Glueball Spectrum from Lattice QCD Study

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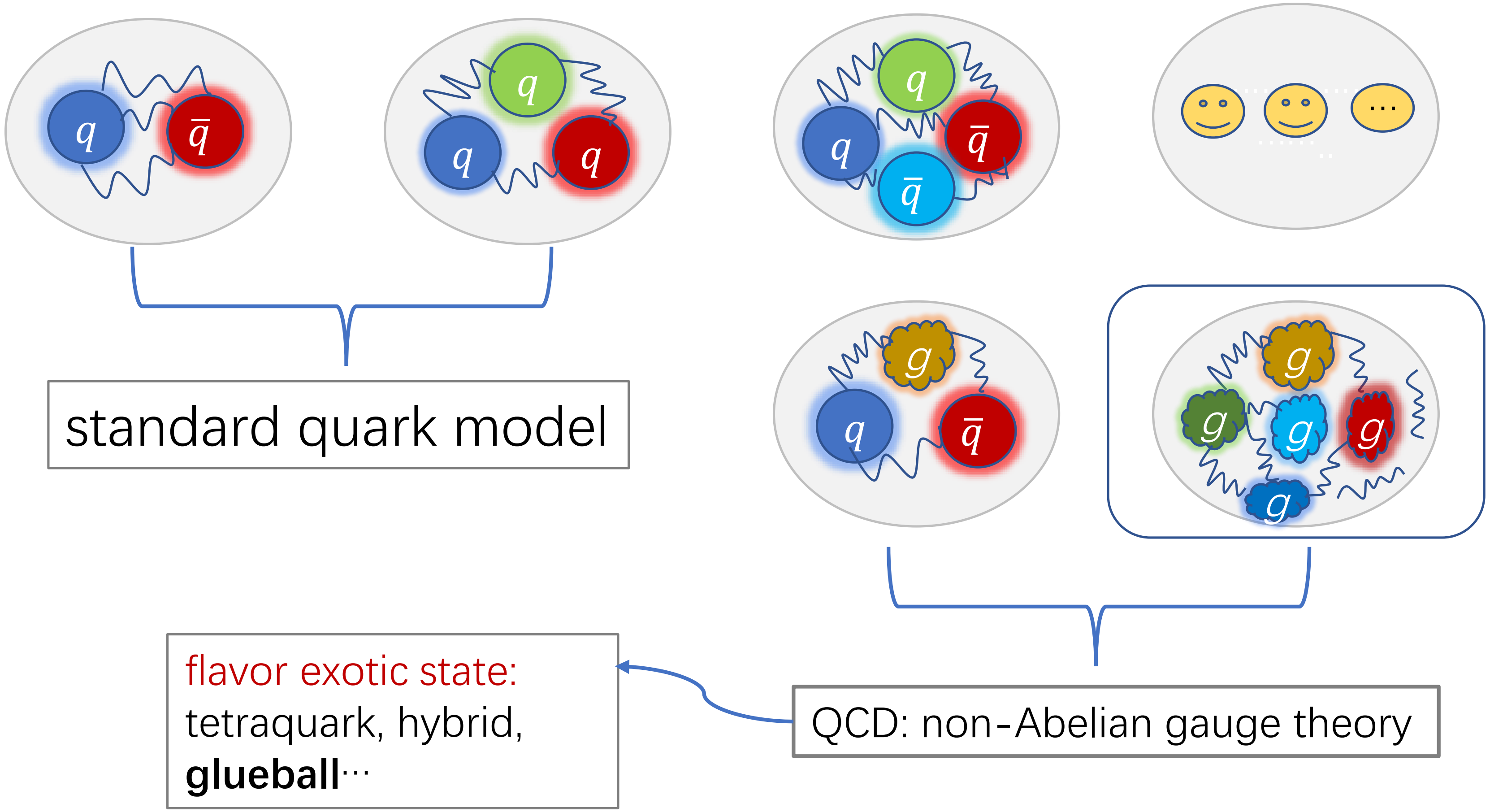
Introduction



Introduction

Scalar

below 1 GeV:
9 states

- I = 1: $a_0(980)$
- I=1/2: $K_0^*(700)(\kappa)$
- **I=0**: $f_0(500)(\sigma), f_0(980)$

above 1 GeV:
10 states

- $a_0(1450)$
- $K_0^*(1430)$
- $f_0(1370), f_0(1500), f_0(1710)$

Experimental status

- **BESIII**
  - J/psi radiative decay
- **COMPASS**
  - proton-proton central production
- **PANDA**
  - proton-antiproton annihilation
- **GlueX**
  - photoproduction

Beijing Spectrometer(BESIII) Experiment

COMPASS

COmmon Muon Proton Apparatus for Structure and Spectroscopy

Anti-Proton ANnihilation at DArmstadt

BESIII

$J/\psi \longrightarrow \gamma X \longrightarrow \gamma\eta\eta$

M.Ablikim et al, *Phys.Rev.***D.87**, 092009 (2013)

Resonance	Mass(MeV/c^2)	Width(MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	8.2 σ
$f_0(1710)$	$1759\pm6^{+14}_{-25}$	$172\pm10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	25.0 σ
$f_0(2100)$	$2081\pm13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	13.9 σ
$f_2'(1525)$	$1513\pm5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	11.0 σ
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	6.4 σ
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	7.6 σ

$J/\psi \longrightarrow \gamma X \longrightarrow \gamma\phi\phi$

M.Ablikim et al, *Phys.Rev.***D.93**, 112011 (2016)

Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.($\times 10^{-4}$)	Sig.
$\eta(2225)$	$2216^{+4}_{-5}{}^{+21}_{-11}$	$185^{+12}_{-14}{}^{+43}_{-17}$	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	28 σ
$\eta(2100)$	$2050^{+30}_{-24}{}^{+75}_{-26}$	$250^{+36}_{-30}{}^{+181}_{-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	22 σ
$X(2500)$	$2470^{+15}_{-19}{}^{+101}_{-23}$	$230^{+64}_{-35}{}^{+56}_{-33}$	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	8.8 σ
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	24 σ
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5 σ
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4 σ
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11 σ
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	6.8 σ

Theoretical research

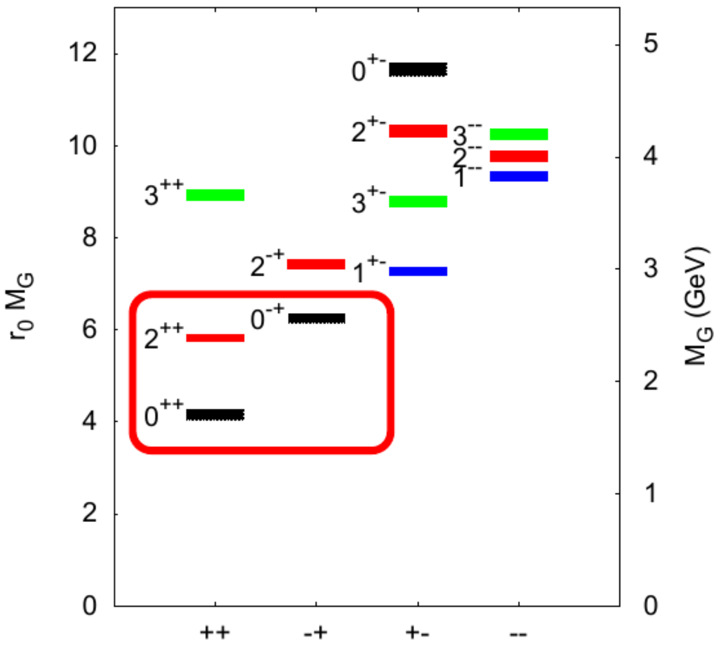
- Flux-tube, bag model, constituent model etc.
- AdS/QCD [Yidian Chen and Mei Huang, Chin.Phys. C40(2016) no.12, 123101]
- QCD sum rule [D.Harnett, et al, Nucl.Phys. A850(2011) 110]
- Lattice QCD
  - Quenched approximation:
    - B.Berg and A.Billoire, *Nuclear Physics* **B221** (1983):109-140
    - C.Morningstar and M.Peardon, *Phys.Rev.***D56**(1997):4043-4061
    - C.Morningstar and M.Peardon, *Phys.Rev.***D60**(1999)034509
    - H.B.Meyer and M.J.Teper, *Phys.Lett.***B605**(2005)344-345
    - Y.Chen et al, *Phys.Rev.***D73**(2006)014516
    - ...
  - Dynamical sea quark:
    - G.S.Bali et al,*Phys.Rev.***D62**(2000)054503
    - A.Hart and M.Teper, *Phys.Rev.***D65**(2002)034502
    - **UKQCD**, *Phys.Rev.***D82**(2010)034501
    - E.Gregory et al, *JHEP* **10**(2012)170

Theoretical research

Lowest-lying glueballs in quenched LQCD

- Lowest states with $J^{PC} = 0^{++},\ 2^{++},\ 0^{-+}$
- Masses around 1.7 GeV, 2.4 GeV and 2.6 GeV respectively.

[Y. Chen et al, *Phys. Rev.* **D 73**, 014516(2006)]



N_f=2 lattice QCD study

[W.S. et al, *Chin.Phys.C* 42(2018) no 9, 093103]

Gauge configuration details

- $N_f = 2$ anisotropic lattice
- Tadpole-improved gauge action
- Clover-improved Wilson fermion action
- Ground state scalar, pseudoscalar, tensor investigated

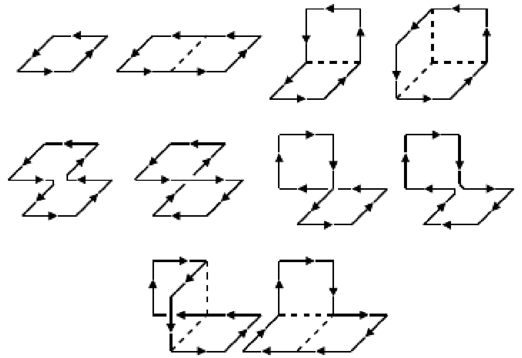
β	$L^3 \times T$	ξ	a_s	m_π	N_{conf}
2.5	$12^3 \times 128$	5	$0.114 fm$	$\sim$ 650 MeV	4800
2.5	$12^3 \times 128$	5	$0.118 fm$	$\sim$ 938 MeV	10400

N_f=2 lattice QCD study

Glueball operators construction

- Particle states denoted by J^{PC} in continuum
- SU(2)(continuum)$\xrightarrow{reduction}$ ^2O(lattice)
- Glueballs are bosons ($^2O \rightarrow O$)
- Octahedral group O has five IRs, A_1, A_2, E, T_1, T_2, denoted by R
- Subduced representation of $SU(2)$ with respect to group O is generally reducible($J \geq 2$)
- $R \not\leftrightarrow J$ ($A_1 \longrightarrow J = 0, J = 4, ...$)
- Assuming that the ground state on the lattice corresponds to the lowest spin state in continuum
- Using different spatial oriented Wilson loops to construct glueball operators with quantum number denoted by R^{PC}

R \ J	0	1	2	3	4	5
A_1	1	0	0	0	1	0
A_2	0	0	0	1	0	0
E	0	0	1	0	1	1
T_1	0	1	0	1	1	2
T_2	0	0	1	1	1	1



[C.J. Morningstar and M.J. Peardon, *Phys. Rev.* **D 60**, 034509(1999)]

N_f=2 lattice QCD study

Glueball operators construction

- Operators with R^{PC} quantum number are linear combinations of Wilson loops
  - $P = \pm,\ C = +$

$$\phi_i^{R^{PC}} = \sum_{g \in O} c_R ReTr[g \circ W_i(\mathbf{x}, t) \pm \mathcal{P} g \circ W_i(\mathbf{x}, t) \mathcal{P}^{-1}]$$

  - $P = \pm,\ C = -$

$$\phi_i^{R^{PC}} = \sum_{g \in O} c_R ImTr[g \circ W_i(\mathbf{x}, t) \pm \mathcal{P} g \circ W_i(\mathbf{x}, t) \mathcal{P}^{-1}]$$

N_f=2 lattice QCD study

Effective mass plateaus

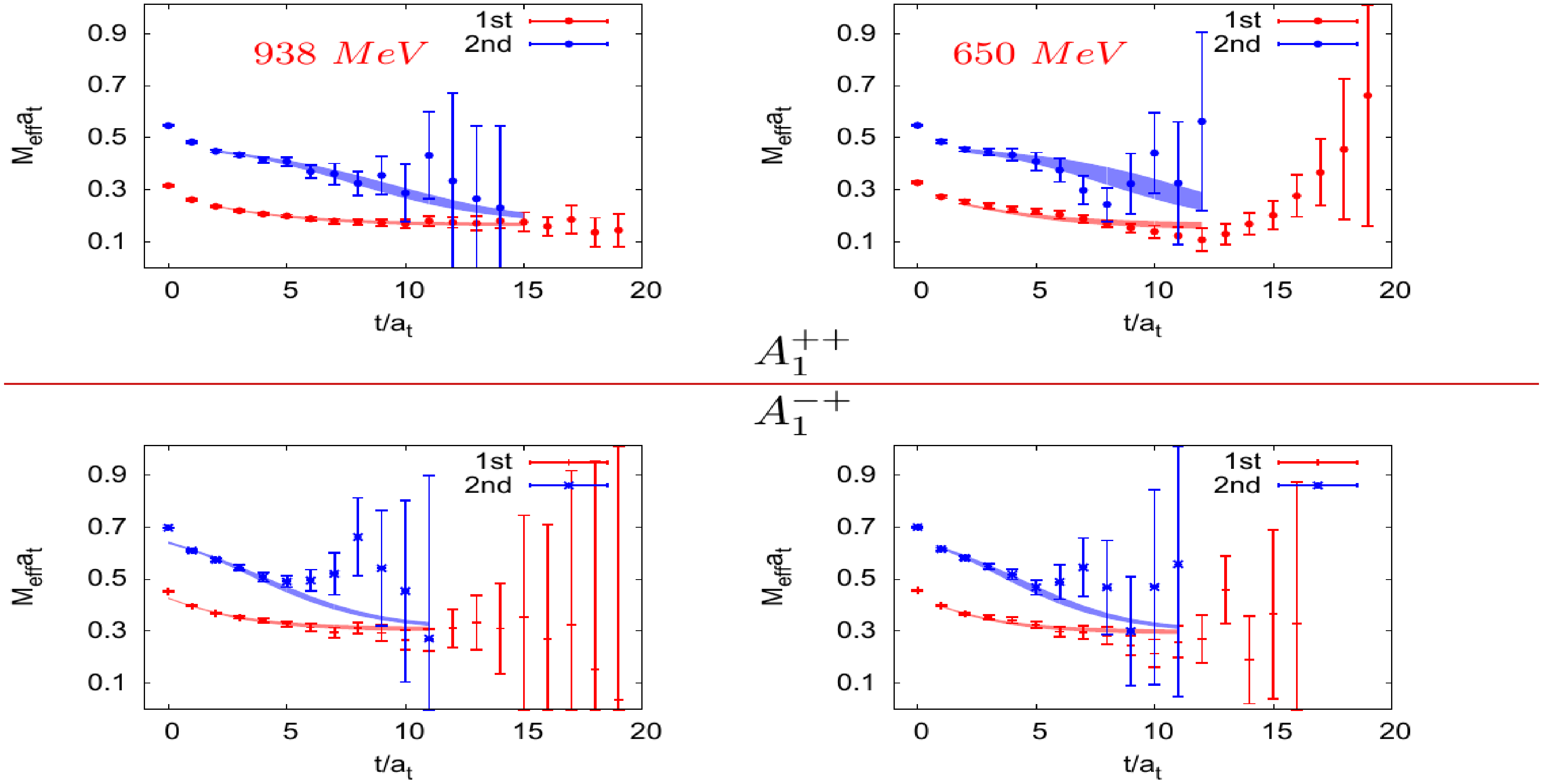
- 24 operators $\phi_\alpha^{(R^{PC})}$ for each R^{PC}
- Use the variational method to get a optimal operator $\Phi_i^{R^{PC}}$
- The optimal correlation function is

$$\tilde{C}_i^{(R^{PC})}(t) = \sum_\tau \langle 0|\Phi_i^{(R^{PC})}(t+\tau)\Phi_i^{(R^{PC})}(\tau)|0\rangle,$$

$$m_{i,\mathrm{eff}}^{(R^{PC})}(t) = \ln\left(\frac{\tilde{C}_i^{(R^{PC})}(t)}{\tilde{C}_i^{(R^{PC})}(t+1)}\right)$$

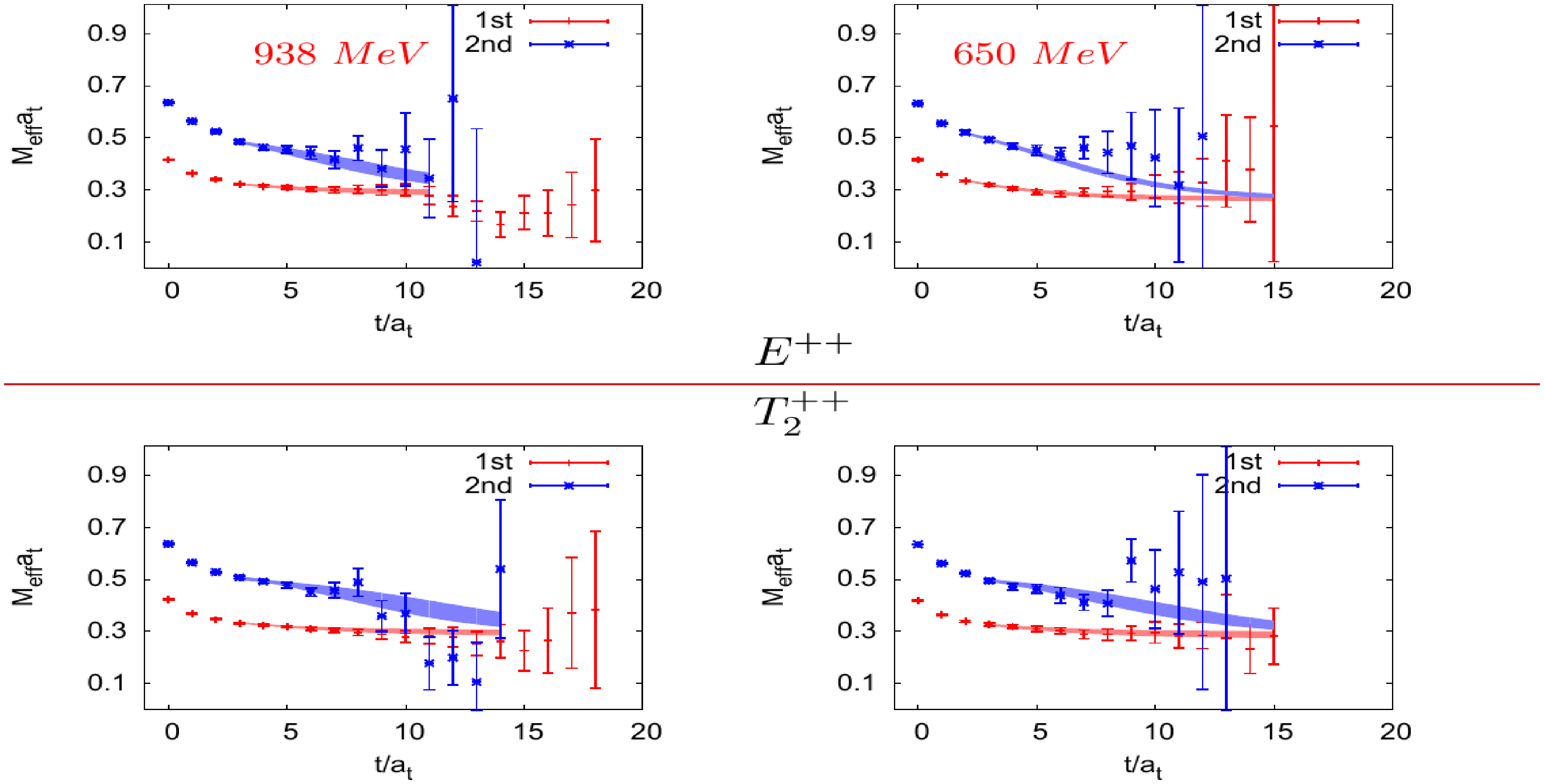
N_f=2 lattice QCD study

Effective mass plateaus (A_1^{++} & A_1^{-+})



N_f=2 lattice QCD study

Effective mass plateaus (E^{++} & T_2^{++})



N_f=2 lattice QCD study

- The optimal correlation function is

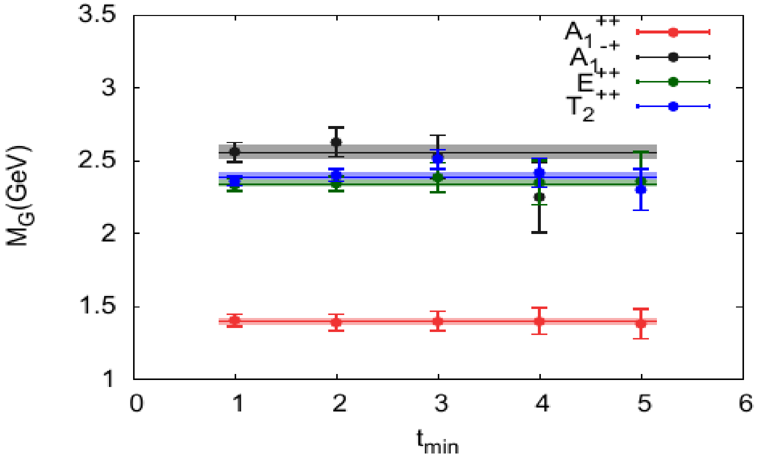
$$\tilde{C}_i^{(R^{PC})}(t) = \sum_{\tau}\langle 0|\Phi_i^{(R^{PC})}(t+\tau)\Phi_i^{(R^{PC})}(\tau)|0\rangle,$$

- Use two state union fit

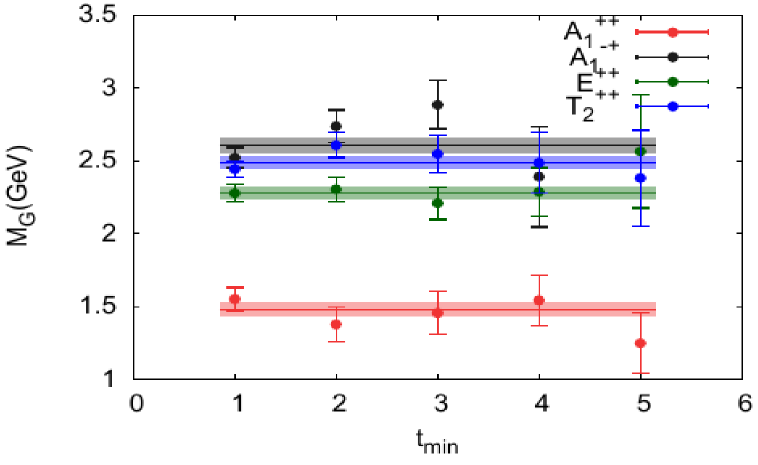
$$\begin{aligned}\tilde{C}_1^{(R^{PC})}(t) &= W_{11}^{(R^{PC})}e^{-m_1 t} + W_{12}^{(R^{PC})}e^{-m_2 t},\\ \tilde{C}_2^{(R^{PC})}(t) &= W_{21}^{(R^{PC})}e^{-m_1 t} + W_{22}^{(R^{PC})}e^{-m_2 t},\end{aligned}$$

N_f=2 lattice QCD study

Fitted results



938 MeV



650 MeV

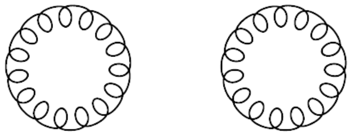
	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$ [this work]	938	1397(25)	2367(35)	2559(50)
	650	1480(52)	2380(61)	2605(52)
$N_f = 2+1$ [E. Gregory]	360	1795(60)	2620(50)	—
quenched [C. Morningstar]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [Y. Chen]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

More about scalar channel

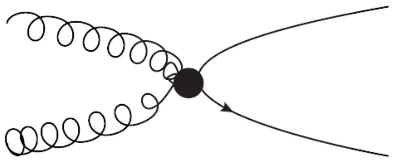
[W.S. et al, *EPJ Web Conf.*175(2018) 05016]

Include isoscalar $q\bar{q}$ operator

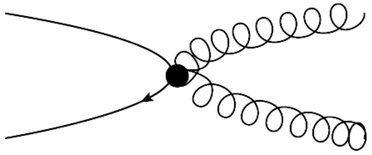
$$\mathcal{C}(\mathbf{p},t)=\begin{pmatrix}\langle O_G^{(1)}(\mathbf{p},t)O_G^{(1)\dagger}(\mathbf{p},0)\rangle & \langle O_G^{(1)}(\mathbf{p},t)O_{q\bar{q}}^{\dagger}(\mathbf{p},0)\rangle\\ \langle O_{q\bar{q}}(\mathbf{p},t)O_G^{(1)\dagger}(\mathbf{p},0)\rangle & \langle O_{q\bar{q}}(\mathbf{p},t)O_{q\bar{q}}^{\dagger}(\mathbf{p},0)\rangle\end{pmatrix}$$



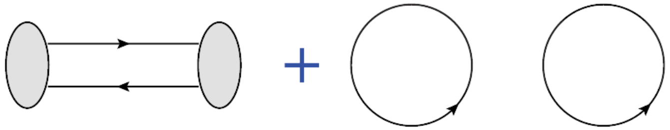
(a) glueball correlator



(b) cross term



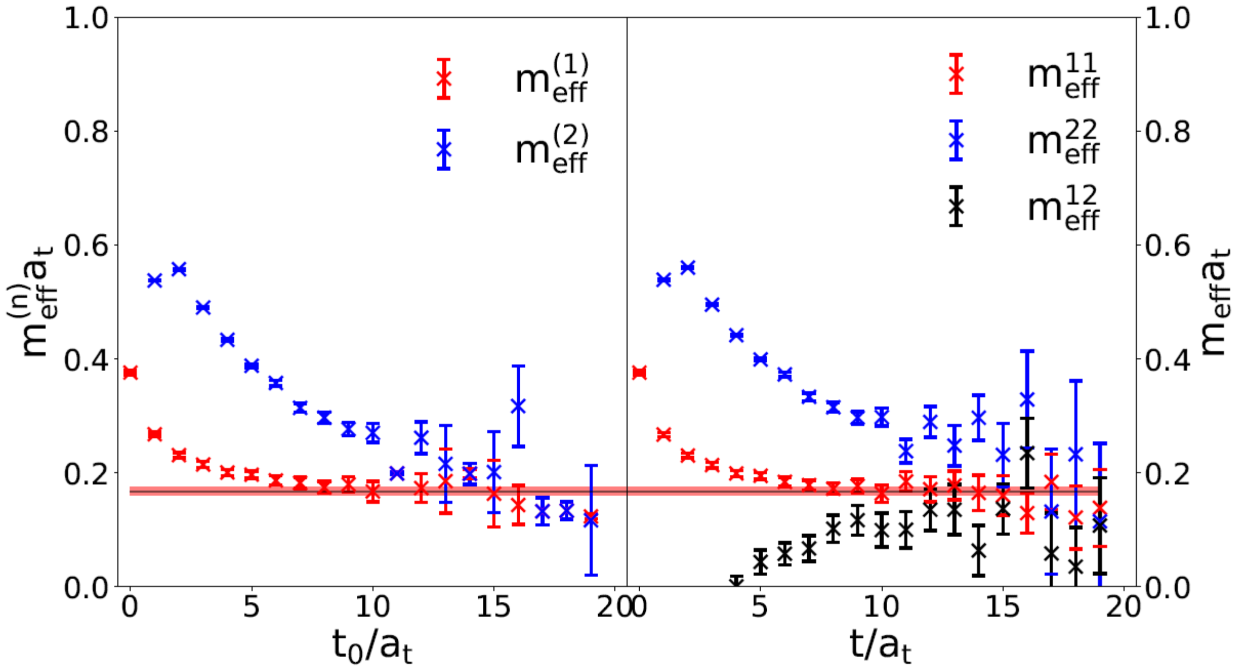
(c) cross term

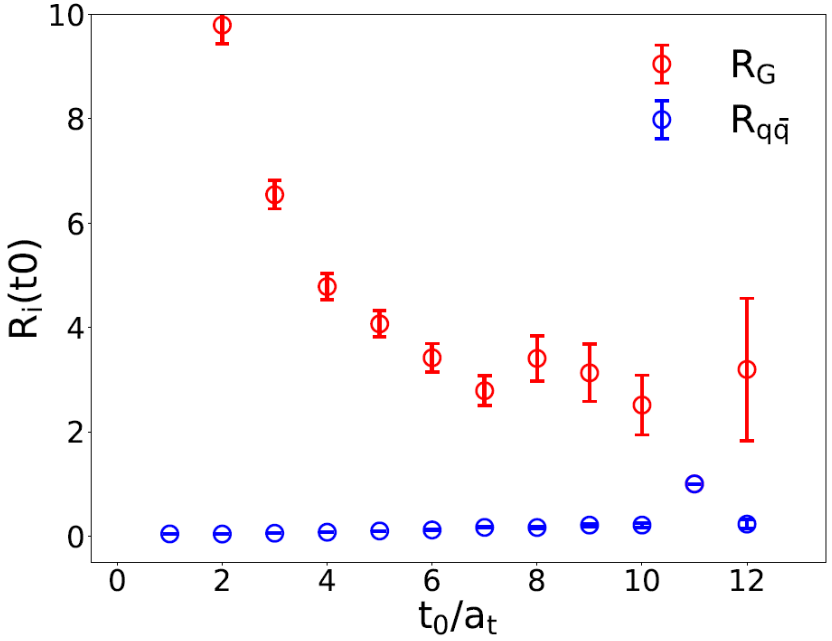


(d) connected and disconnected

More about scalar channel

$$R_i(t_0) = \frac{\langle 0|O_i|1^{st}(t_0)\rangle}{\langle 0|O_i|2^{nd}(t_0)\rangle},\ O_i = O_G^{(1)},\ O_{q\bar{q}}$$





I=0 scalar channel is much more complicated!

Pseudoscalar and topological charge density

[W.S. et al, *Chin.Phys.C* 42(2018) no 9, 093103]

$\eta(1295)$ $\eta(1405)$ $\eta(1475)$

[A.Masoni *et al*, *J.Phys.* **G 32**, R293(2006)]...

may contain glueball state

[J.-J. Wu *et al*, *Phys.Rev.Lett* **108**, 081803(2012)]

same state $\eta(1440)$

Pseudoscalar and topological charge density

[W.S. et al, *Chin.Phys.C* 42(2018) no 9, 093103]

- $U(1)_A$ anomaly gives

$$\partial_\mu A^\mu(x) = 2mP(x) - N_f q(x)$$

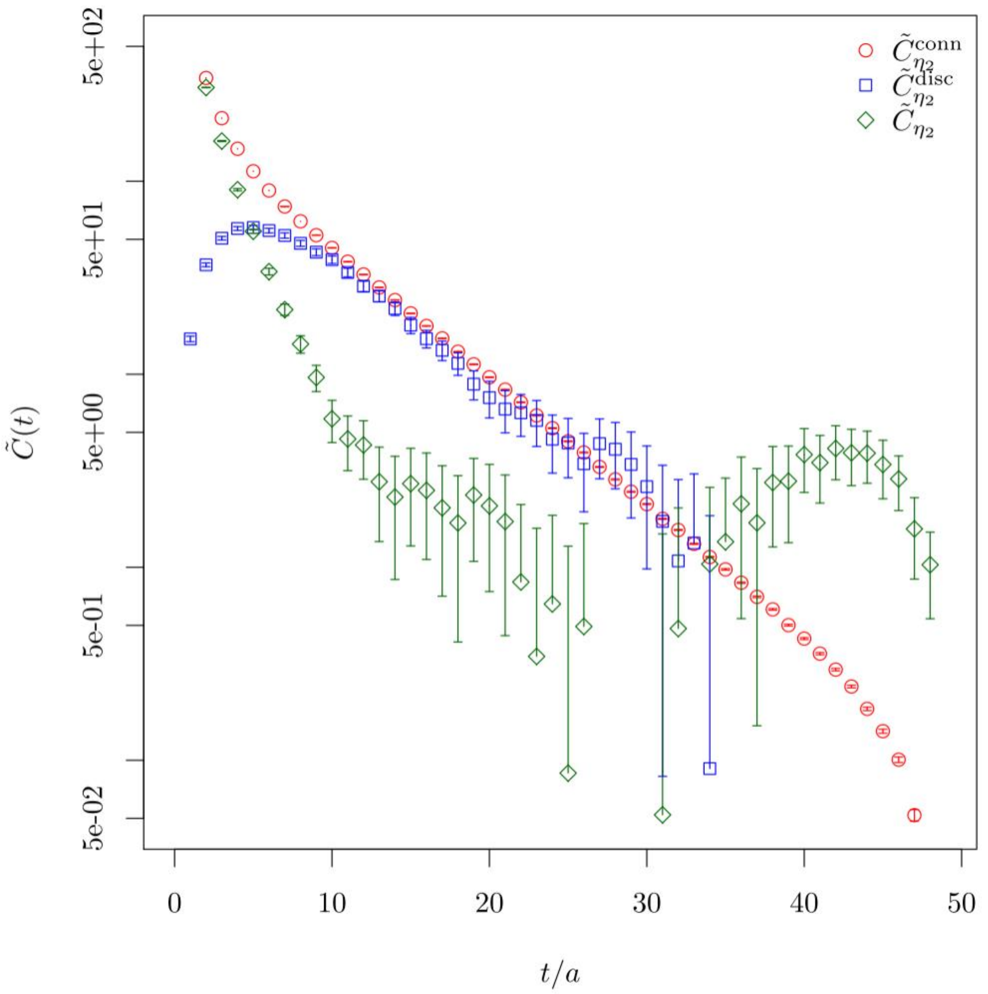
- P(x):flavor singlet pseudoscalar density

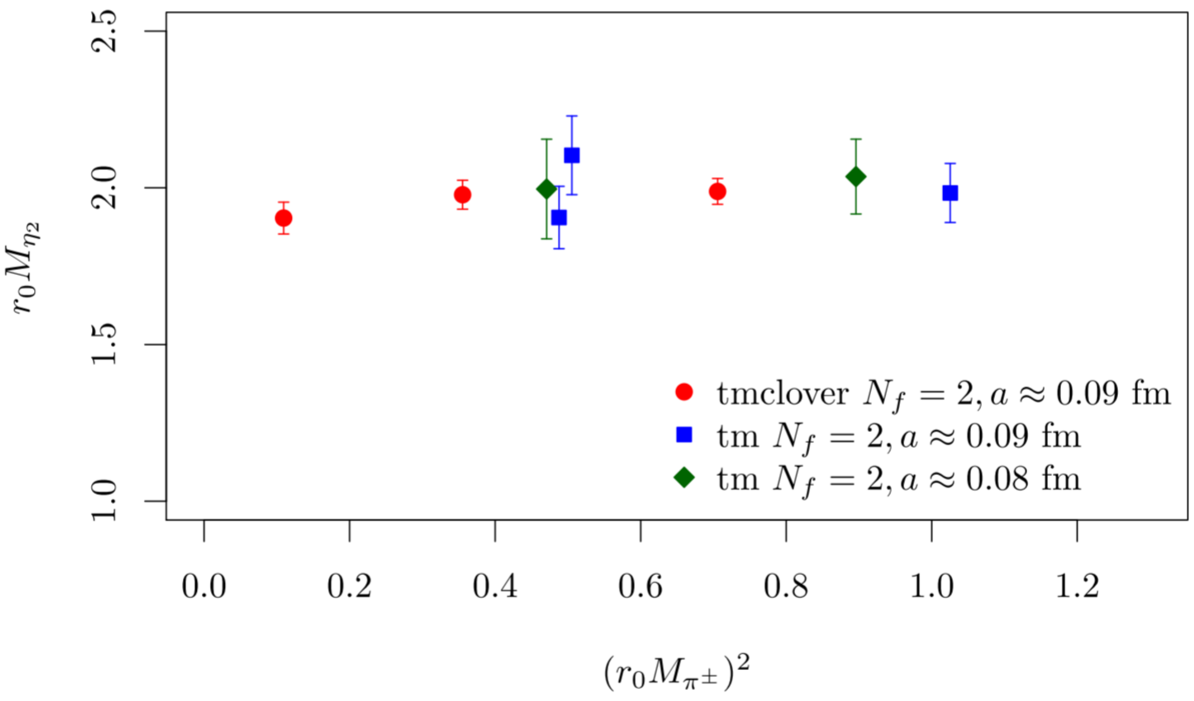
$$P(x) = \psi(\bar{x})\gamma_5\psi(x)$$

- q(x):topological charge density

$$q(x) = \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}TrF_{\mu\nu}F_{\rho\sigma}$$

Correlation of P(x) (N_f=2) [C.Helmes et al, *EPJ Web Conf.*175(2018)05025]



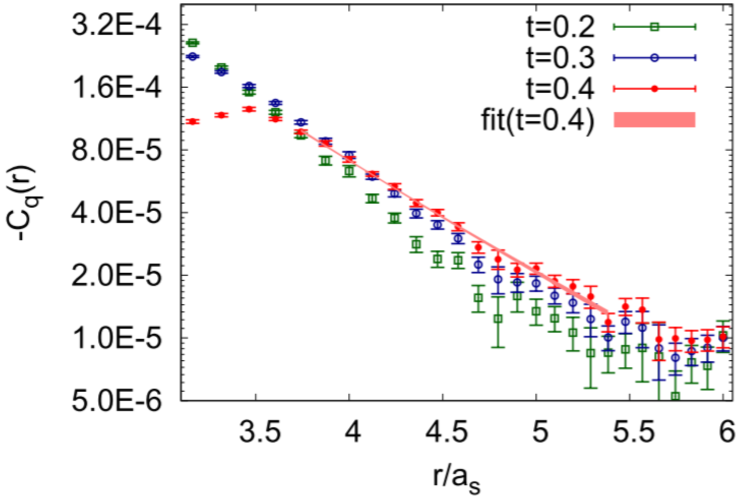


- $r_0 = 0.4907(5)$ fm gives

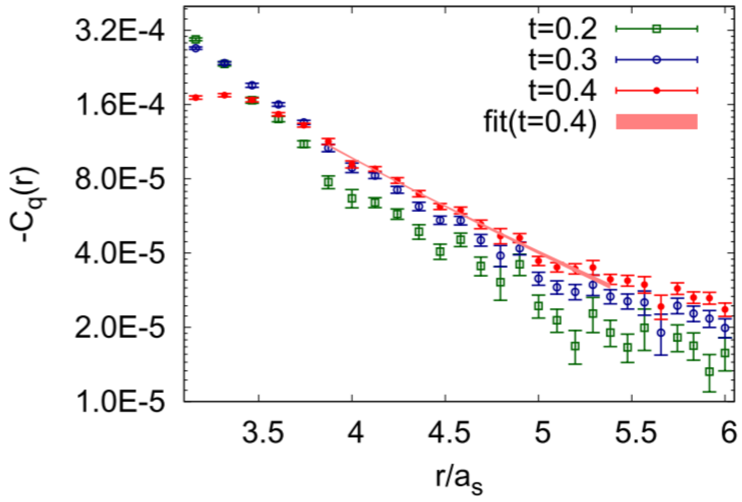
$M_{\eta_2} = 768(24) MeV$

Correlation of q(x) ($N_f = 2$)

[W.S. et al, *Chin.Phys.C* 42(2018) no 9, 093103]



(a)$m_\pi \sim 938$ MeV



(b)$m_\pi \sim 650$ MeV

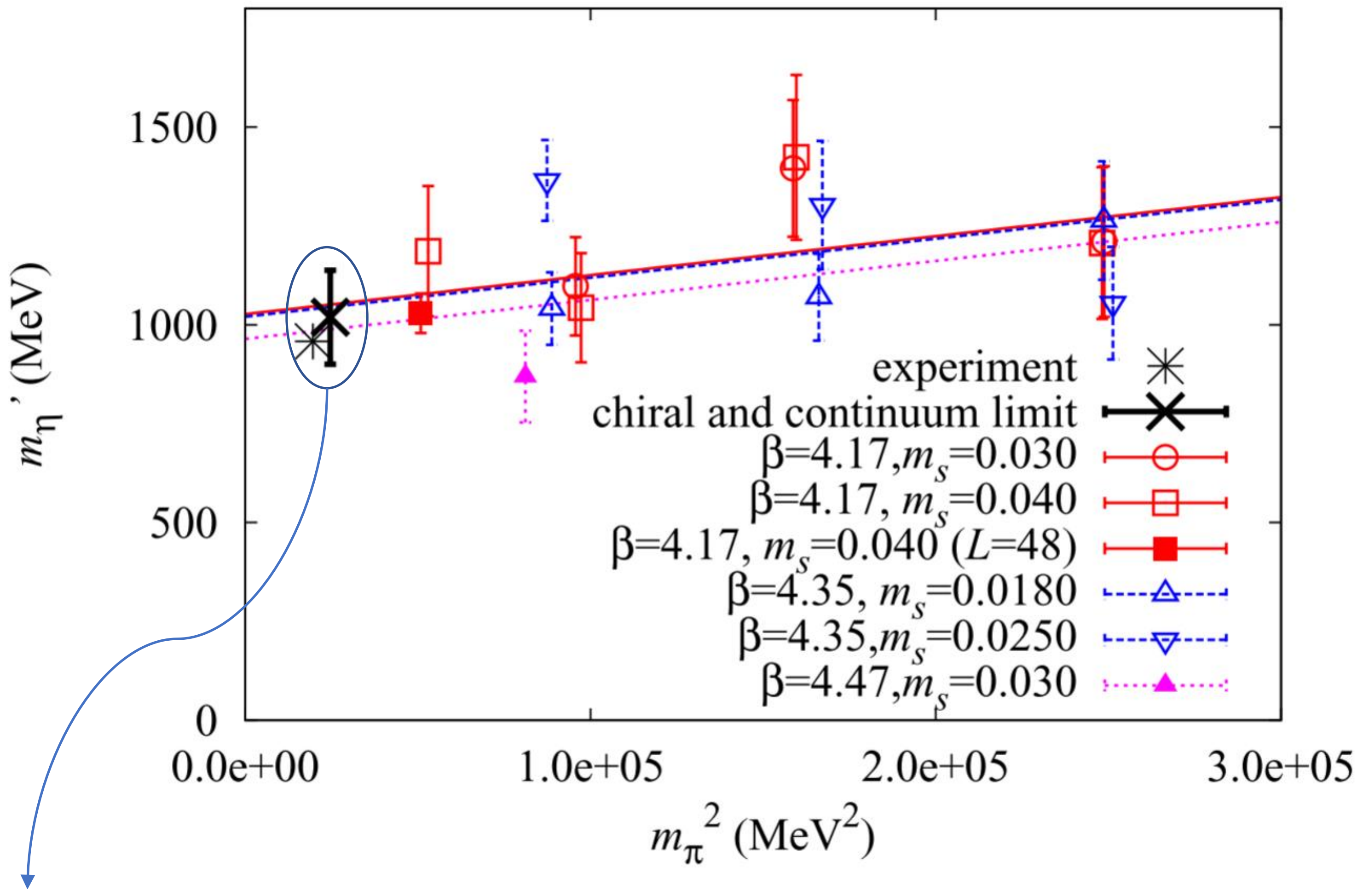
$$C_q(x-y) = \langle q(x) q(y) \rangle = A\delta^4(x-y) - \bar{C}_q(x-y)$$

$$\bar{C}_q(r) = N\frac{m_{\text{PS}}}{4\pi^2 r} K_1(m_{\text{PS}} r)$$

use Wilson gradient flow

m_π	fit range(a_s)	$m_{\eta'} a_s$	$m_{\eta'}$(MeV)	χ^2/dof
938 MeV	3.74-5.92	0.856(21)	1481(36)	1.01
650 MeV	3.87-5.48	0.514(22)	890(38)	1.43

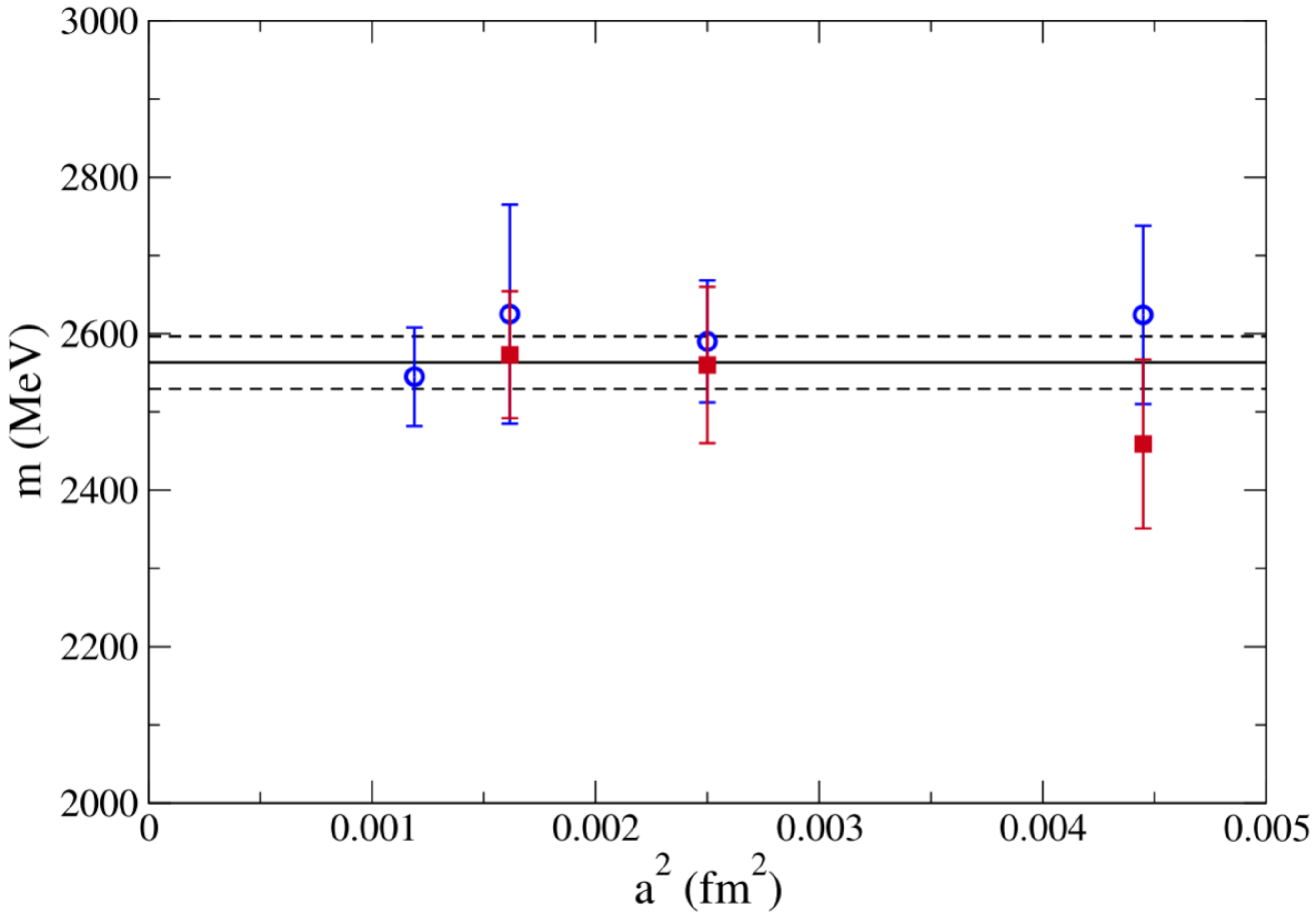
Correlation of q(x) (N_f=2+1) [H. Fukaya et al, *Phy. Rev.* **D92 (R)**, 111501 (2015)]



agrees with physical η'

Correlation of q(x) (quenched)

[A. Chowdhury et al., *Phys. Rev.* **D 91** (2015)074507]



agrees with quenched pseudoscalar glueball

Summary of pseudoscalar results

	$P(x)$	$q(x)$	O_G
$N_f = 0$	——	2563(34)MeV A.Chowdhury, PRD91(2015)	2590(40)(130)MeV Y.Chen, PRD73(2006)
$N_f = 2$	768(24)MeV C.Urbach, Lattice2017	890(38)MeV this work $m_\pi = 650$MeV	2605(52)MeV this work $m_\pi = 650$MeV
$N_f = 2+1$	947(142)MeV N.Christ, PRL105(2010)	1019(119)MeV JLQCD, PRD92(2015)	——
$N_f = 2+1+1$	1006(54)(38)MeV C.Michael, PRL111(2013)	——	——

- P(x): $\bar{\psi}\gamma_5\psi$
- q(x): topological charge density
- O_G: glueball operators

Continuum form of operators

- The continuum form of our pseudoscalar glueball operator is

$$\phi^{A_1^{-+}}(\mathbf{x},t) \sim \epsilon_{ijk} Tr B_i(\mathbf{x},t) D_j B_k(\mathbf{x},t) + O(a_s^2)$$

- Topological charge density operator goes like

$$q(x) \propto \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \propto \mathbf{E}(x) \cdot \mathbf{B}(x)$$

- The large difference of our glueball and $\eta'(\eta_2)$ mass can be explained.

Summary

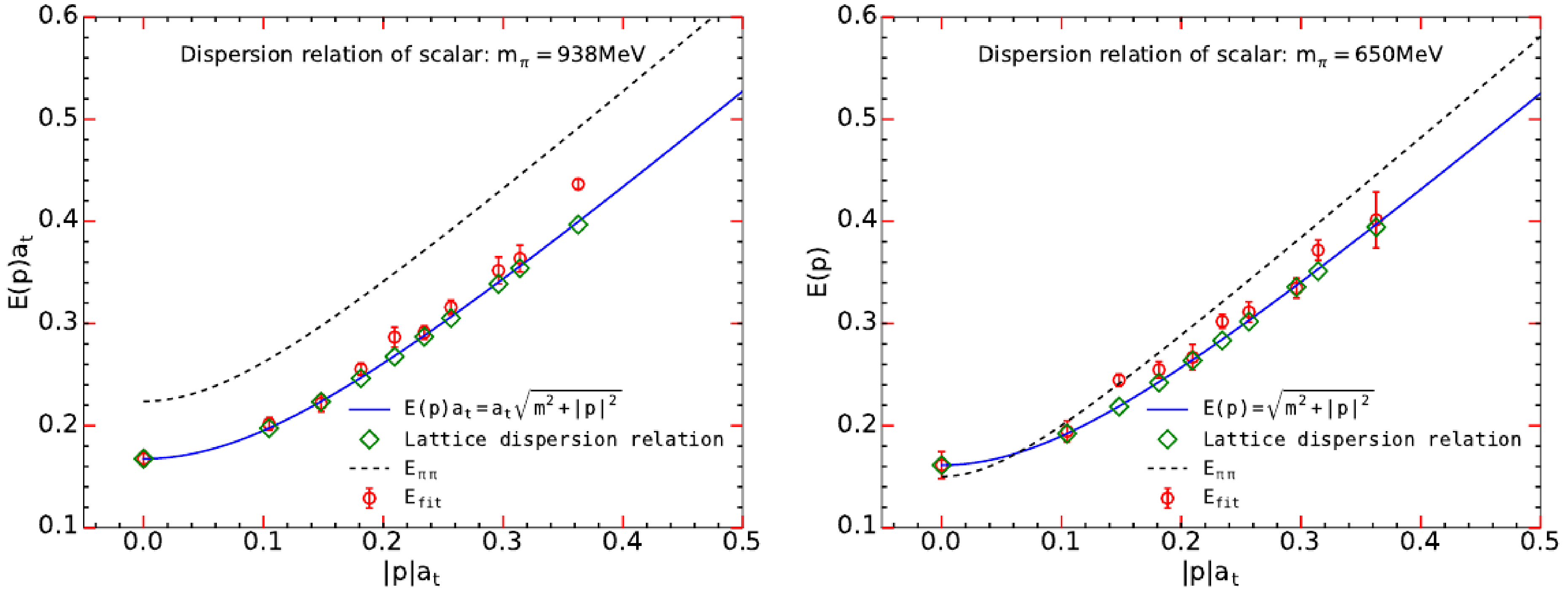
- Ground state I=0 scalar, pseudoscalar and tensor states investigated by “glueball” operator
- large difference between topological charge density and pseudoscalar glueball operator
- Still no definite answer on the precise spectrum
- **Need next:**
  - **Control systematic errors(continuum limit, infinite volume, physical quark mass…)**
  - **Include multi-hadron operators(especially for scalar channel)**

Thank you!

Backup

[W.S. et al, *EPJ Web Conf.*175(2018) 05016]

- Single particle or multi-particle state?
- Dispersion relation of one-particle and lowest free two pion state



backup

Lattice study on J/ψ radiatively decay into glueballs

- From experimental $J/\psi \longrightarrow \gamma X \longrightarrow \gamma\pi\pi$, $X \longrightarrow \pi\pi$ we can estimated

$$Br(J/\psi \longrightarrow \gamma f_0(1710)) = 2.5 \times 10^{-3}$$

$$Br(J/\psi \longrightarrow \gamma f_0(1500)) = 3.1 \times 10^{-4}$$

- Quenched LQCD predicted the J/ψ radiatively decay into scalar glueball with branching ratio :

  [L.C Gui et al, CLQCD Collaboration, *Phys.Rev.Lett.* **110** (2013) no.2, 021601]

$$Br(J/\psi \longrightarrow \gamma G_{0^{++}}) = 3.8(9) \times 10^{-3},$$

  which suggested the $f_0(1710)$ as scalar glueball candidate.

- J/ψ radiative decay into tensor glueball gives

  [Yi-Bo Yang et al, CLQCD Collaboration, *Phys. Rev. Lett.* **111**, 091601 (2013)]

$$Br(J/\psi \longrightarrow \gamma G_{2^{++}}) = 1.1(2) \times 10^{-2}$$