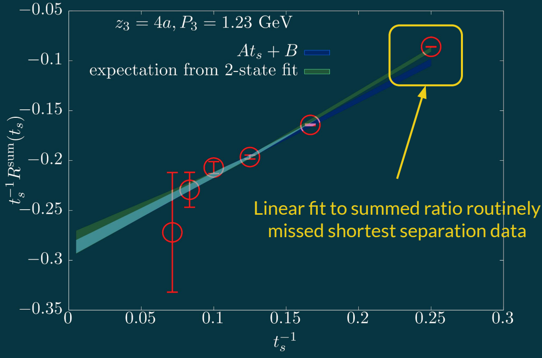


Matrix elements extracted from ratio of 2-state fits

$$C_2(p_z, T) = \langle \mathcal{N}(-p_z, T_f)\overline{\mathcal{N}}(p_z, T_0)\rangle = \sum_n |\mathcal{A}_n|^2 e^{-E_n T}$$

$$C_3(p_z, T, \tau; z_3) = V_3 \langle \mathcal{N}(-p_z, T_f)\mathring{\mathcal{O}}^{[\gamma_4]}_{\mathrm{WL}}(z_3, \tau)\overline{\mathcal{N}}(p_z, T_0)\rangle$$

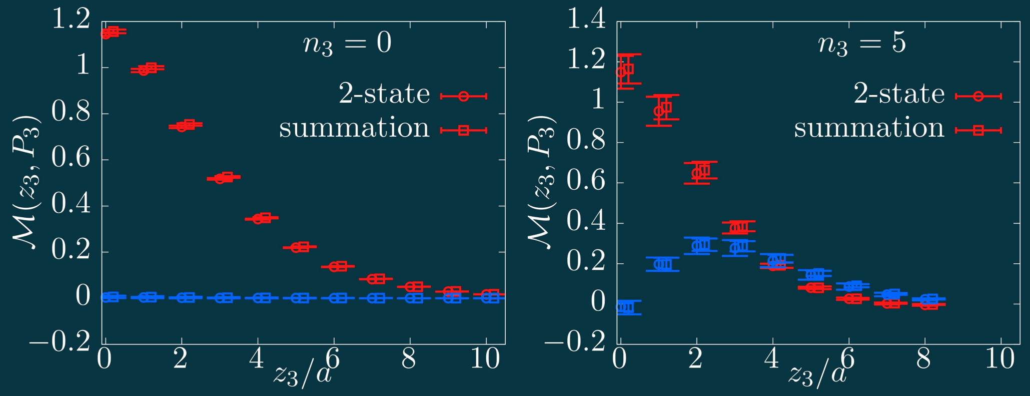
$$= V_3 \sum_{n,n'} \langle \mathcal{N}|n'\rangle\langle n|\overline{\mathcal{N}}\rangle\langle n'|\mathring{\mathcal{O}}^{[\gamma_4]}_{\mathrm{WL}}(z_3, \tau)|n\rangle e^{-E_{n'}(T-\tau)}e^{-E_n T}$$



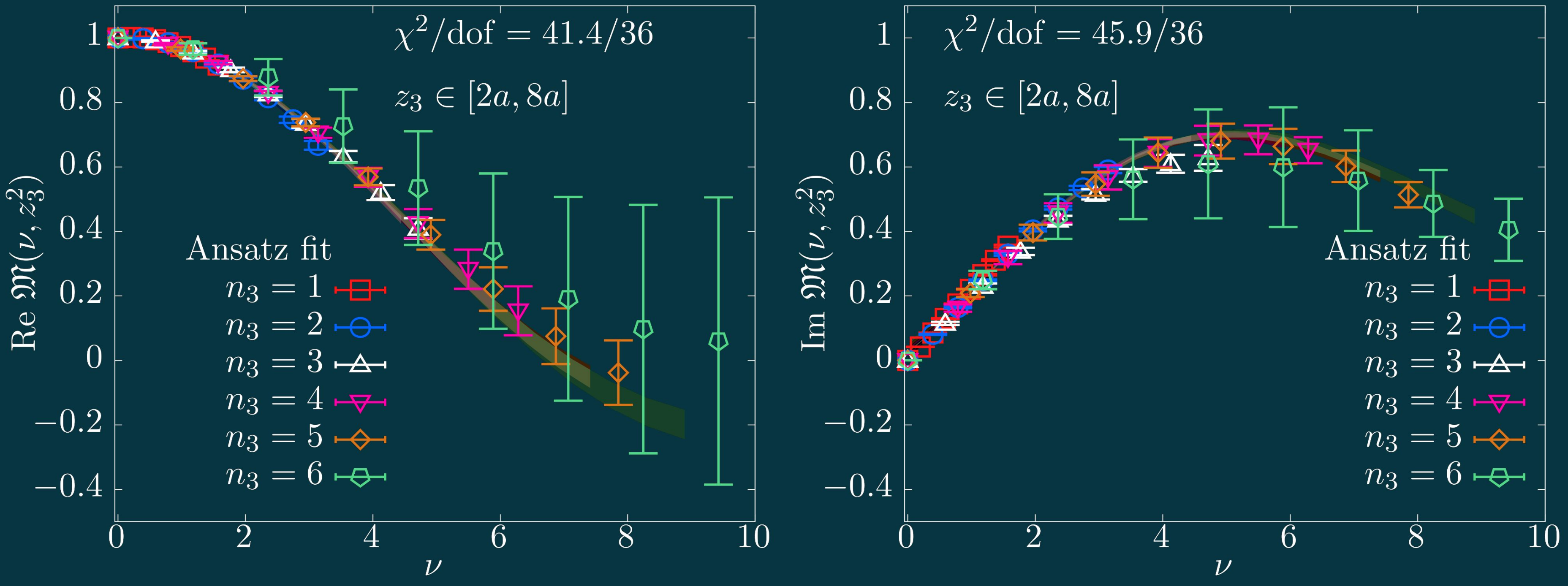
Linear fit to summed ratio routinely missed shortest separation data

$T/a \in [6, 14]$ $\quad \tau \in [2a, (T-2)a]$ ← Used in analysis

Matrix elements consistently determined across momenta



Transversity Reduced Pseudo-ITD



Pheno.-type parameterization

- ➢ convolution as Taylor series in Ioffe-time
- ➢ plus leading discretization/higher-twist

$$g_T^{-1} h_{\pm}(x) = N_{\pm} x^{\alpha_{\pm}} (1-x)^{\beta_{\pm}} \left(1 + \gamma_{\pm}\sqrt{x} + \delta_{\pm} x\right)$$

Estimating Model Dependence

Jacobi basis (α,β) selected from 4-param PDF ansatz

- expand ansatz in basis per jackknife bin - mean/error estimates of Jacobi expansion coeffs.
  - form Bayesian priors

Analogous strategy as before, w/ trivially different Wilson coefficients

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!} c_{2k}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+1,\beta+1\right)\nu^{2k}$$

$$\eta_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \sum_{j=0}^{n}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!} c_{2k+1}\left(z^2\mu^2\right)\omega_{n,j}^{(\alpha,\beta)} B\left(\alpha+2k+j+2,\beta+1\right)\nu^{2k+1}$$

Akaike Information Criterion (AIC)

H. Akaike, IEEE Transactions on Automatic Control, vol.19, no.6, 716-723 (1974)

- models w/ too many parameters disfavored
- combine model estimates per jackknife bin into single mean/error estimator

$$h_\pm^{\mathrm{AIC}}(x) = \sum_{m\in\mathrm{fit}} w^{(m)} h_\pm^{(m)}(x) \qquad \Delta_\pm^{\mathrm{AIC}}(x) = \sqrt{\sum_{m\in\mathrm{fit}} w^{(m)}\left[h_\pm^{(m)}(x) - h_\pm^{\mathrm{AIC}}(x)\right]^2}$$

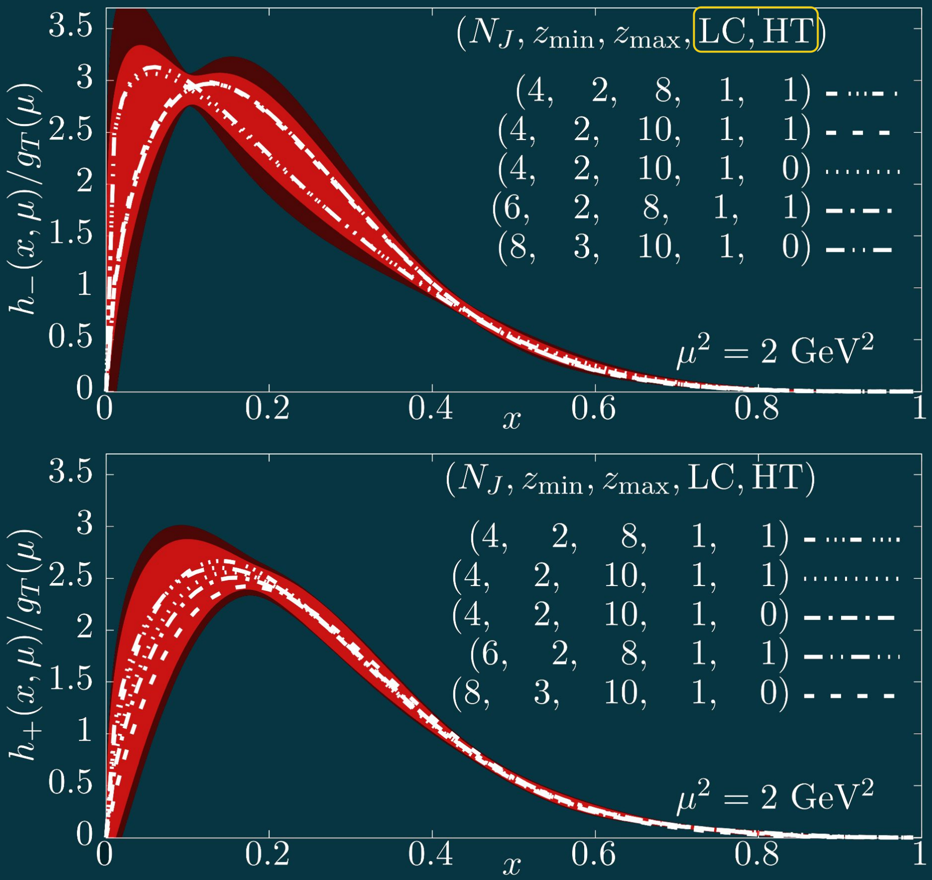
$$w^{(m)} = \frac{e^{-\frac{1}{2}\mathrm{AIC}(m)}}{\sum_{n\in\mathrm{fit}} e^{-\frac{1}{2}\mathrm{AIC}(n)}} \qquad \mathrm{AIC}(n) = \mathcal{L}_n + 2p_n + \frac{2p_n\left(p_n+1\right)}{\left(d_n - p_n - 1\right)}$$

Nth fit corrected AIC

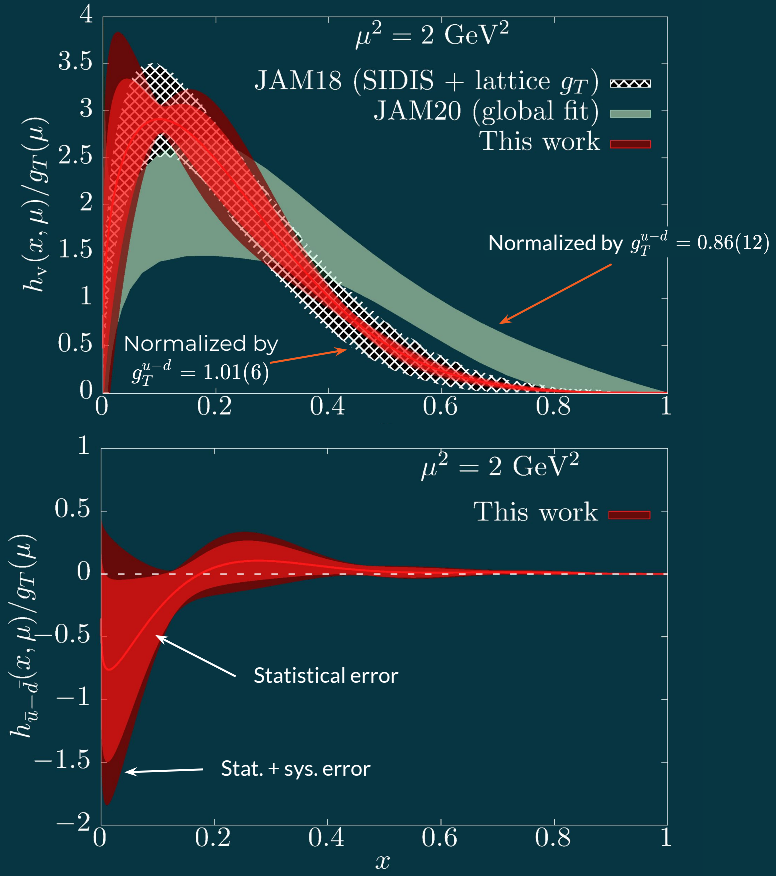
num fit parameters

num data fit w/ nth fit

Single discretization/higher-twist correction



Final Transversity Quark Distributions



CE, J. Karpie, N. Karthik et al., Phys.Rev.D 105 (2022) 034507

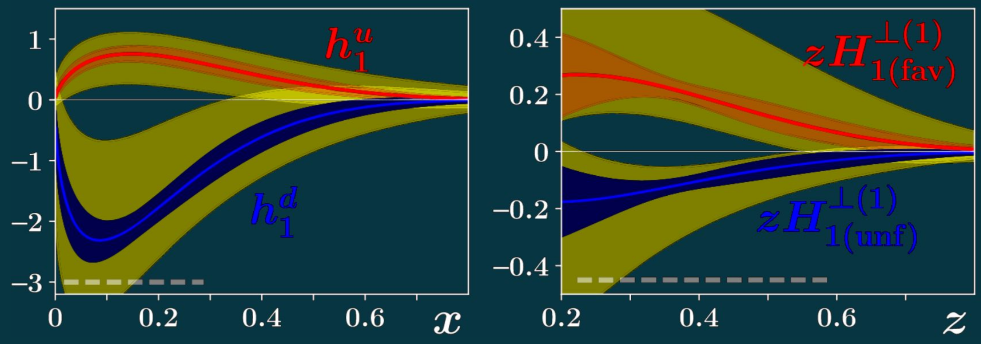
[JAM20]: SIDIS + transverse SSAs via SIA (e+e-) & pp-collisions

[JAM20] J. Cammarota, L. Gamberg, Z.-B. Kang et al., Phys.Rev.D 102 054002 (2020)

[JAM18]: 1st global analysis of nucleon quark transversity distribution

H.-W. Lin, W. Melnitchouk, A. Prokudin et al., Phys.Rev.Lett.120 152502 (2018)

- ➢ single-transverse spin asymmetries in pion production off proton/deuteron targets [PDF & Collins FFs]
- ➢ constraints from lattice QCD



Impact on phenomenology if isolated transversity PDF were included in a global analysis?

See CE, J. Karpie, N. Karthik et al., Phys.Rev.D 105 (2022) 034507 for an equivalent analysis using Mellin moments

Twist-2 PDF Checklist

ID	a (fm)	m_π (MeV)	β	c_{SW}	$L^3 \times T$	N_{cfg}
E1	0.094(1)	358(3)	6.3	1.205	$32^3 \times 64$	349
E2	0.094(1)	278(4)	6.3	1.205	$32^3 \times 64$	259
E3	0.091(2)	170(5)	6.3	1.205	$48^3 \times 96$	1370

ID	N_{vec}	N_{srcs}	T/a	$p_z \times \left(\frac{2\pi}{L}\right)$	z/a
E2	64	4	$4, 6, \cdots, 14$ $0.38, \cdots, 1.32$ fm	$0, \pm1, \cdots, \pm6$ $0, 0.41, \cdots, 2.47$ GeV	$0, \pm1, \cdots, \pm8$ $0, 0.094, \cdots, 0.75$ fm

ID	N_{vec}	N_{srcs}	T/a	$p_z \times \left(\frac{2\pi}{L}\right)$	z/a
E3	128	4	$4, 6, \cdots, 14$ $0.36, \cdots, 1.27$ fm	$0, \pm1, \cdots, \pm8$ $0, 0.28, \cdots, 2.27$ GeV	$0, \pm1, \cdots, \pm8$ $0, 0.091, \cdots, 0.73$ fm

Results will enable extrapolation to physical pion mass

Genprops on lattice ensembles w/ larger volumes and finer lattice spacings underway

		$f_{q/h}(x, \mu^2)$	$g_{q/h}(x, \mu^2)$	$h_{q/h}(x, \mu^2)$
E1	Single local interpolating field 3 phase smearings	Published	Preliminary	Published
E2	Single local interpolating field 3 phase smearings	Analyzing	Preliminary	Analyzing
E3	Expanded interpolator basis (16 ops) 5-7 phase smearings	Ongoing	Ongoing	Ongoing

A Multi-Dimensional Description of Hadronic Structure

"How does subatomic matter organize itself and what phenomena emerge?"

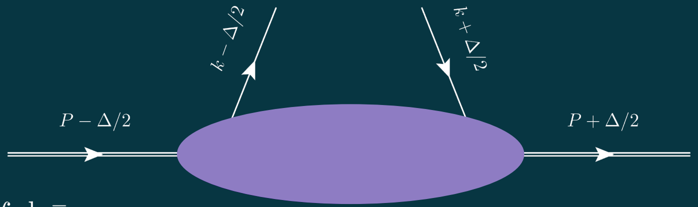
"What are the static and dynamical properties of matter?"

NSAC / NuPECC Long Range Plans (2015 / 2017)

Generalized Parton Distributions (GPDs)

A. V. Radyushkin, Phys. Rev.D56, 5524 (1997)
X.-D. Ji, Phys. Rev.D55, 7114 (1997)
M. Diehl, Phys. Rept.388, 41 (2003),

Off-forward Integrated parton correlations



$$\mathcal{G}^{[\gamma^+]}_{q/h} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle h(p_f)|\bar{q}\left(-\tfrac{z}{2}\right)\gamma^+\Phi^{(f)}_{\hat{z}^-}\left(\{-\tfrac{z}{2},\tfrac{z}{2}\}\right) q\left(\tfrac{z}{2}\right)|h(p_i)\rangle\, |_{z^+=0,\mathbf{z}_T=\mathbf{0}}$$

GPDs of fundamental importance

- unify familiar PDFs/FFs + impact parameter densities

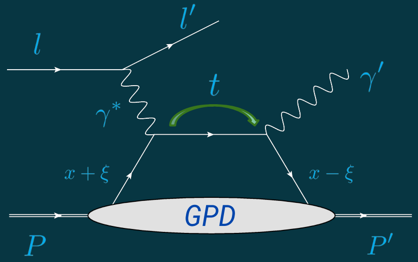
$$\int_{-1}^{1} dx \{H,\tilde{H},E,\tilde{E}\}^q(x,\xi,t) = \{F_1,F_2,G_A,G_P\}^q(t)$$

- quark/gluon EMT appears in OPE of off-forward matrix element of two external currents - forward limit of GFFs

$$J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H^q(x,\xi,t=0)+E^q(x,\xi,t=0)\right]$$

Variety of exclusive channels/observables

- DVCS/DVMP Eur.Phys.J.A 52 (2016) 6, 157; Eur.Phys.J.A 52 (2016) 6, 151 Eur.Phys.J.A 52 (2016) 6, 158
  - e.g. E12-06-113, E12-11-003 - JLab's Hall A [HRS] & B [CLAS12]



Challenge to extract GPDs from experimental data

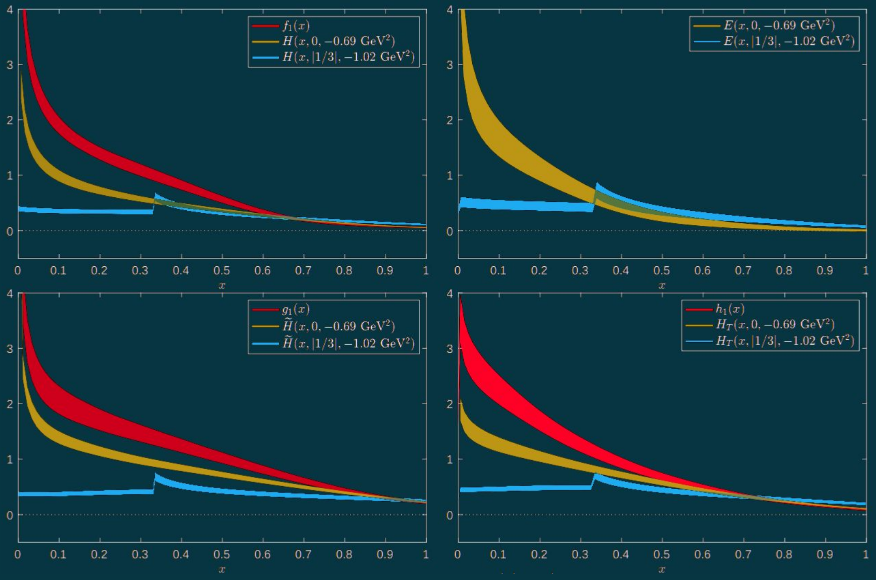
- DVCS observables & Compton Form Factors

$$CFF \sim \int_{-1}^{1} dx \frac{GPD(x,\xi,t)}{x-\xi+i\epsilon} + \cdots$$

"[GPDs] will transform the current picture of hadronic structure"
NSAC 2015

CENTER for NUCLEAR FEMTOGRAPHY

Recent Efforts from Lattice QCD to Resolve GPDs



A. Scapellato, C. Alexandrou, K. Cichy et al., 2201.06519 [hep-lat]

Generalized Quasi-distributions (quasi-GPDs) - generally zero skewness

- ➢ (mostly) nucleon

C. Alexandrou et al., Phys.Rev.Lett 125 (2020) 26, 262001
C. Alexandrou et al., Phys.Rev.D 105 (2022) 3, 034501

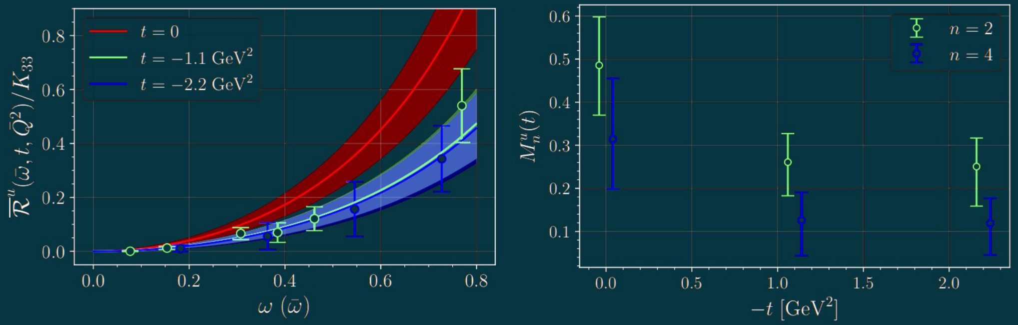
- ➢ pion J.-W. Chen et al., Nucl.Phys.B 952 (2020) 114940

LaMET has been extended to off-forward regime, giving access to unpolarized/helicity/transversity GPDs

X. Ji et al., Phys. Rev. D 92, 014039 (2015)
X. Xiong and J.H. Zhang, Phys. Rev. D 92, no.5, 054037 (2015)
Y.S. Liu et al., Phys. Rev. D 100, no.3, 034006 (2019)

Mellin moments of GPDs at leading-twist

- ➢ Off-forward Compton amplitude from Feynman-Hellman techniques



A. Hannaford-Gunn, K. U. Can, R. Horsley et al., arXiv: 2202.03662 [hep-lat]

GFFs of the nucleon - total quark angular momentum/transverse spin densities

G. Bali et al., Phys.Rev.D 100, 014507 (2019)

Pseudo-Distributions in the Off-Forward Regime

A. Radyushkin, Phys. Rev. D100, 116011 (2019)
A. Radyushkin, arXiv: Int.J.Mod.Phys.A 35 (2020) 05, 2030002

isovector projection

$$\mathbb{M}^{\alpha}(p_f, p_i, z) \equiv \langle N(p_f)|\overline{\psi}(-z/2)\frac{\tau^3}{2}\gamma^{\alpha}W(-z/2, z/2; A)\psi(z/2)|N(p_i)\rangle$$

$$= e^{i(\nu_i-\nu_f)/2}\langle N(p_f)|\overline{\psi}(0)\gamma^{\alpha}W(0, z; A)\psi(z)|N(p_i)\rangle$$

$$= \langle\langle\gamma^{\alpha}\rangle\rangle M(\nu_f, \nu_i, t, z^2) + \langle\langle\mathbb{1}\rangle\rangle z^{\alpha} N(\nu_f, \nu_i, t, z^2) + \mathcal{O}(\Delta^{\alpha})\text{-terms}$$

Matching performed here

$$\begin{cases} \langle\langle\Gamma\rangle\rangle = \bar{u}_N(p_f)\Gamma u_N(p_i) \\ \nu_k = p_k \cdot z \\ t = \Delta^2 = (p_i - p_f)^2 \end{cases}$$

$$\alpha = + \quad z^{\alpha} = (0, z^-, \mathbf{0}_\perp) \qquad \xi = \frac{p_i^+ - p_f^+}{p_i^+ + p_f^+}$$

A first implementation in Lattice QCD:

ID	$N_{\rm vec}$	$N_{\rm srcs}$	T/a	$p_z \times \left(\frac{2\pi}{L}\right)$	z/a
E1	64	4	$4, 6, \cdots, 14$ $0.38, \cdots, 1.32$ fm	$0, \pm1, \cdots, \pm6$ $0, 0.41, \cdots, 2.47$ GeV	$0, \pm1, \cdots, \pm12, \cdots$ $0, 0.094, \cdots, 1.13$ fm

+19 Momentum Transfers ⇢ Same set of inversions as forward case

$$\alpha = 4$$

$$p_i^{\alpha} = (\mathbf{p}_\perp^i, p_z^i, E_i)$$

$$p_f^{\alpha} = (\mathbf{p}_\perp^f, p_z^f, E_f)$$

$$z^{\alpha} = (\mathbf{0}_\perp, z_3, 0)$$

$$\xi = \frac{(p_i z) - (p_f z)}{(p_i z) + (p_f z)} \qquad \nu = \frac{\nu_f + \nu_i}{2}$$

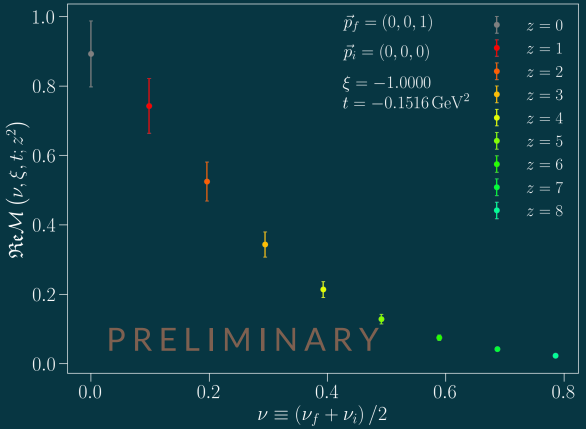
Zero skewness limit particularly interesting

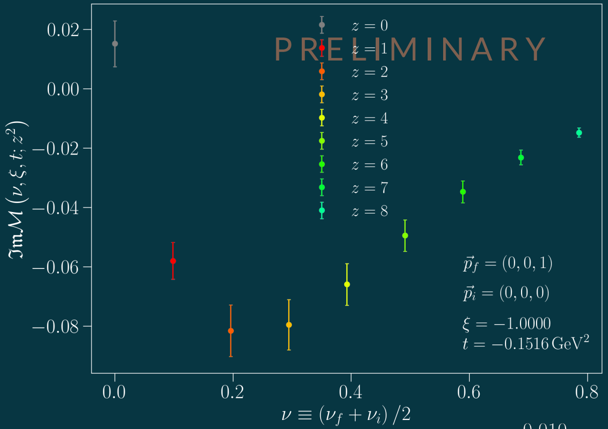
- ➢ mapping skewness/momentum transfer dependence in isolation a challenge on a discrete lattice
- ➢ verify extrapolation of GPD to zero momentum transfer → PDF

➢ slightly different ratio to remove unknown overlap factors, time dependencies

$$R_{\Gamma}(\vec{p}_f, \vec{p}_i; T, \tau) = \frac{C_{\rm 3pt}^{\Gamma}(\vec{p}_f, \vec{p}_i; T, \tau)}{C_{\rm 2pt}(\vec{p}_f, T)}\sqrt{\frac{C_{\rm 2pt}(\vec{p}_i; T-\tau)\, C_{\rm 2pt}(\vec{p}_f; \tau)\, C_{\rm 2pt}(\vec{p}_f; T)}{C_{\rm 2pt}(\vec{p}_f; T-\tau)\, C_{\rm 2pt}(\vec{p}_i; \tau)\, C_{\rm 2pt}(\vec{p}_i; T)}}$$

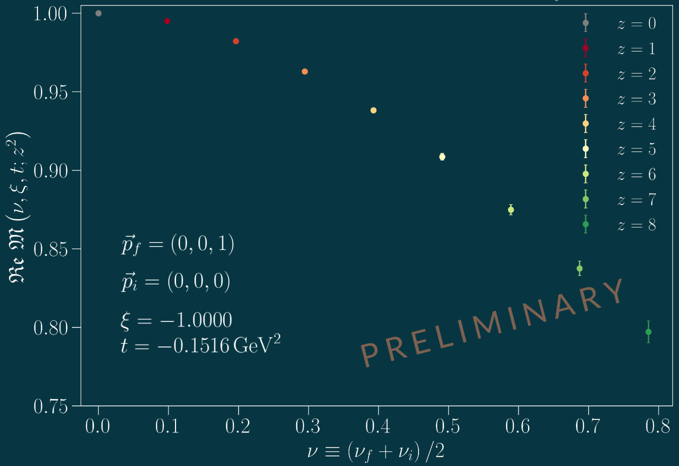
An Unpolarized Double Ioffe-Time Distribution Slice

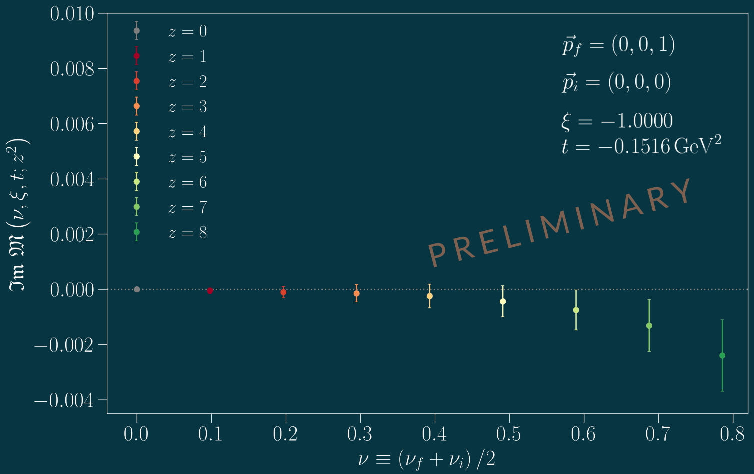




$$\mathfrak{M}\left(\nu,\xi,t;z_3^2\right) \equiv \frac{\mathcal{M}\left(\nu,\xi,t;z_3^2\right)}{\mathcal{M}\left(0,0,0;z_3^2\right)}$$

$$= \frac{\mathcal{M}\left(\nu,\xi,t;z_3^2\right)}{\mathcal{M}\left(\nu,\xi,t;0\right)\,|_{z_3=0}} \times \frac{\mathcal{M}\left(0,0,0;0\right)\,|_{\vec{p}_f=\vec{p}_i=0,z_3=0}}{\mathcal{M}\left(0,0,0;z_3^2\right)\,|_{\vec{p}_f=\vec{p}_i=0}}$$





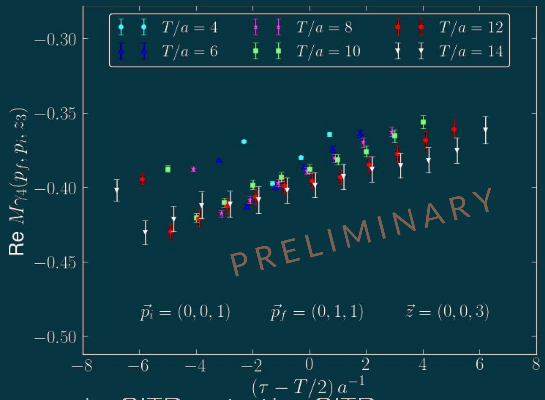
Next Steps in Constructing the Pseudo-GITD

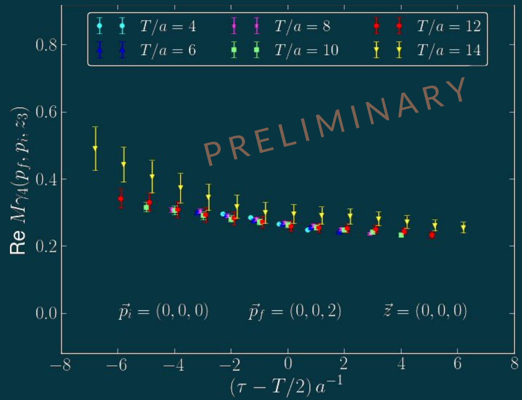
Consistent treatment of operator constructions/subductions

Subductions

$$\langle \vec{p}_f, \Lambda_f, \mu_f | \, j^{\Lambda_\gamma,\mu_\gamma} \, | \vec{p}_i, \Lambda_i, \mu_i \rangle = \sum_{\lambda_f,\lambda_\gamma,\lambda_i} S^{\Lambda_f,\mu_f}_{J_f,\lambda_f} \left[S^{\Lambda_\gamma,\mu_\gamma}_{J_\gamma=1,\lambda_\gamma}\right]^* \left[S^{\Lambda_i,\mu_i}_{J_i,\lambda_i}\right]^* \sum_\ell \mathcal{K}_\ell \left(h_f, J_f\left[\lambda_f,\vec{p}_f\right]; h_i, J_i\left[\lambda_i,\vec{p}_i\right]\right) A_\ell\left(\nu_f,\nu_i;t;z^2\right)$$

Kinematic factors; Wigner-D's





Matching the pseudo-GITD onto the GITD

$$\widetilde{\mathcal{I}}\left(\nu,\xi,t,\mu^2\right) = \widetilde{\mathfrak{M}}\left(\nu,\xi,t,z^2\right) + \frac{\alpha_s C_F}{2\pi}\int_0^1 du\, \widetilde{\mathfrak{M}}\left(u\nu,\xi,t,z^2\right)\left\{\ln\left[\frac{e^{2\gamma_E+1}}{4}z^2\mu^2\right] B_G\left(u,\bar{u},\xi,\nu\right) + L_G\left(u,\bar{u},\xi,\nu\right)\right\} + \mathcal{O}\left(z^2\Lambda^2_{\mathrm{QCD}}\right)$$

$$e^{-i\xi\nu}\mathcal{M}\left(\nu,\xi,t,z^2\right) = \int_{-1}^1 dx\, e^{ix\nu}\mathcal{G}\left(x,\xi,t,z^2\right)$$

$$\left[\frac{2u}{1-u}\right]_+ \cos\left(\bar{u}\xi\nu\right) + \frac{\sin\left(\bar{u}\xi\nu\right)}{\xi\nu} - \frac{1}{2}\delta\left(\bar{u}\right)$$

$$4\left[\frac{\ln\left(1-u\right)}{1-u}\right]_+ \cos\left(\bar{u}\xi\nu\right) - 2\frac{\sin\left(\bar{u}\xi\nu\right)}{\xi\nu} + \delta\left(\bar{u}\right)$$

Express as kernel relation between pseudo-GITD and GPD

- parameterization of underlying GPD still needed

GPD Parameterizations

$$\int_{-1}^{1} dx \begin{Bmatrix} H^q \\ \widetilde{H}^q \end{Bmatrix} (x,\xi,t) = \begin{Bmatrix} F_1^q \\ G_A^q \end{Bmatrix}(t)$$

$$\int_{-1}^{1} dx \begin{Bmatrix} E^q \\ \widetilde{E}^q \end{Bmatrix} (x,\xi,t) = \begin{Bmatrix} F_2^q \\ G_P^q \end{Bmatrix}(t)$$

Functional dependence constrained by:

- several physical BCs
  - Reduction to elastic form factors
  - Recover twist-2 PDFs in forward limit

$$H^{q/h}(x,0,0) = f_{q/h}(x) \qquad \widetilde{H}^{q/h}(x,0,0) = \Delta f_{q/h}(x)$$

➢ polynomiality

$$\int_{-1}^{1} dx\, x^n \begin{Bmatrix} H^q \\ E^q \end{Bmatrix} (x,\xi,t) = \sum_{i=0;\,\mathrm{even}}^{n} (2\xi)^i \begin{Bmatrix} A_{n+1,i}^q \\ B_{n+1,i}^q \end{Bmatrix}(t) \pm \mathrm{mod}(n,2)\,(2\xi)^{n+1}\, C_{n+1}^q(t)$$

D. Müller, D. Robaschik, B. Geyer et al., Fortsch. Phys. 42, 101 (1994)
A. V. Radyushkin, Phys. Rev. D 56, 5524 (1997)
X. D. Ji, Phys. Rev. Lett. 78, 610 (1997)

But how should these important distributions be parameterized?

➢ kernel relation (cf. jacobi polynomials for PDFs) - model bias?
➢ Neural-networks?

H. Dutrieux et al., arXiv:2112.10528 [hep-ph]
L. D. Debbio, et al., arXiv: 2010.03996 [hep-ph]
K. Cichy, L. D. Debbio, T. Giani, JHEP 10 (2019) 137

➢ Double-distributions (DDs)

A. V. Radyushkin, Phys. Rev. D 59, 014030 (1999)
D. Müller et al., Fortschr.Phys. 42, 101 (1994)

Double-distributions

$$\langle p'|\overline{\psi}(-z/2)\not{z}\psi(z/2)|p\rangle|_{z^2=0} = \overline{u}(p')\not{z}u(p)\int d\beta d\alpha\, e^{-i\beta(P\cdot z)+i\alpha(\Delta\cdot z)/2} f_q(\beta,\alpha,t)$$
$$+\,\overline{u}(p')\frac{i\sigma^{\mu\nu}z_\mu\Delta_\nu}{2m}u(p)\int d\beta d\alpha\, e^{-i\beta(P\cdot z)+i\alpha(\Delta\cdot z)/2} k_q(\beta,\alpha,t) + \cdots$$

$$|\beta|+|\alpha|\leq 1$$

- initial momenta/momentum transfer treated equally (cf. FT of parton momentum fraction)
- DDs generate GPDs which automatically satisfy polynomiality

Closing Remarks

Hadronic structure accessible from certain lattice calculable matrix elements

- short-distance factorization

Twist-2 Nucleon valence (plus) quark PDFs

- distillation (+phasing) - precise pseudo-ITDs & PDFs
  - phased distillation now central to HadStruc
- systematic effects can be reliably addressed
- Impact on phenomenology
  - combine data w/ a global analysis of exp. cross sections?
  - Soffer bound?

GPDs - a vast/open landscape wherein LQCD can provide guidance

HadStruc Collaboration

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Jianwei Qiu, David Richards, Eloy Romero, Frank Winter [1]

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Carl Carlson, Chris Chamness, Tanjib Khan,
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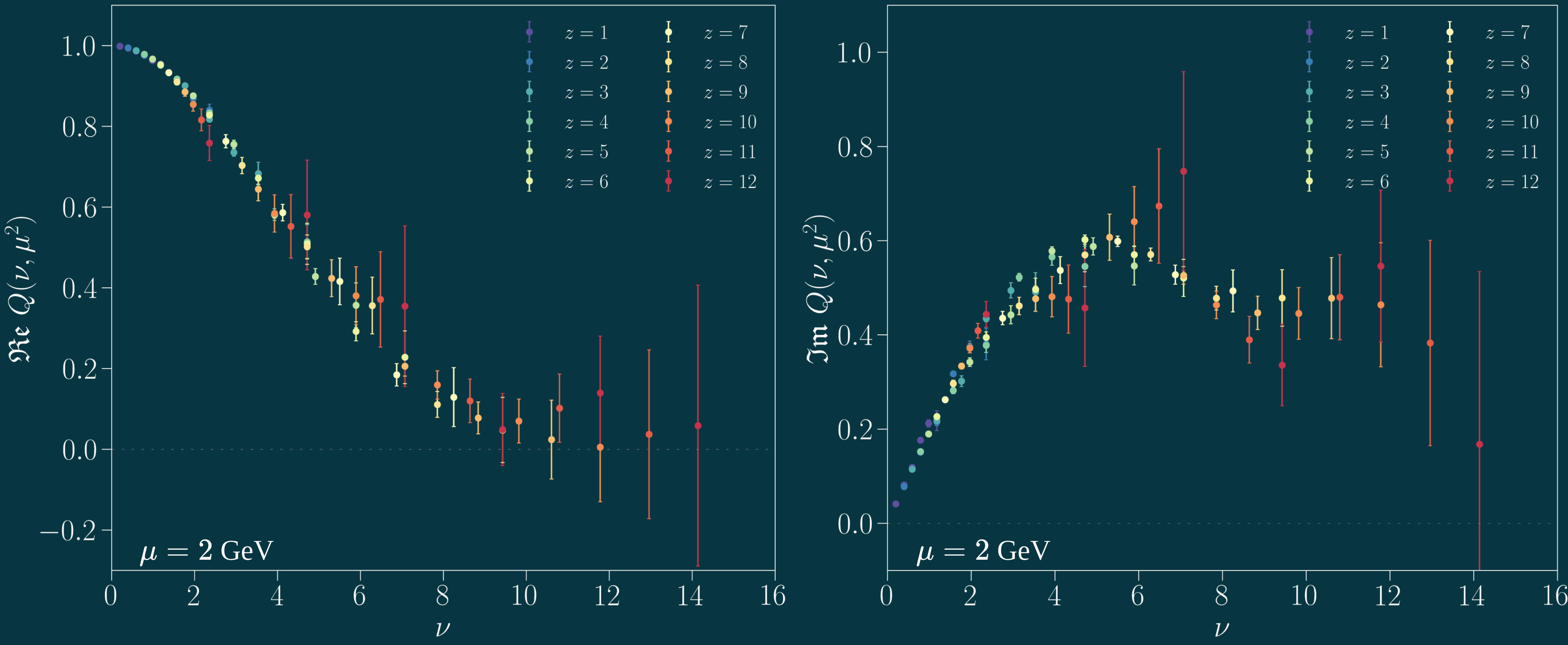
Joe Karpie [5]

Savvas Zafeiropoulos [6]

Yan-Qing Ma [7]

Jefferson Lab [1], Oak Ridge [2], William and Mary [3], Old Dominion University [4], Columbia University [5], Aix Marseille University [6], Peking University [7]

Unpolarized Ioffe-time Distribution



CE, R. Edwards, C. Kallidonis et al., JHEP 11 (2021) 148

Quasi-GPD Matching (Example: Transversity)

$$C_G\left(\sigma^{3j};x,\xi,\frac{p_3}{\mu},\frac{p_3}{(\mu_0)_3},r\right)=\delta(x-1)+\frac{\alpha_s C_F}{2\pi}\begin{cases}G_1(\sigma^{3j};x,\xi)_+ & x<-\xi\\ G_2(\sigma^{3j};x,\xi,p_3/\mu)_+ & |x|<\xi\\ G_3(\sigma^{3j};x,\xi,p_3/\mu)_+ & \xi<x<1\\ -G_1(\sigma^{3j};x,\xi)_+ & x>1\end{cases}$$

$$-\frac{\alpha_s C_F}{2\pi}\left|\frac{p_3}{(\mu_0)_3}\right| f_{\mathcal{P}_T}\left(\sigma^{3j};\frac{p_3}{(\mu_0)_3}(x-1)+1,r\right)_+ +\frac{\alpha_s C_f}{4\pi}\delta(x-1)\ln\left(\frac{\mu^2}{(\mu_0)_3^2}\right)$$

C. Alexandrou, K. Cichy, M. Constantinou et al., Phys. Rev. D 105 (2022) 3, 034501

$$G_1(\sigma^{3j};x,\xi)=-\frac{x+\xi}{(x-1)(1+\xi)}\ln\frac{x-1}{x+\xi}+(\xi\to-\xi),$$

$$G_2(\sigma^{3j};x,\xi)=\frac{x+\xi}{(1-x)(1+\xi)}\left[\ln\frac{4(1-x)^2(x+\xi)p_3^2}{(\xi-x)\mu^2}-1\right]+\frac{2\xi}{1-\xi^2}\ln\frac{\xi-x}{1-x},$$

$$G_3(\sigma^{3j};x,\xi)=\frac{2(x-\xi^2)}{(1-x)(1-\xi^2)}\left[\ln\frac{4\sqrt{x^2-\xi^2}(1-x)p_3^2}{\mu^2}-1\right]+\frac{\xi}{1-\xi^2}\ln\frac{x+\xi}{x-\xi}$$

Nucleon Dispersion

Discretized continuum-like interpolators of definite permutational symmetries

$$\left(N_M \otimes \left(\tfrac{1}{2}^+\right)^1_M \otimes D^{[2]}_{L=1,A}\right)^{J^P=\frac{1}{2}^+} \equiv N^2 P_A \tfrac{1}{2}^+ \qquad N^{(2S+1)} L_{\mathcal{P}} J^P$$

Rest

$$\mathcal{B}_{\vec{p}=\vec{0}} = \{N^2 S_S \tfrac{1}{2}^+, N^2 S_M \tfrac{1}{2}^+, N^2 S'_S \tfrac{1}{2}^+, N^2 P_A \tfrac{1}{2}^+, N^2 P_M \tfrac{1}{2}^+, N^4 P_M \tfrac{1}{2}^+, N^4 D_M \tfrac{1}{2}^+\}$$

Dominate low-lying spectrum

R. Edwards et al., Phys. Rev. D84, 074508 (2011)
J. Dudek & R. Edwards, Phys. Rev. D85, 054016 (2012)

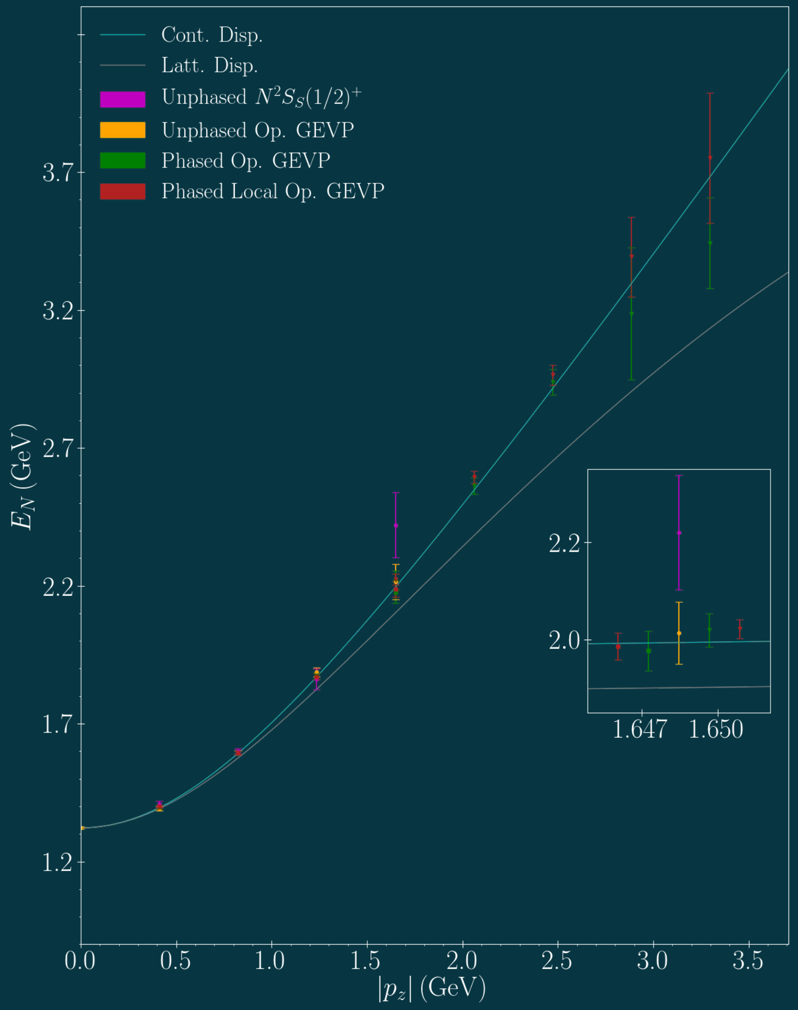
Boosted

$$\mathcal{B}_{\vec{p}\neq\vec{0}} = \{N^2 S_S \tfrac{1}{2}^+, N^2 S_M \tfrac{1}{2}^+, N^2 P_A \tfrac{1}{2}^+, N^2 P_M \tfrac{1}{2}^+, N^4 P_M \tfrac{1}{2}^+, N^4 D_M \tfrac{1}{2}^+, N^4 S_M \tfrac{3}{2}^+, N^2 D_S \tfrac{5}{2}^+, N^2 P_M \tfrac{1}{2}^-, N^4 P_M \tfrac{1}{2}^-, N^2 P_M \tfrac{3}{2}^-, N^4 P_M \tfrac{3}{2}^-, N^4 P_M \tfrac{5}{2}^-, N^2 D_S \tfrac{3}{2}^+, N^4 D_M \tfrac{3}{2}^+, N^2 D_M \tfrac{3}{2}^+\}$$

Broken parity - high spin states relevant

Repeat energy extractions with and without phasing

➔ GEVP: precision/agreement with continuum dispersion



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