A Better Angle on Hadron Transverse Momentum Distributions At the EIC

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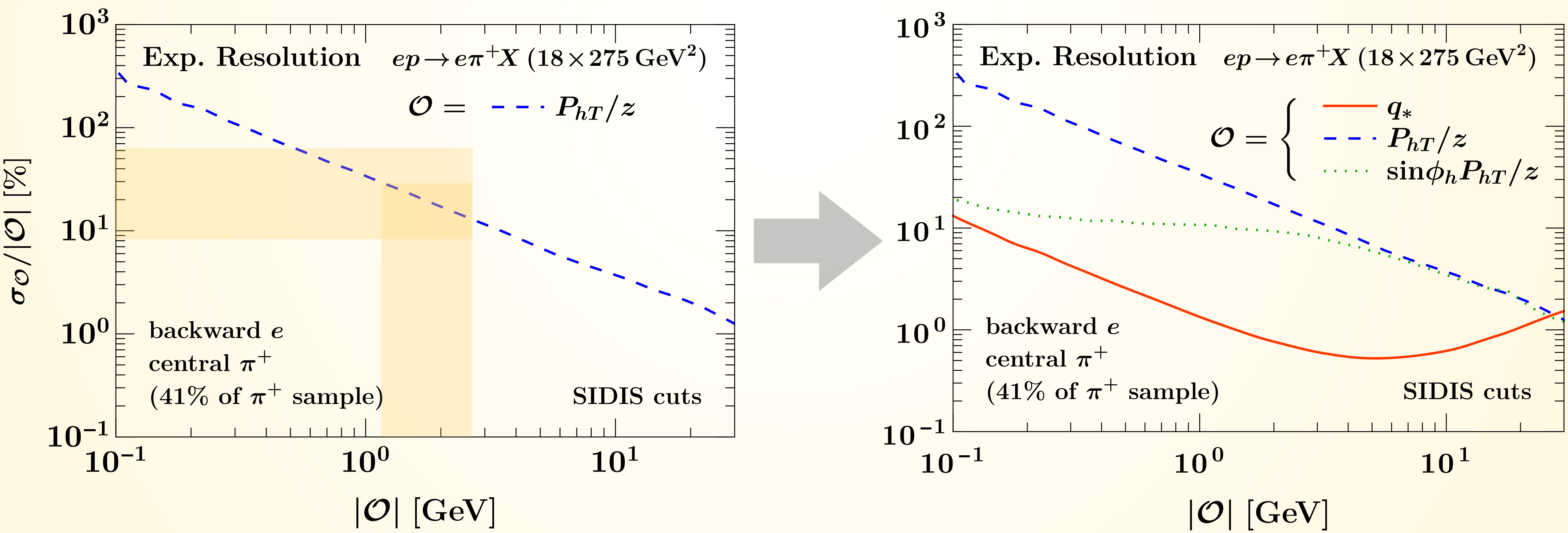
2209.11211

Jefferson Lab, November 7th, 2022

PUNCHLINE

- A new observable q_* for **SIDIS** to study **TMDs** with order of magnitude improvement in experimental resolution!



Outline

- Motivation & introduction to TMDs
- How to study TMDs
- Construction of a new observable q_*
- Factorization theorem for q_* cross section
- Experimental sensitivity and robustness
- Power corrections
- Summary

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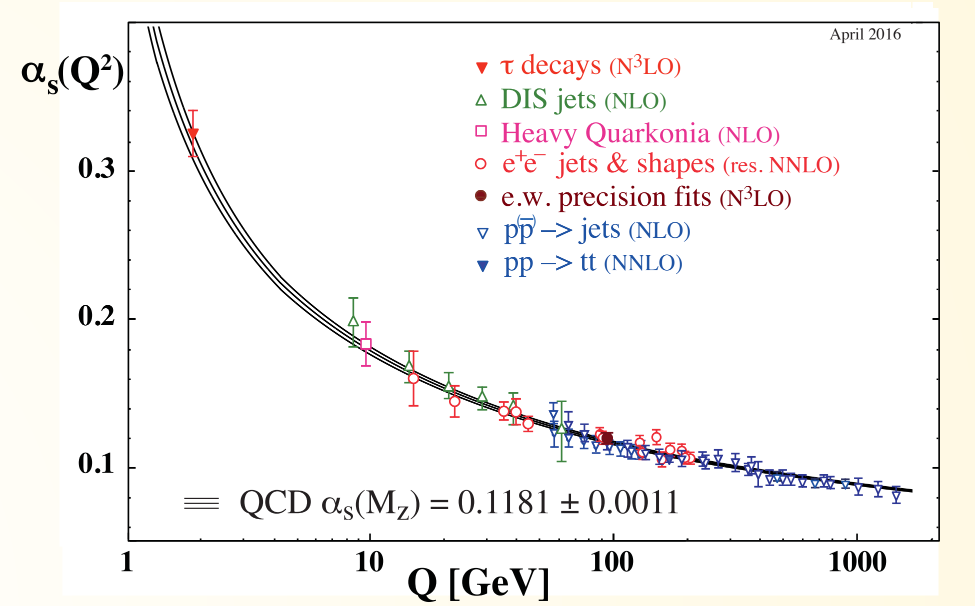
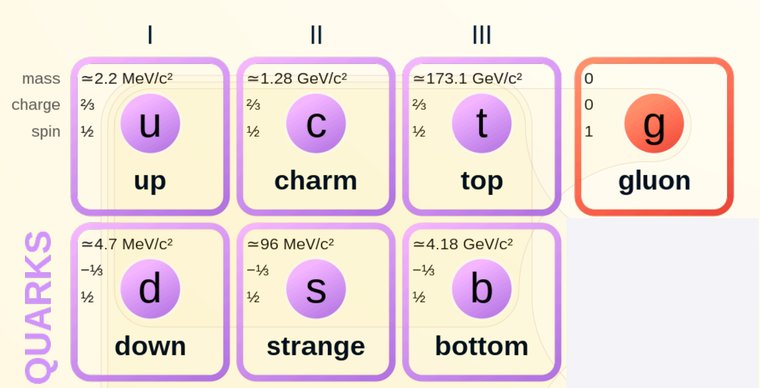
QCD IS HARD

- Nonperturbative:
  coupling $\alpha_s(Q)$ explodes at low energy
  perturbative calculation fails



- Confinement:
  we see no free quarks or gluons
  can not measure their properties directly

- Hadronization:
  we do not fully understand the real-time dynamic of how quarks and gluons become bound states



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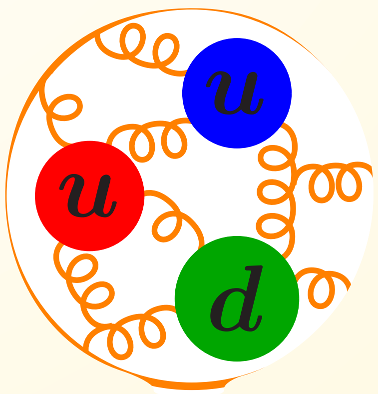
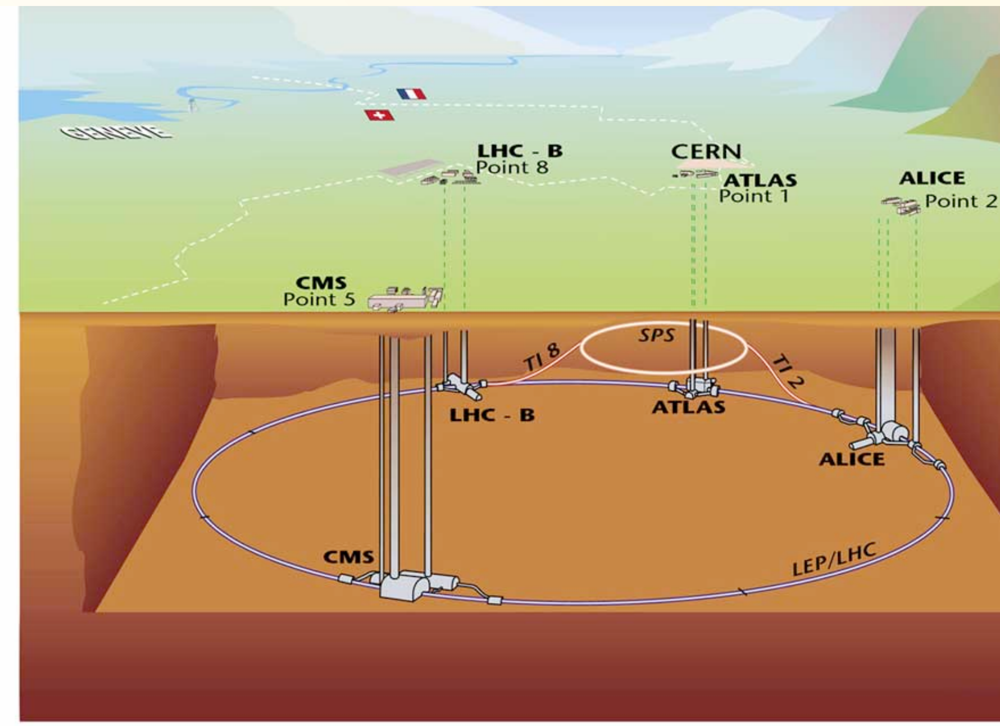


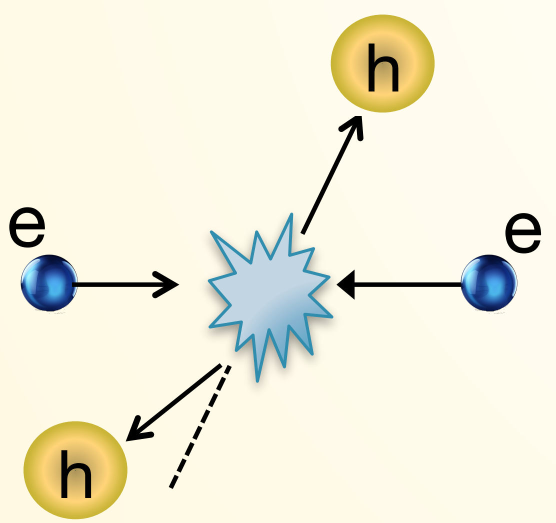
One of our Best Tools: Colliders!

e^+e^- (LEP expt.)

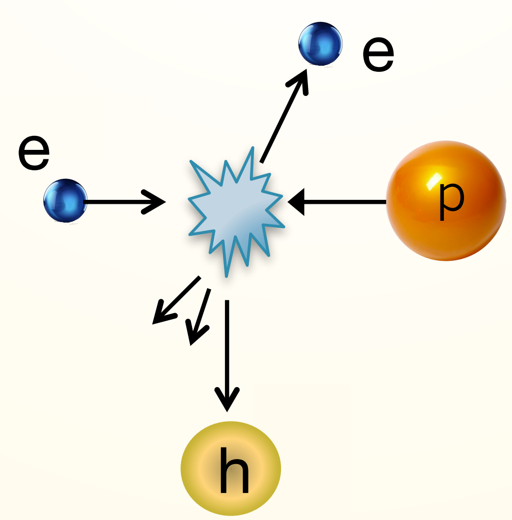
e^-p (Jefferson lab)

pp (Large Hadron Collider)

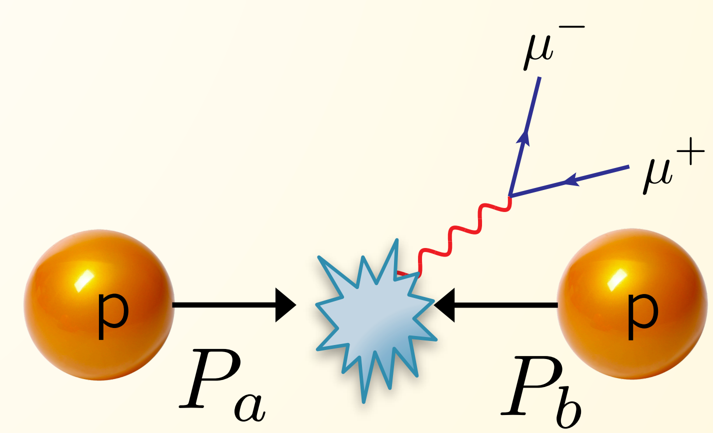




Dihadron in e+e-



Semi-Inclusive DIS

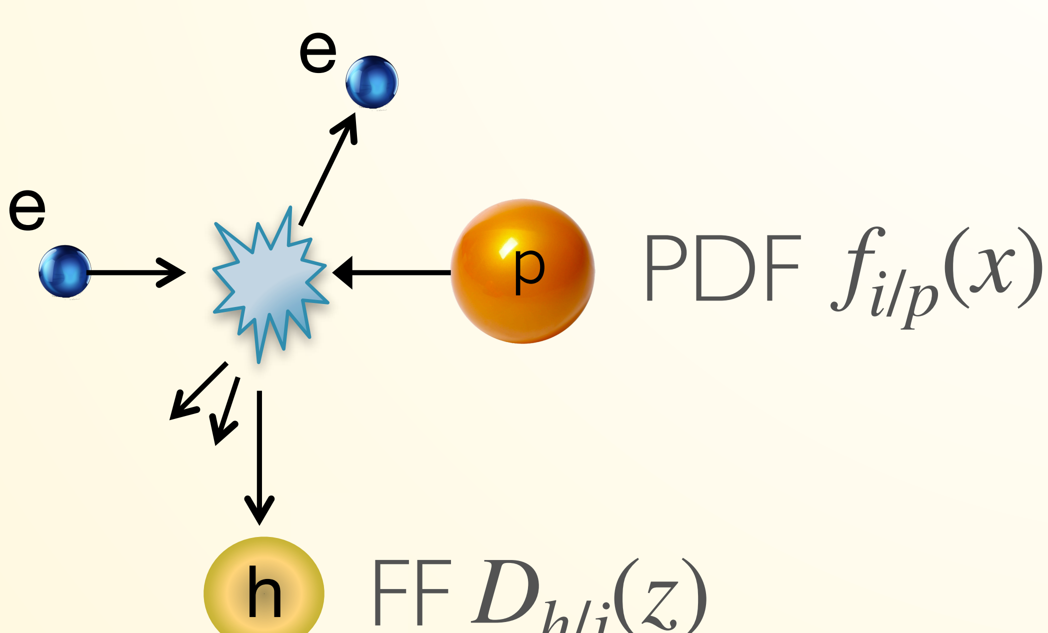


Drell-Yan

Parton Distribution Functions & Fragmentation Functions

- Parton distribution function (**PDF**):
  probability of finding a parton i with collinear momentum fraction x inside a hadron H

- Fragmentation function (**FF**):
  probability that a parton i with collinear momentum fraction z hadronizes to a hadron H

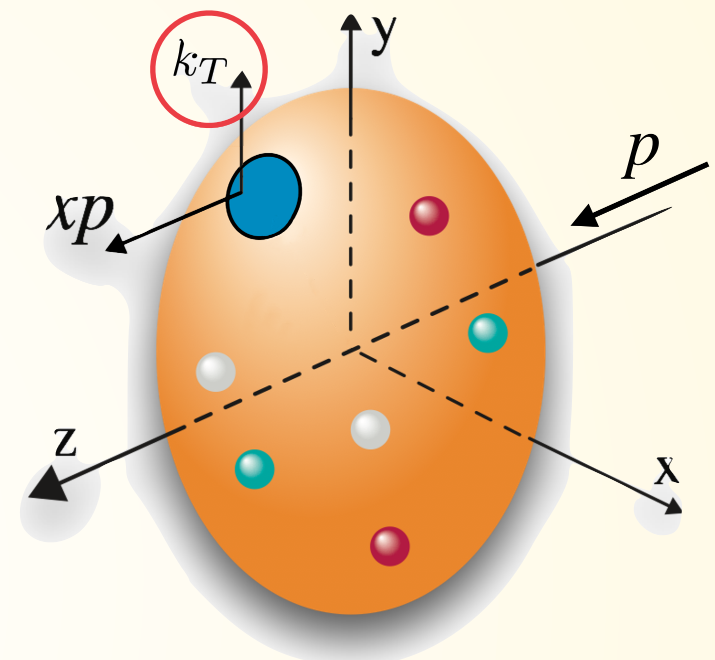


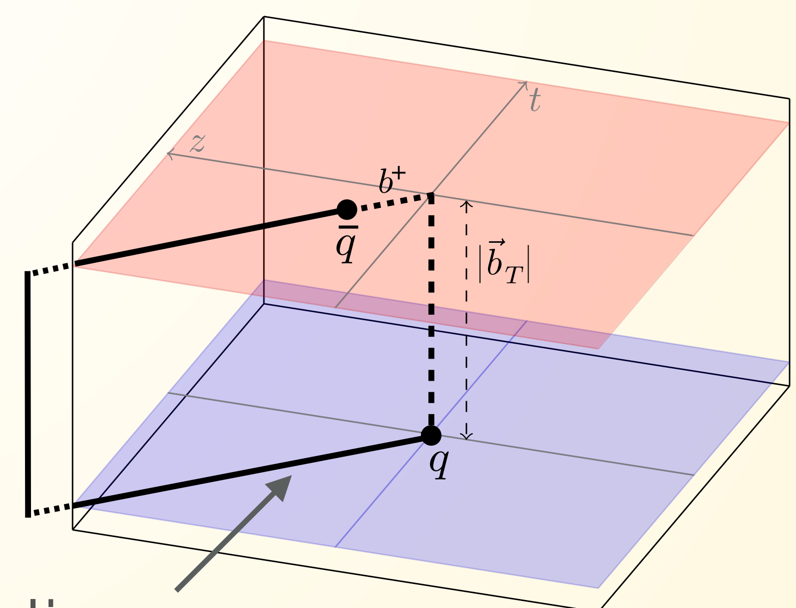
- Can **factorize** cross section of a physical process (e.g. SIDIS) in terms of PDFs and FFs

$$\sigma \sim f_{i/p}(x)\, D_{h/j}(z)$$ (more later)

Transverse Momentum Dependent PDFs/FFs

- Same interpretation, but now the distribution also depends on the **transverse momentum** k_T of the parton
- Describes the **3D structure** of hadrons
- Can be rigorously defined by matrix elements of operators



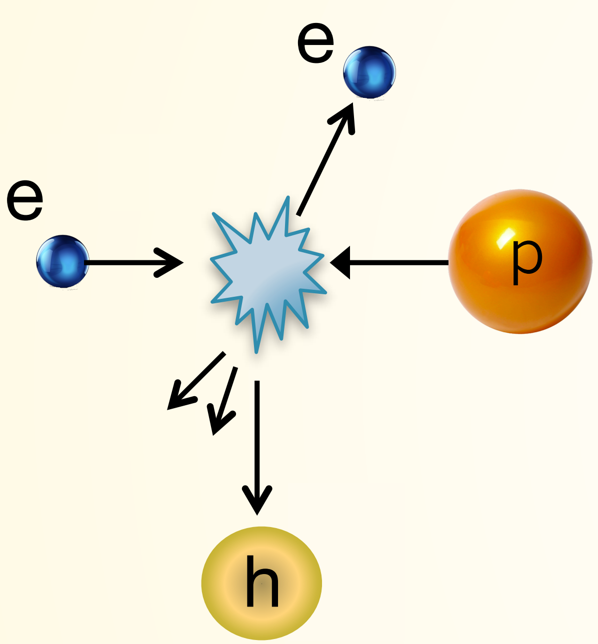


Wilson lines

TMDs in Different Processes

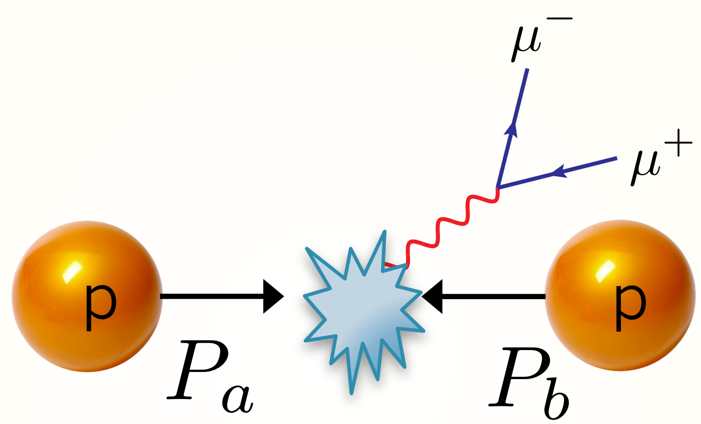
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(z, k_T)$$



Drell-Yan

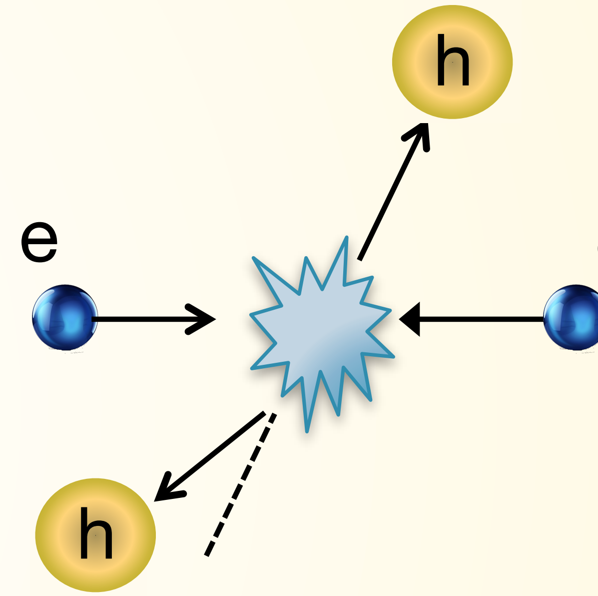
$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



$$q_T \ll Q$$

Dihadron in e+e-

$$\sigma \sim D_{h/q}(z, k_T) D_{h/q}(z, k_T)$$

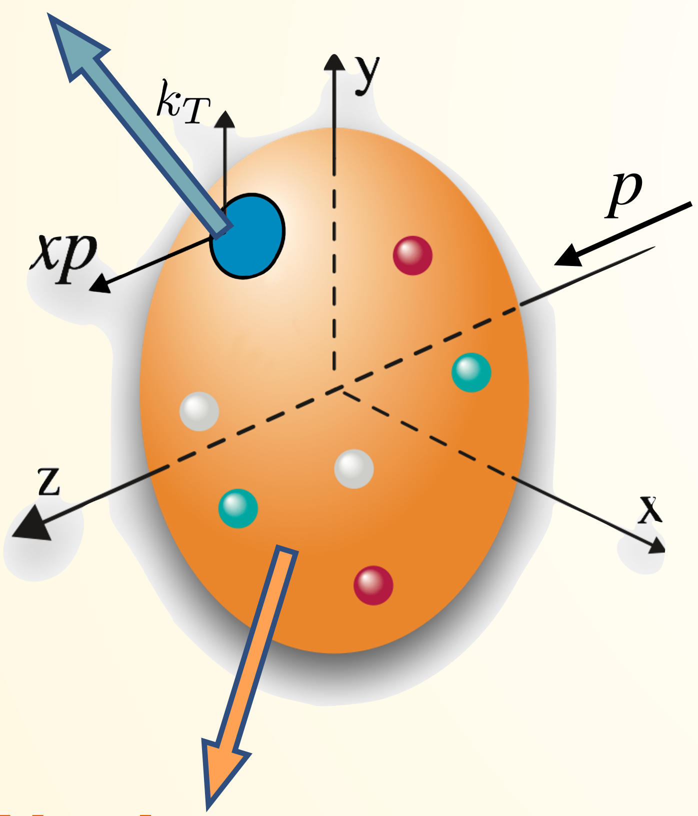


- TMDs are **universal** across processes
- **Two scales** q_T, Q allows natural power counting

Spin-dependent TMDs

- 8 TMD PDFs with polarizations, similar for TMD FFs

Quark Polarization Γ



Nucleon Polarization

$S_L, \vec{S}_T$

Leading Quark TMDPDFs (Nucleon Spin, Quark Spin)

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

Why study TMDs

- Understand the 3D structure of hadrons
- Understand the nonperturbative structures of QCD as a field theory (confinement, hadronization, …)
- Improve QCD theory uncertainty in other processes (W mass measurement, …)
- Precision in experiments allow discovery of new physics (Higgs transverse momentum spectrum, …)
- ……

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SIDIS Setup

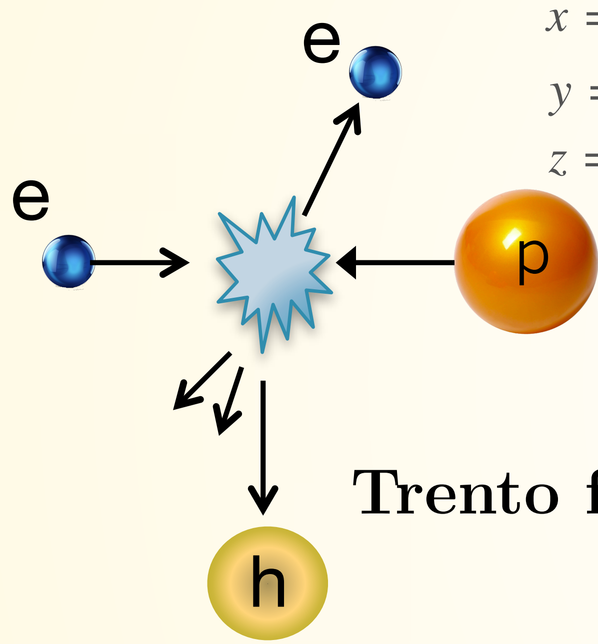
Semi-Inclusive DIS

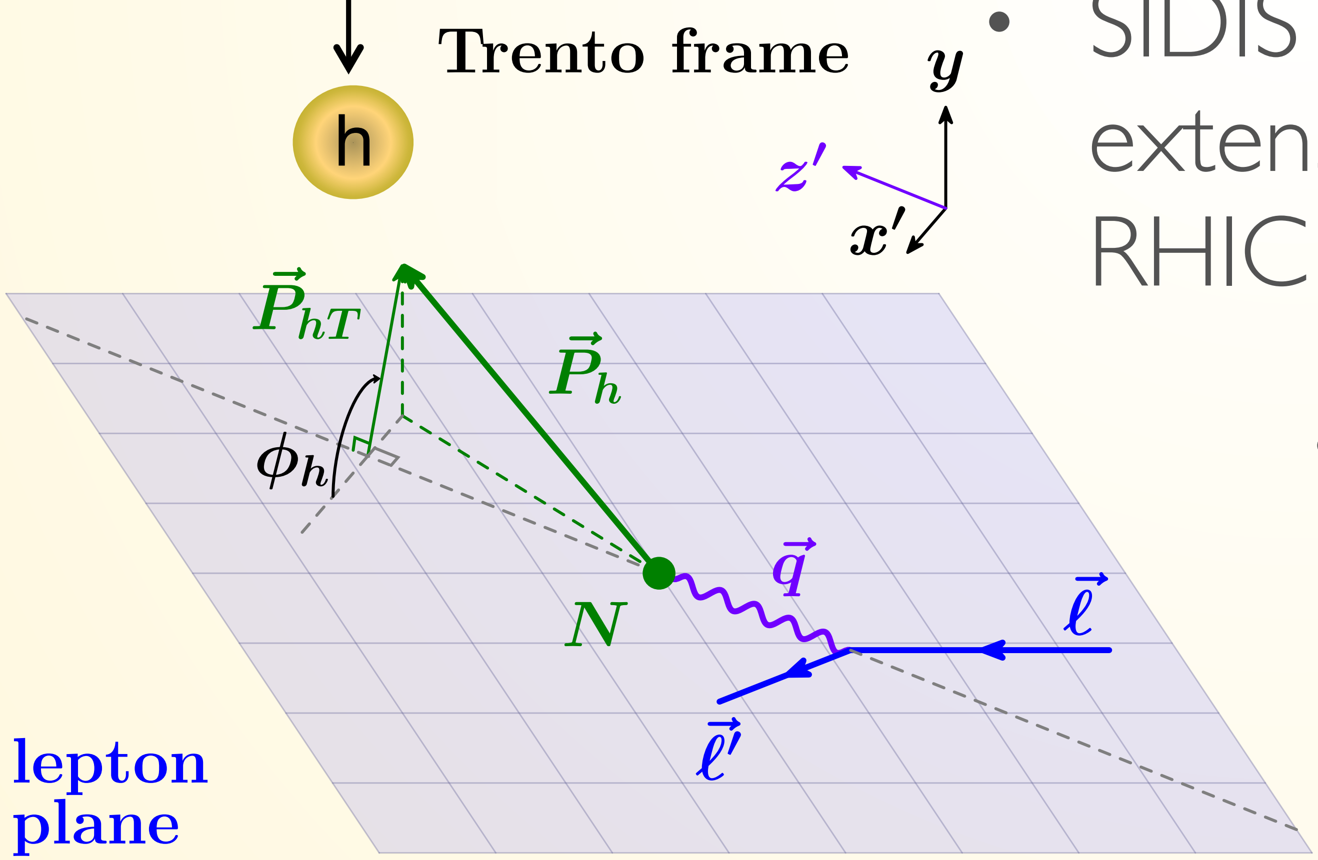
$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(z, k_T)$$

$x = Q^2/(2P \cdot q)$

$y = P \cdot q / P \cdot \ell$

$z = P \cdot P_h / P \cdot q$





- $e^-(\ell) + N(P) \rightarrow e^-(\ell') + h(P_h) + X$
- Interested in the **transverse momentum** of the **outgoing hadron** with respect to the photon, P_{hT}
- SIDIS cross section is studied extensively at HERMES, COMPASS, RHIC, and JLab
    - The upcoming Electron-Ion Collider (EIC) promises **improved precision**

How TMDs are studied

- In the limit $P_{hT}/z = q_T \ll Q$, can **factorize** the cross section into hard function and TMDs:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}^2\vec{P}_{hT}} = \sigma_0 2z \sum_{f,\bar{f}} \mathcal{H}_f(Q^2) \int_0^\infty \frac{\mathrm{d}b_T\, b_T}{2\pi} e^{i\vec{b}_T\cdot\vec{q}_T} f_{1f}(x,b_T) D_{1f}(z,b_T) \times [1+\mathcal{O}(\frac{q_T^2}{Q^2})]$$

(unpolarized)

- TMDs have perturbative and **nonperturbative** parts:

$$\tilde{f}_{1f}(x,b_T,\mu,\zeta) = \tilde{f}_{1f}(x,b^*(b_T),\mu,\zeta)\, \tilde{f}_1^{\mathrm{NP}}(x,b_T)$$
$$\tilde{D}_{1f}(z,b_T,\mu,\zeta) = \tilde{D}_{1f}(z,b^*(b_T),\mu,\zeta)\, \tilde{D}_1^{\mathrm{NP}}(z,b_T)$$

Perturbative calculation & evolution

Nonperturbative **model**

Leading Power Factorization Theorem

X.-d. Ji, J.-P. Ma, and F. Yuan arXiv:hep- ph/0404183, arXiv:hep-ph/0405085
A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders, and M. Schlegel, arXiv:hep-ph/0611265

- Factorization theorem relates **cross sections to TMDs**

$$\frac{d\sigma}{dx\,dy\,dz\,d^2\vec{P}_{hT}} = \sigma_0\Big\{W_{UU,T} + \lambda_e S_L\sqrt{1-\epsilon^2}\,W_{LL} + \epsilon\cos(2\phi_h)W_{UU}^{\cos(2\phi_h)} + S_L\epsilon\sin(2\phi_h)W_{UL}^{\sin(2\phi_h)} + S_T\sin(\phi_h-\phi_S)W_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon S_T\Big[\sin(\phi_h+\phi_S)\times W_{UT}^{\sin(\phi_h+\phi_S)} + \sin(3\phi_h-\phi_S)W_{UT}^{\sin(3\phi_h-\phi_S)}\Big] + \lambda_e S_T\sqrt{1-\epsilon^2}\cos(\phi_h-\phi_S)W_{LT}^{\cos(\phi_h-\phi_S)}\Big\}$$

Notation: M. A. Ebert, A. Gao, and I. W. Stewart, arXiv:2112.07680

λ_e lepton beam helicity
$S^\mu = (0, S_T\cos\phi_S, S_T\sin\phi_S, -S_L)$ nucleon spin vector in Trento frame $\quad y = P\cdot q/P\cdot\ell$
$\epsilon = (1-y)/(1-y+y^2/2)$
$\sigma_0 \equiv \alpha^2\pi y\kappa_\gamma/[zQ^2(1-\varepsilon)]$

Structure functions
⇕
TMD PDFs/FFs

- **For example,**

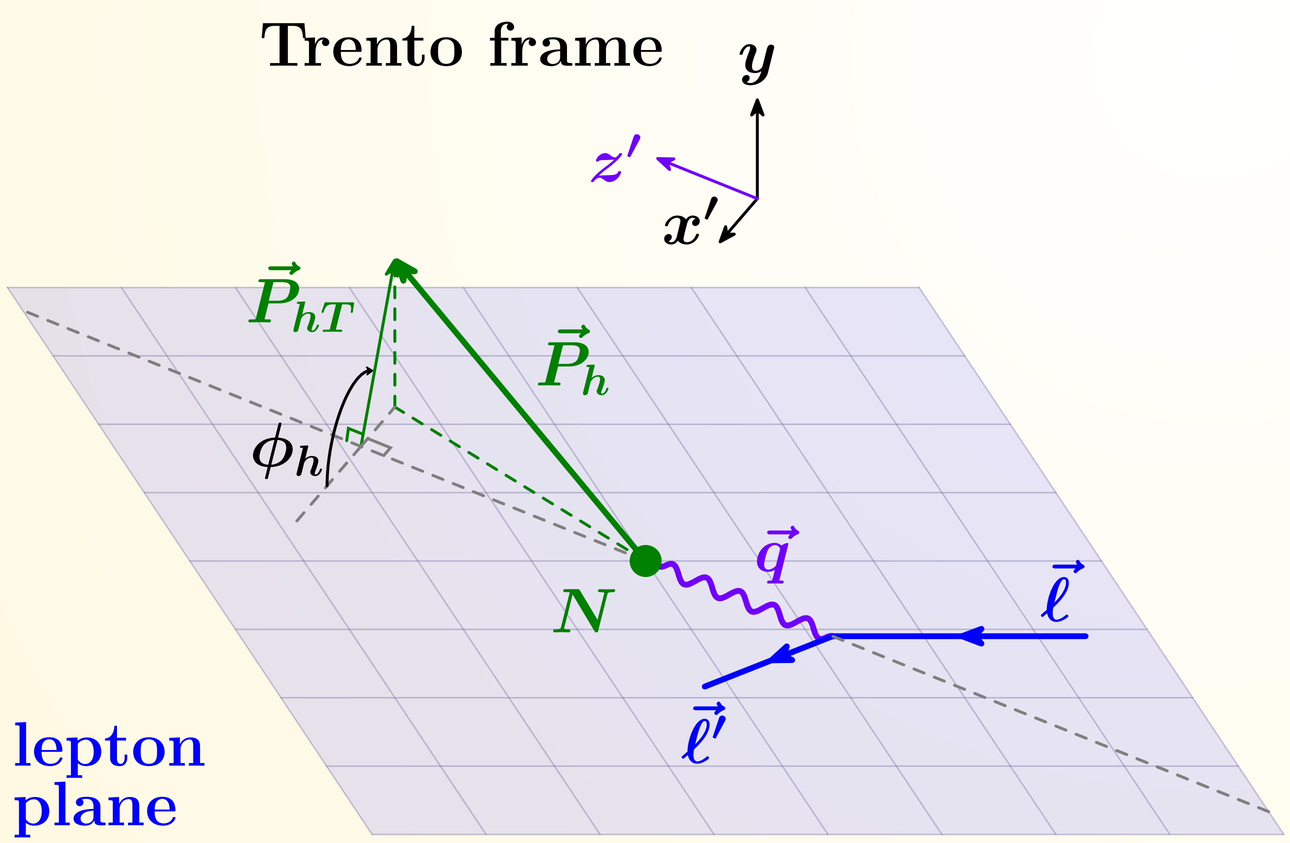
D. Boer, L. Gamberg, B. Musch, and A. Prokudin, arXiv:1107.5294

$$W_{UU}^{\cos(2\phi_h)} = -2z\int_0^\infty \frac{db_T b_T}{2\pi}\,\mathcal{I}\big[\mathcal{H}\,\tilde{h}_1^{\perp(1)}\,\tilde{H}_1^{\perp(1)}\big]\,J_2(b_T P_{hT}/z)$$

where $\mathcal{I}[\mathcal{H}\,\tilde{g}^{(n)}\,\tilde{D}^{(m)}] \equiv (Mb_T)^n(-M_h b_T)^m\sum_f \mathcal{H}_f\,\tilde{g}_f^{(n)}\,\tilde{D}_f^{(m)}$

Challenges of Studying TMDs

- In SIDIS, need to reconstruct $\vec{q}$ from $\vec{\ell}\,'$ to measure $\vec{P}_{hT}$
- **Momentum** resolution at detectors introduces large uncertainty in reconstruction of $P_{hT} \sim \Lambda_{\text{QCD}} \ll Q \sim |\vec{\ell}\,'|$



- **As an example**, consider SIDIS kinematics.

$$|\vec{\ell}\,'| = (20 \pm 0.5)\,\text{GeV}$$

$$\Downarrow$$

$$|\vec{P}_{hT}| = (1 \pm 0.5)\,\text{GeV}$$

which is 50% uncertainty!

Challenges of Studying TMDs

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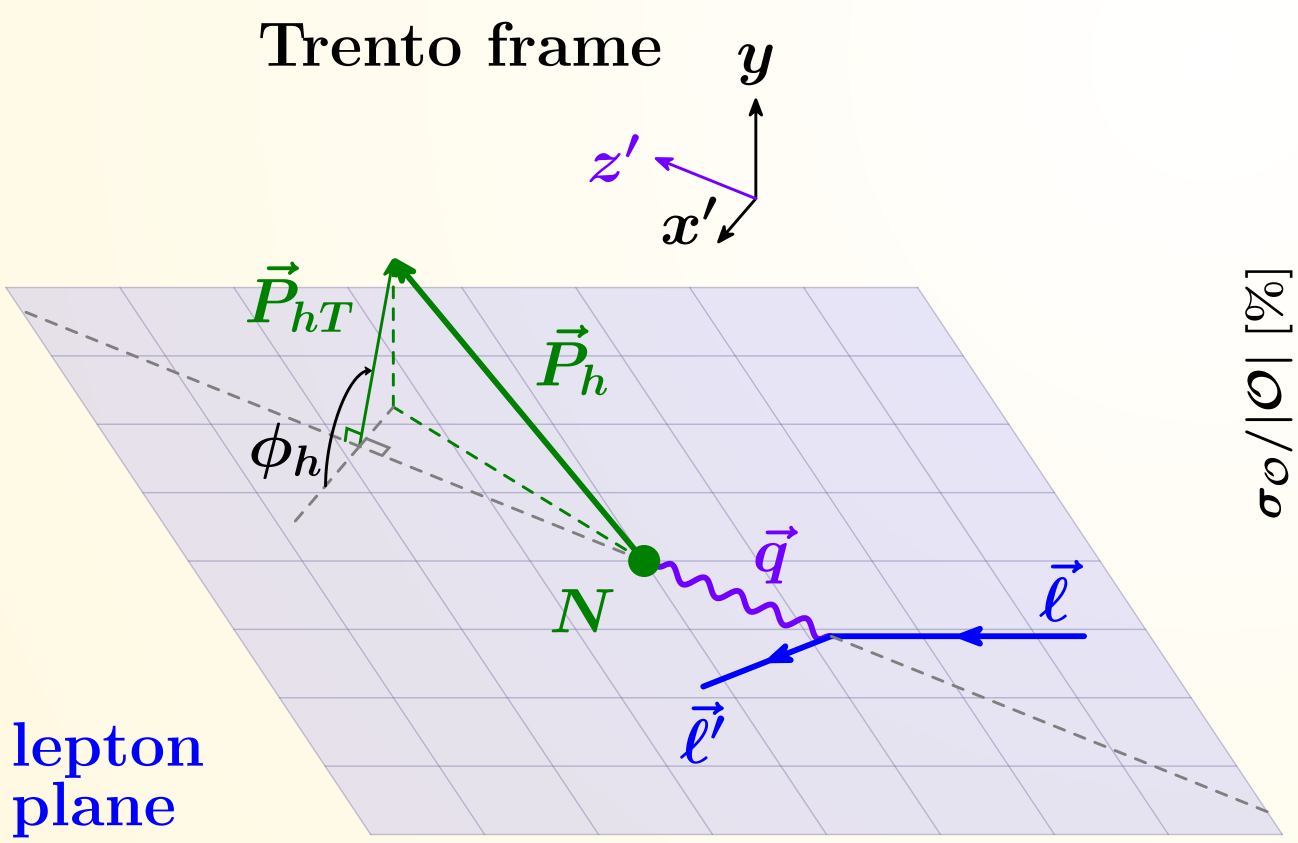
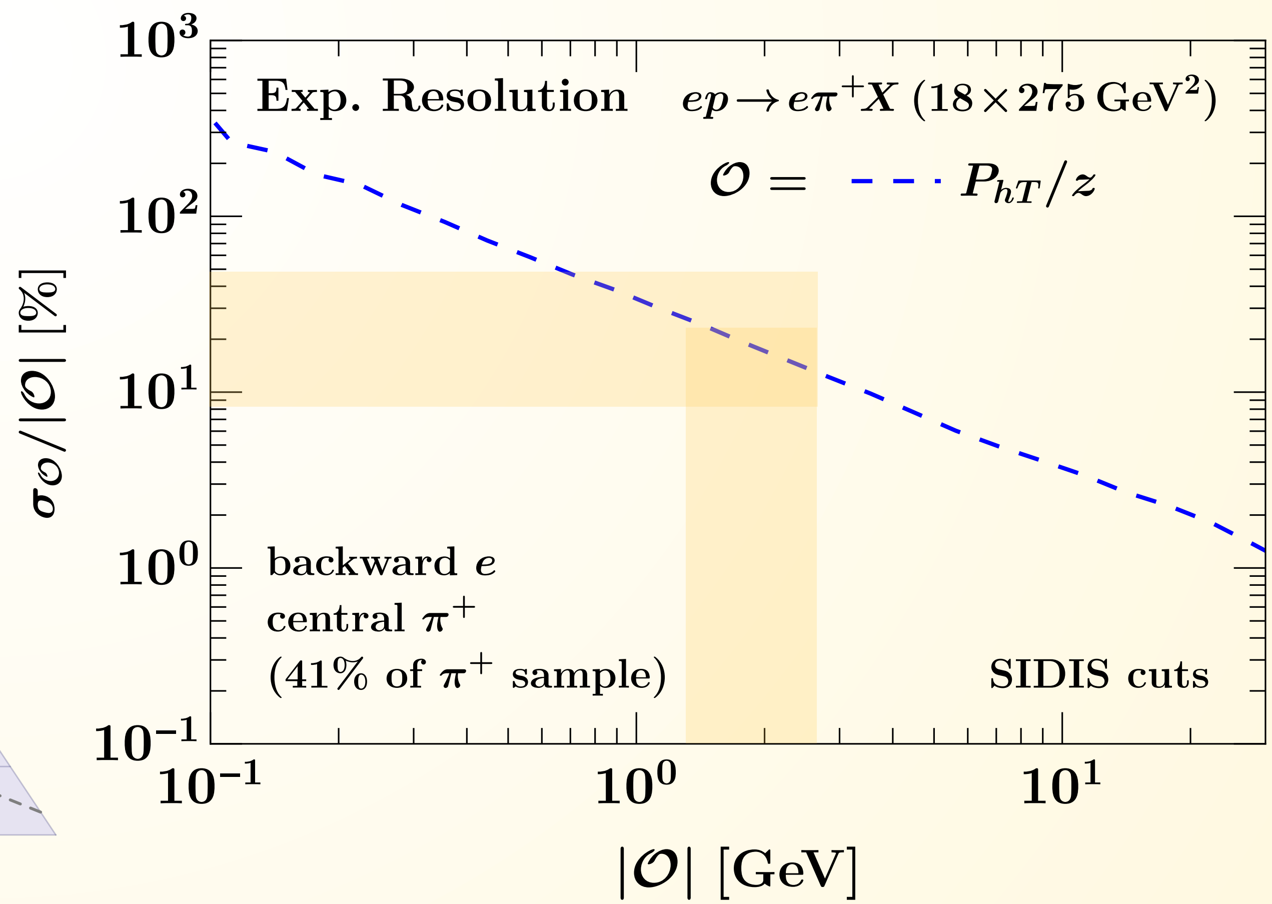
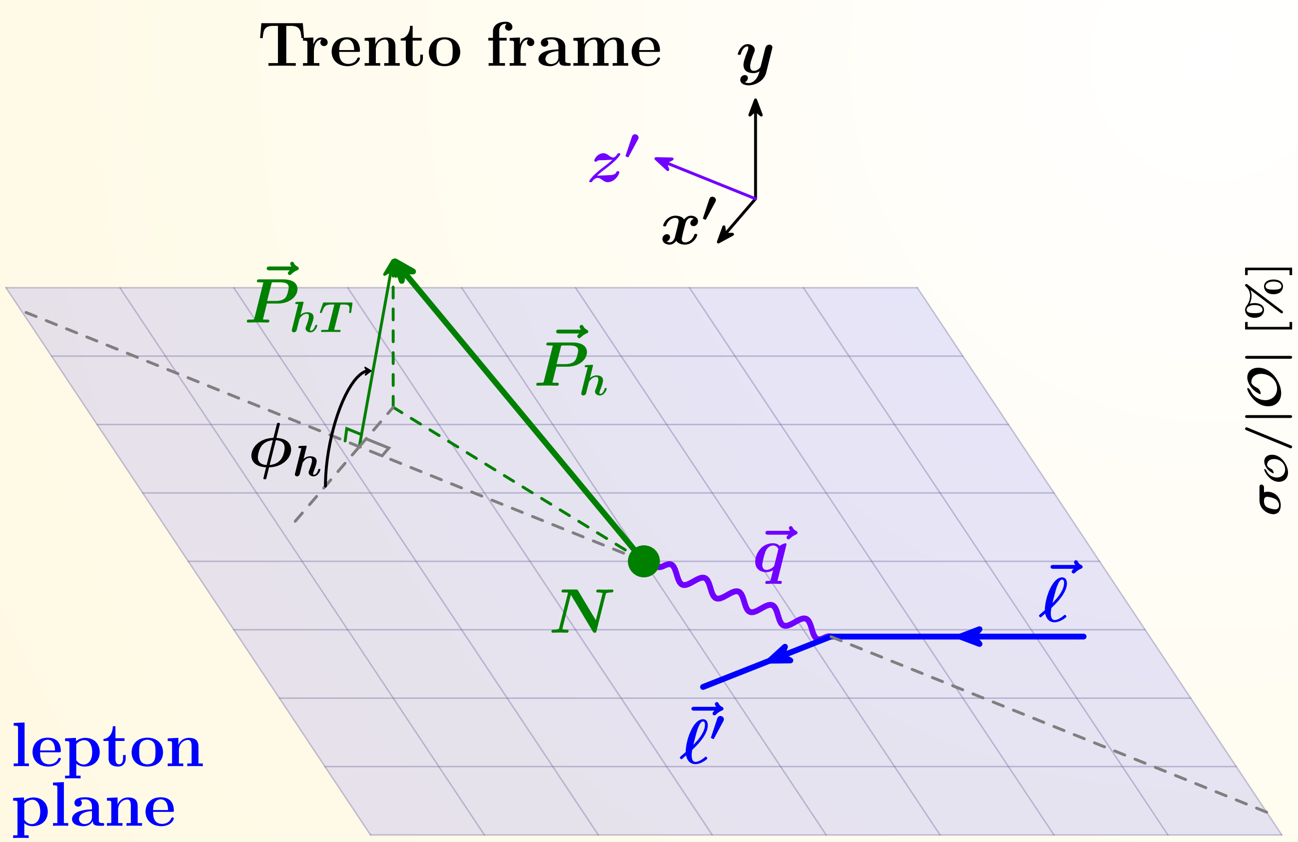


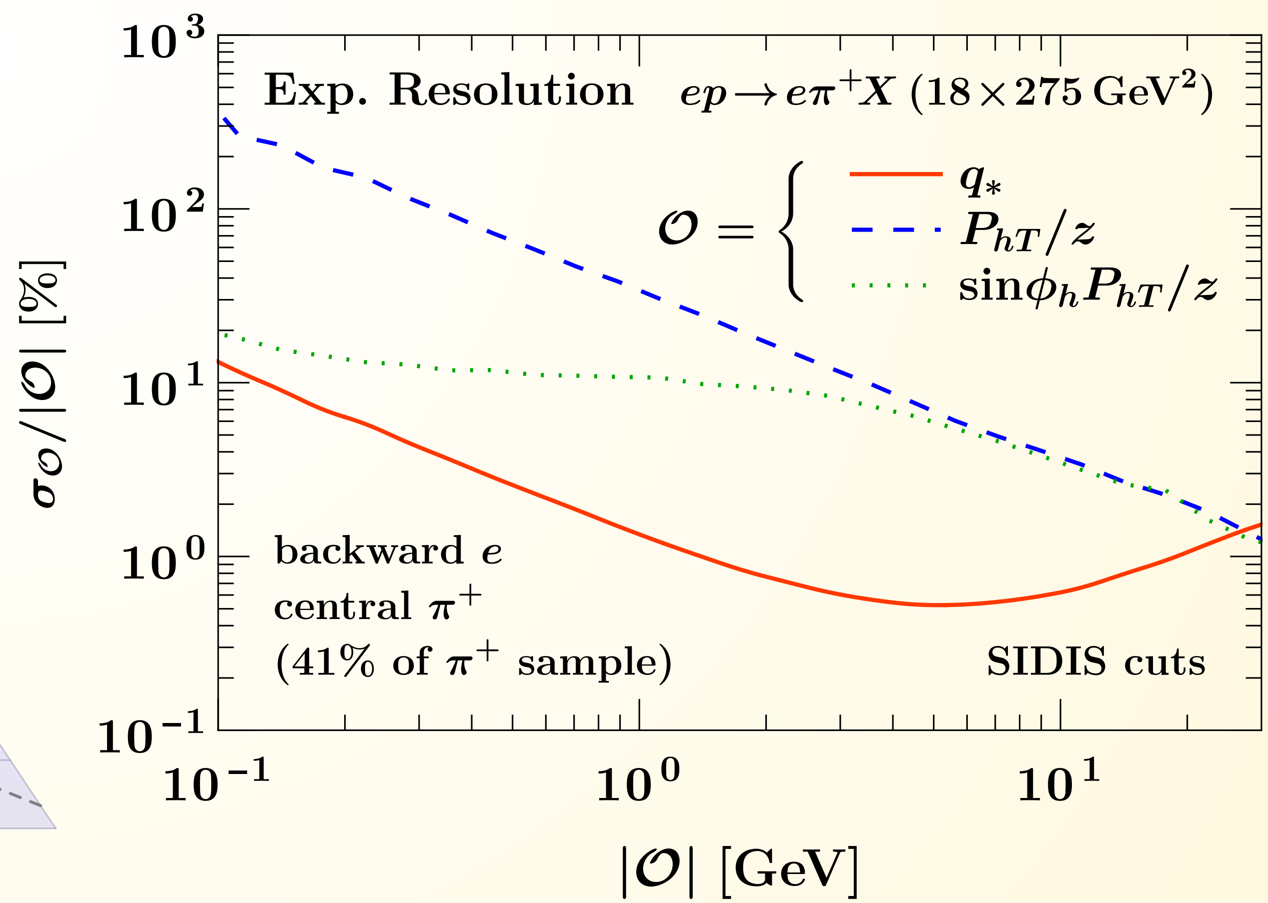


Challenges of Studying TMDs

- In SIDIS, nee
- **Momentum**
  uncertainty

Promise:
New observable q_* gives
Order of Magnitude Improvement
in Resolution!





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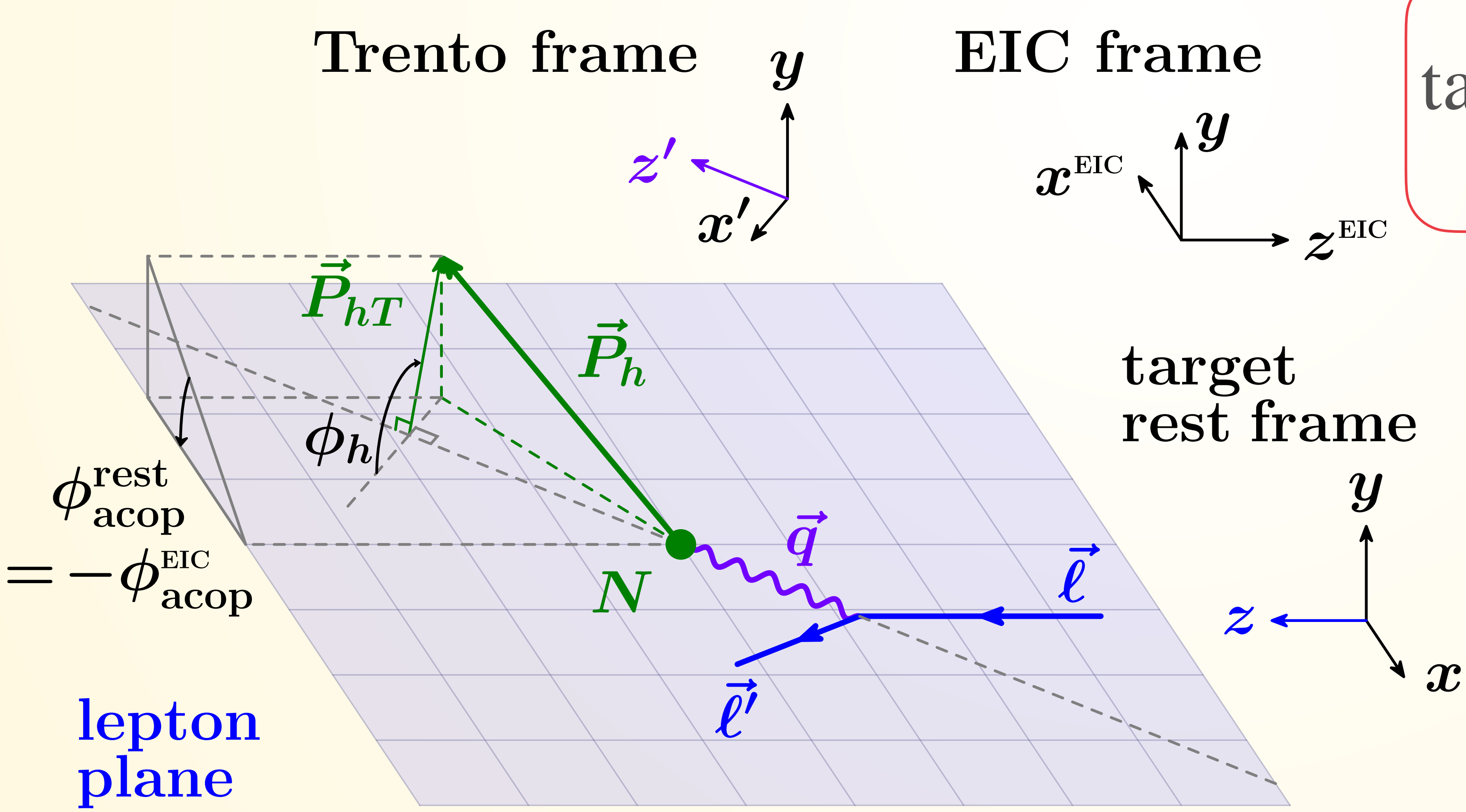
CONSTRUCTING q_*

(In the light target limit $M \ll Q$)

- **Angular** resolution at detectors is much better!
- Notice the acoplanarity angle $\tan \phi_{\rm acop} = -P_{h,y}/P_{h,x} \propto \sin \phi_h P_{hT}$ in the target rest frame



$$\tan \phi_{\rm acop}^{\rm rest} = \frac{\sin \phi_h P_{hT}}{zQ\sqrt{1-y}} + \mathcal{O}(\lambda^2)$$

$\lambda \sim P_{hT}/(zQ) \ll 1$

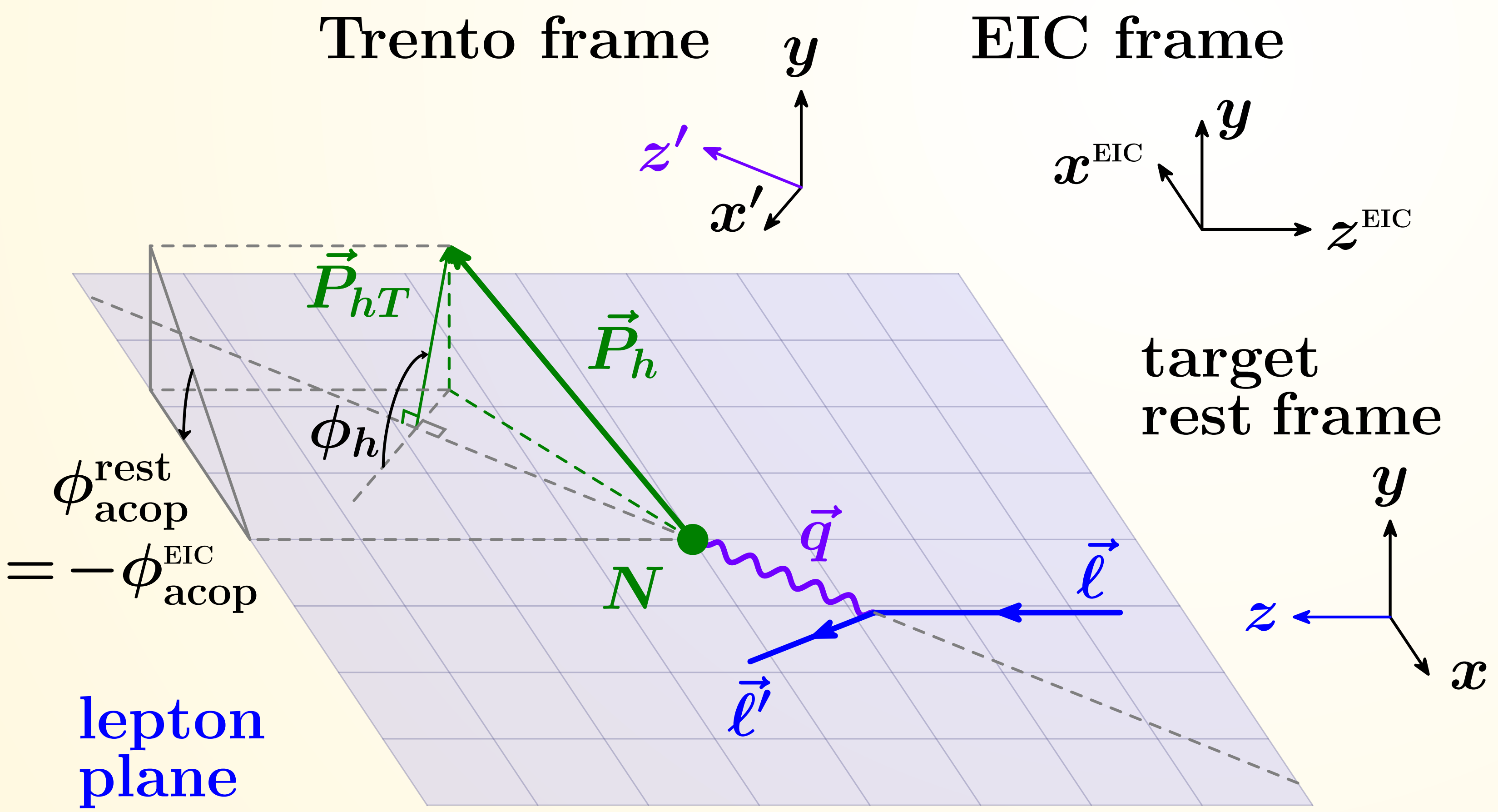
$Q^2 = -q^2$

$z = P \cdot P_h / P \cdot q$

$y = P \cdot q / P \cdot \ell$

CONSTRUCTING q_*

- Want to construct an optimized observable that **only uses angular measurements**
- Already have $\tan\phi_{\rm acop}^{\rm rest} = \dfrac{\sin\phi_h P_{hT}}{zQ\sqrt{1-y}} + \mathcal{O}(\lambda^2)$

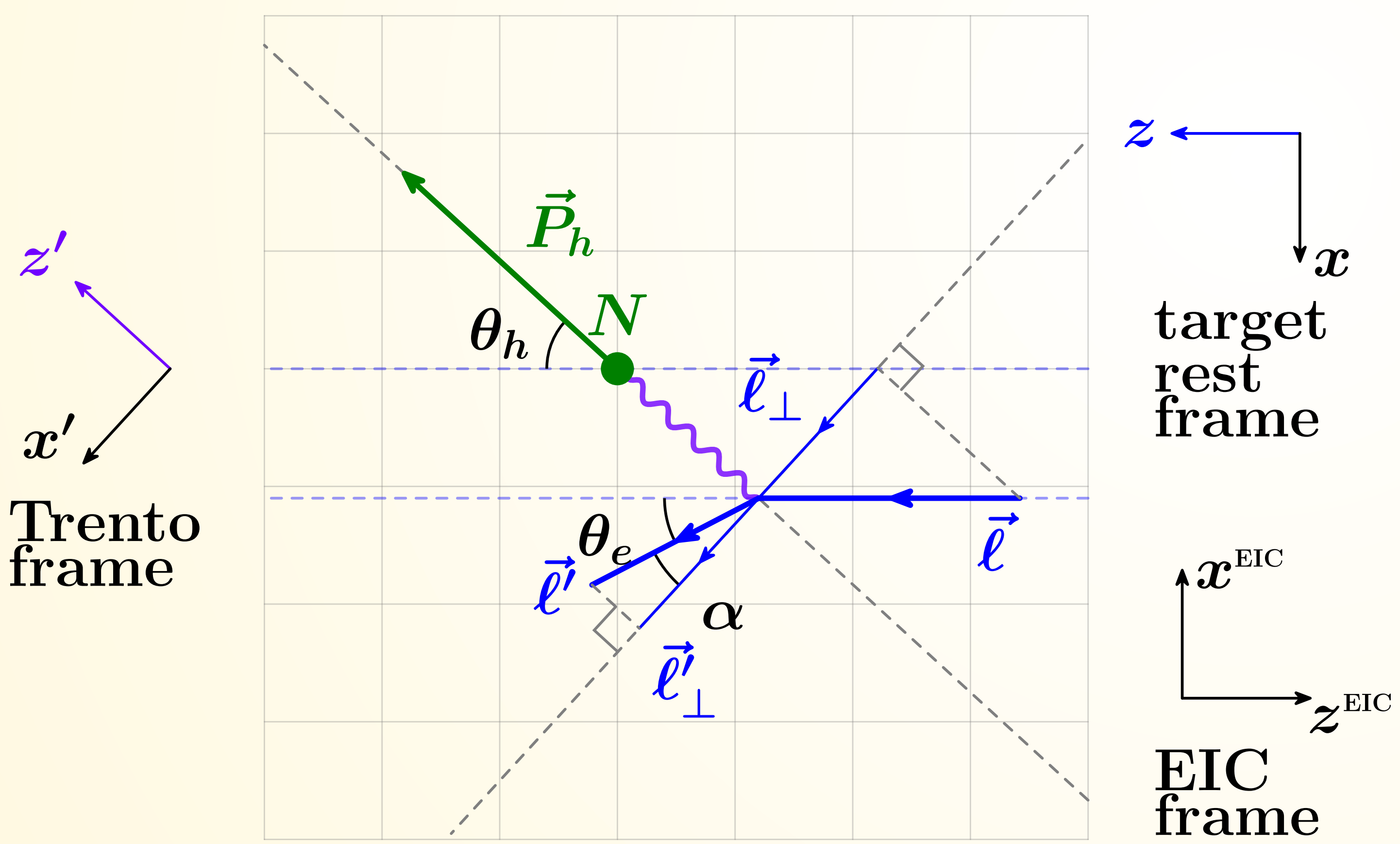


- Identify $P_{hT}/z = q_T$ is the parton transverse momentum
- Only need to write the **prefactor** $1/Q\sqrt{1-y}$ in terms of angles as well

CONSTRUCTING q_*

- Consider the in-plane leading power kinematics ($\overrightarrow{P}_h$ collinear to $\vec{q}$)

$$P_{hT} \ll Q \Rightarrow \theta_h + \theta_e + \alpha = \frac{\pi}{2}$$



- In terms of θ_h, θ_e , we find

$$y = 1 - \frac{\sin\theta_h}{\cos\alpha}$$

$$Q^2 = (E_\ell^{\mathrm{rest}})^2\left[\frac{\sin^2\theta_e}{\cos^2\alpha} - \left(1 - \frac{\sin\theta_h}{\cos\alpha}\right)^2\right]$$

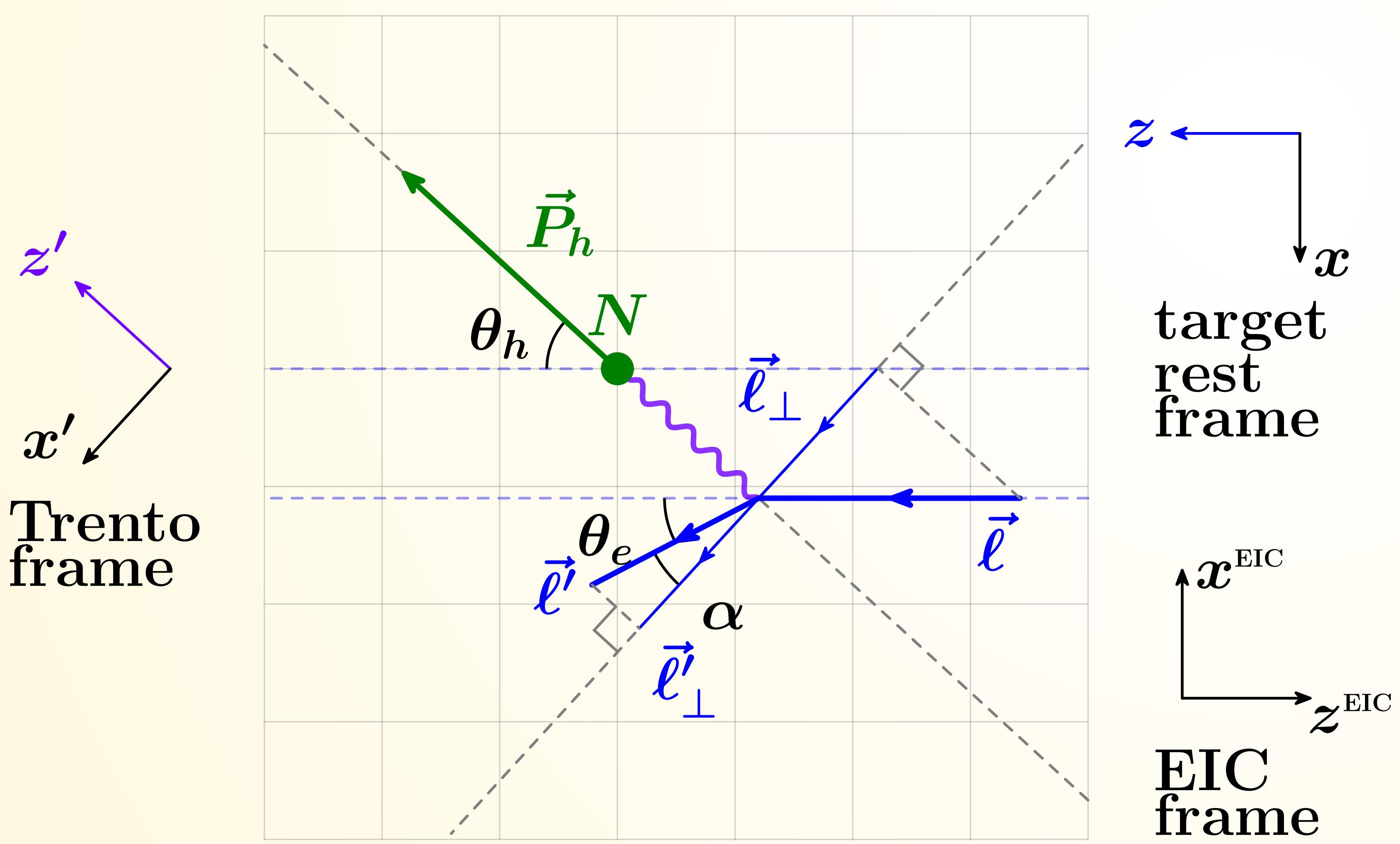
- Can easily translate θ_h, θ_e into EIC lab frame angles/ rapidities

CONSTRUCTING q_*

- Boosting to EIC lab frame and continue working in the $M \ll Q$ limit, we find in terms of lab frame rapidities η_h, η_e :

$\Delta\eta = \eta_h - \eta_e$

$$P_{hT} \ll Q \Rightarrow \theta_h + \theta_e + \alpha = \frac{\pi}{2}$$



$$y = \frac{1}{1+e^{\Delta\eta}} + \mathcal{O}(\lambda^2)$$

$$Q^2 = (2E_N)^2 \frac{e^{\eta_e+\eta_h}}{1+e^{\Delta\eta}} + \mathcal{O}(\lambda)$$

$$\Rightarrow \quad Q\sqrt{1-y} = 2E_N \frac{e^{\eta_h}}{1+e^{\Delta\eta}} + \mathcal{O}(\lambda)$$

- E_N is the energy of the nucleus in the EIC frame, which is known exactly, the rest of the quantities are **all angular**!

CONSTRUCTING q_*

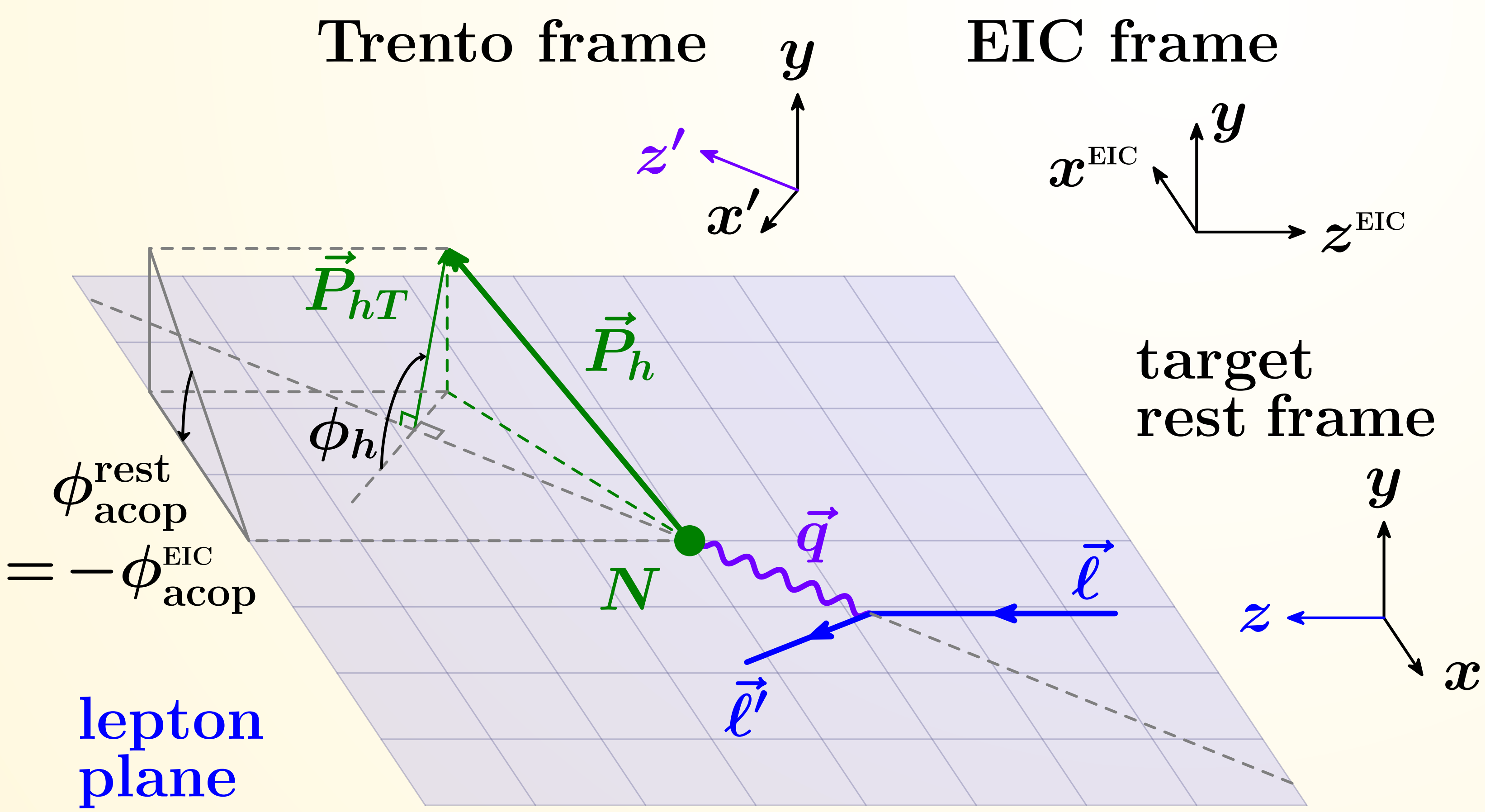
(In the light target limit $M \ll Q$)

- Now combine all the ingredients, we have the **definition** of our new observable q_*

$$\tan\phi_{\rm acop}^{\rm rest} = \frac{\sin\phi_h P_{hT}}{zQ\sqrt{1-y}} + \mathcal{O}(\lambda^2)$$

$$Q\sqrt{1-y} = 2E_N\frac{e^{\eta_h}}{1+e^{\Delta\eta}} + \mathcal{O}(\lambda)$$

$$q_* \equiv 2E_N\frac{e^{\eta_h}}{1+e^{\eta_h-\eta_e}}\tan\phi_{\rm acop}^{\rm EIC}$$



- q_* has a simple **leading power relation**, which allows for easy factorization:

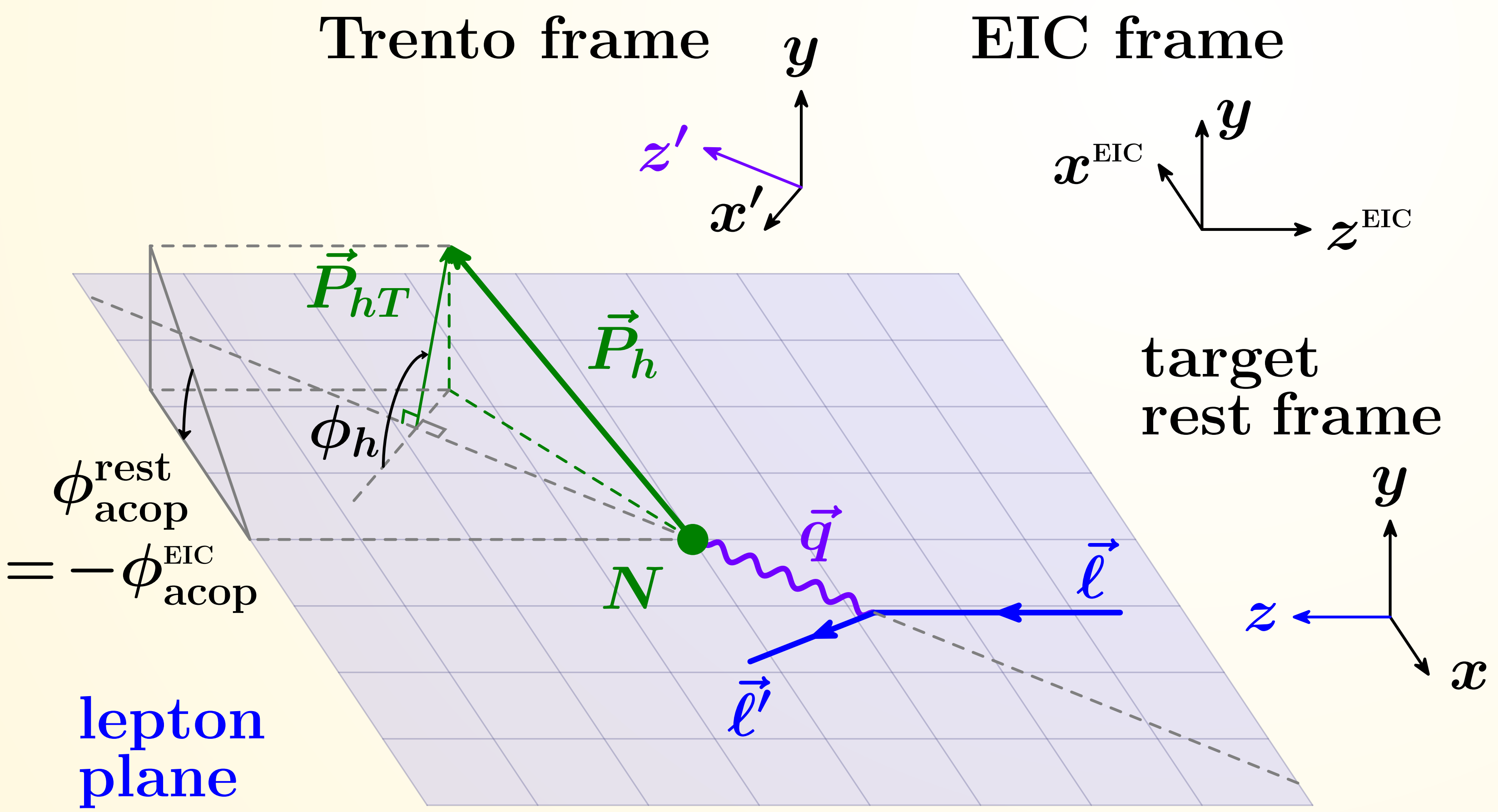
$$q_* \overset{\rm LP}{=} -\sin\phi_h\frac{P_{hT}}{z}$$

(more later on power corrections)

A DIMENSIONLESS VARIABLE

- Can also define a dimensionless observable ϕ^*_{SIDIS} similar to ϕ^*_η in unpolarized Drell-Yan [Banfi et al., EPJC 71, 1600 (2011), arXiv:1009.1580]

$$\phi^*_{\mathrm{SIDIS}} \equiv \sqrt{\frac{e^{\eta_h-\eta_e}}{1+e^{\eta_h-\eta_e}}}\tan\phi^{\mathrm{EIC}}_{\mathrm{acop}}$$

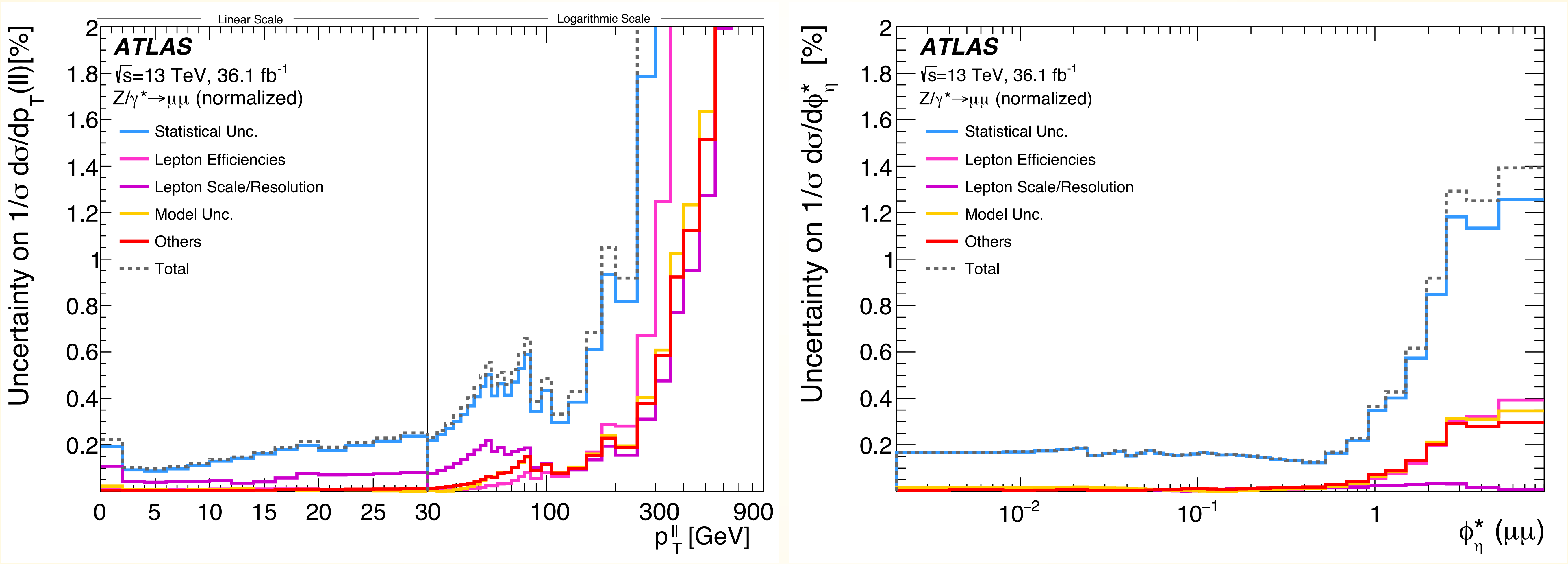


- ϕ^*_{SIDIS} can be related to q_* in leading power relation:

$$\phi^*_{\mathrm{SIDIS}} \overset{\mathrm{LP}}{=} \frac{q_*}{Q}$$

A DIMENSIONLESS VARIABLE

- ϕ_η^* in unpolarized Drell-Yan known to eliminate experimental systematics



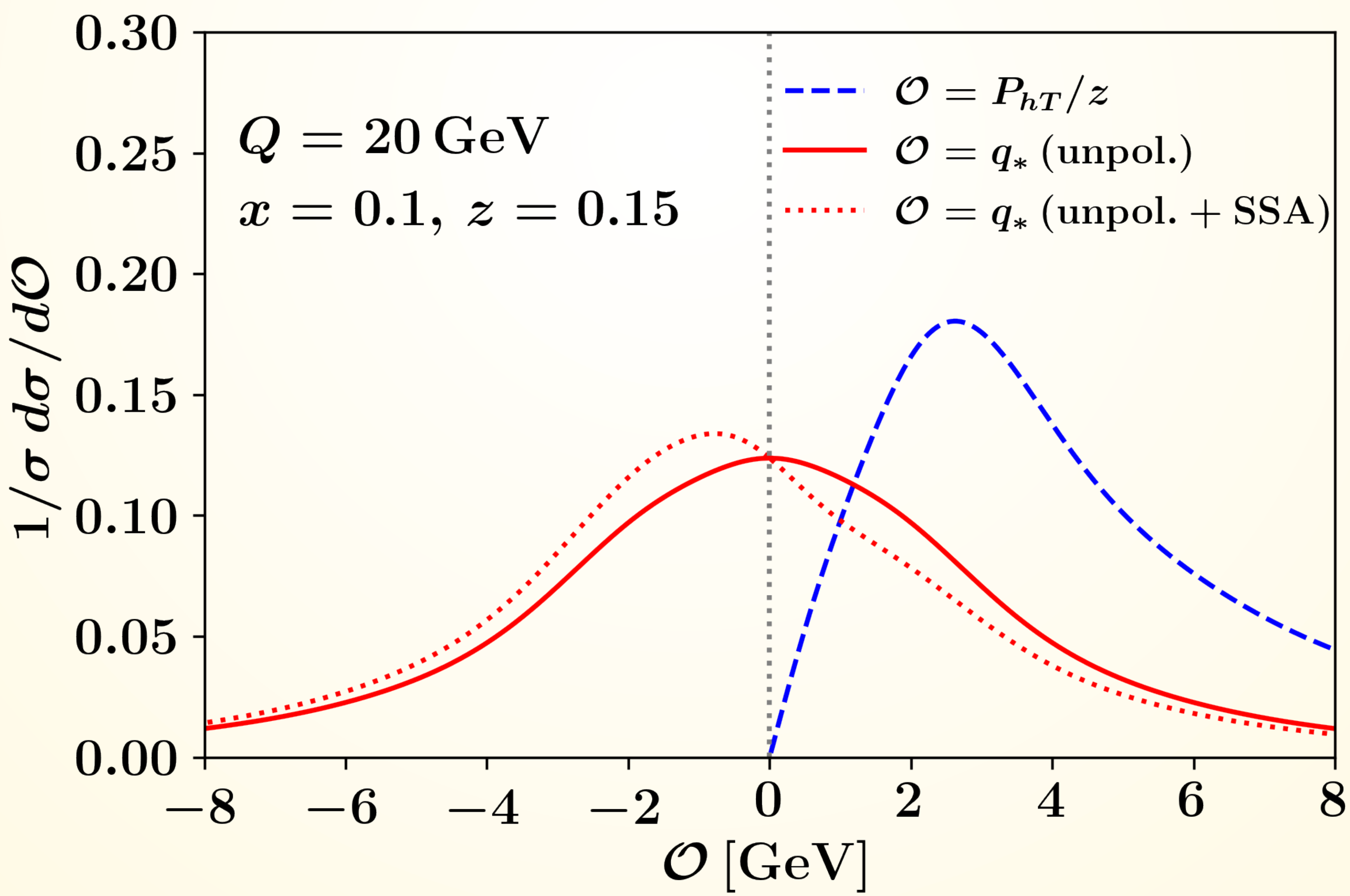
[ATLAS, EPJC 80 (2020) 7, 616, 1912.02844]

Spectrum comparison

$$q_* \equiv 2E_N \frac{e^{\eta_h}}{1+e^{\eta_h-\eta_e}} \tan\phi_{\text{acop}}^{\text{EIC}}$$

$$q_* \overset{\text{LP}}{=} -\sin\phi_h \frac{P_{hT}}{z}$$

- q_* is a signed observable
- Even and peak at 0 for unpolarized spectrum
- Single spin asymmetry (SSA) introduces odd contribution



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Factorization for q_* cross section

- Recall the leading power decomposition of SIDIS cross section in terms of structure functions:

$$\frac{d\sigma}{dx\,dy\,dz\,d^2\vec{P}_{hT}} = \sigma_0\Big\{W_{UU,T} + \lambda_e S_L\sqrt{1-\epsilon^2}\,W_{LL}$$
$$+\,\epsilon\cos(2\phi_h)W_{UU}^{\cos(2\phi_h)} + S_L\epsilon\sin(2\phi_h)W_{UL}^{\sin(2\phi_h)}$$
$$+\,S_T\sin(\phi_h-\phi_S)W_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon S_T\Big[\sin(\phi_h+\phi_S)$$
$$\times\,W_{UT}^{\sin(\phi_h+\phi_S)} + \sin(3\phi_h-\phi_S)W_{UT}^{\sin(3\phi_h-\phi_S)}\Big]$$
$$+\,\lambda_e S_T\sqrt{1-\epsilon^2}\cos(\phi_h-\phi_S)W_{LT}^{\cos(\phi_h-\phi_S)}\Big\}$$

λ_e lepton beam helicity
$S^\mu = (0, S_T\cos\phi_S, S_T\sin\phi_S, -S_L)$ nucleon spin vector in Trento frame
$\epsilon = (1-y)/(1-y+y^2/2)$
$\sigma_0 \equiv \alpha^2\pi y\kappa_\gamma/[zQ^2(1-\varepsilon)]$

- We can insert the leading power relationship $q_* \overset{\text{LP}}{=} -\sin\phi_h\frac{P_{hT}}{z}$ as a δ-function and do the $\vec{P}_{hT}$ integral to get the factorization theorem for q_* cross section

FACTORIZATION FOR q_* CROSS SECTION

- **As an example**, consider the contribution from

$$W_{UU}^{\cos(2\phi_h)} = -2z \int_0^\infty \frac{\mathrm{d}b_T b_T}{2\pi} \mathcal{I}[\mathcal{H}\, \tilde{h}_1^{\perp(1)} \tilde{H}_1^{\perp(1)}] J_2(b_T P_{hT}/z)$$

- The contribution to $\dfrac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}q_*}$ from $\epsilon \cos(2\phi_h) W_{UU}^{\cos(2\phi_h)}$ is

$$\int_0^\infty \mathrm{d}P_{hT}\, P_{hT} \int_0^{2\pi} \mathrm{d}\phi_h\, \delta\big(q_* + \sin\phi_h P_{hT}/z\big)\, \epsilon \cos(2\phi_h)$$
$$\times (-2z) \int \frac{\mathrm{d}b_T b_T}{2\pi} \mathcal{I}[\mathcal{H}\, \tilde{h}_1^{\perp(1)} \tilde{H}_1^{\perp(1)}]\, J_2(b_T P_{hT}/z)$$

⇐ Inserting δ-function & integrate over $d^2 \overrightarrow{P}_{hT}$

$$= -\frac{2z^3\epsilon}{\pi} \int \mathrm{d}b_T\, \mathcal{I}[\mathcal{H}\, \tilde{h}_1^{\perp(1)} \tilde{H}_1^{\perp(1)}]$$
$$\times \int_0^{2\pi} \frac{\mathrm{d}\phi_H}{\sin^2\phi_h} \Theta\Big(-\frac{q_*}{\sin\phi_h}\Big) \cos(2\phi_h)\, \frac{b_T |q_*|}{2}\, J_2\Big(\frac{b_T q_*}{\sin\phi_h}\Big)$$
$$= -\frac{2z^3\epsilon}{\pi} \int \mathrm{d}b_T\, \mathcal{I}[\mathcal{H}\, \tilde{h}_1^{\perp(1)} \tilde{H}_1^{\perp(1)}]\, \cos(q_* b_T)$$

⇐ Simple nontrivial kernel from ϕ_h dependence

where $\mathcal{I}[\mathcal{H}\, \tilde{g}^{(n)} \tilde{D}^{(m)}] \equiv (M b_T)^n (-M_h b_T)^m \sum_f \mathcal{H}_f \tilde{g}_f^{(n)} \tilde{D}_f^{(m)}$

FACTORIZATION FOR q_* CROSS SECTION

- We get factorization theorem for q_* cross section in terms of **standard TMD PDFs and FFs**

λ_e lepton beam helicity

$S^\mu = (0, S_T \cos\phi_S, S_T \sin\phi_S, -S_L)$ nucleon spin vector in Trento frame

$\epsilon = (1-y)/(1-y+y^2/2)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}q_*} = \frac{2z^3}{\pi}\sigma_0 \int_0^\infty \mathrm{d}b_T \Big\{ \cos(q_* b_T) \Big(\mathcal{I}[\mathcal{H}\,\tilde{f}_1\,\tilde{D}_1] - \epsilon\,\mathcal{I}[\mathcal{H}\,\tilde{h}_1^{\perp(1)}\tilde{H}_1^{\perp(1)}] + \lambda_e S_L \sqrt{1-\epsilon^2}\,\mathcal{I}[\mathcal{H}\,\tilde{g}_{1L}\,\tilde{D}_1] \Big)$$

$$+ \cos\phi_S \sin(q_* b_T)\, S_T \Big(\mathcal{I}[\mathcal{H}\,\tilde{f}_{1T}^{\perp(1)}\,\tilde{D}_1] + \epsilon\,\mathcal{I}[\mathcal{H}\,\tilde{h}_1\,\tilde{H}_1^{\perp(1)}] + \frac{\epsilon}{4}\mathcal{I}[\mathcal{H}\,\tilde{h}_{1T}^{\perp(2)}\,\tilde{H}_1^{\perp(1)}] \Big)$$

$$- \sin\phi_S \sin(q_* b_T)\, \lambda_e S_T \sqrt{1-\epsilon^2}\,\mathcal{I}[\mathcal{H}\,\tilde{g}_{1T}^{\perp(1)}\,\tilde{D}_1] \Big\}$$

- Can **extract** different contributions by unique* dependence on $q_*, \lambda_e, S^\mu, \epsilon$

Extracting TMDs from Factorization

- Practically, we take **asymmetries** with opposite beam polarizations (λ_e, S^μ) and by measuring cross sections as a function of y (ϵ)

- **As an example,** consider taking the double asymmetry $q_* \to -q_*$ and $\lambda_e \to -\lambda_e$, we single out the contribution

$$-\sin\phi_S \sin(q_* b_T)\, \lambda_e S_T \sqrt{1-\epsilon^2}\, \mathcal{I}\big[\mathcal{H}\, \tilde{g}_{1T}^{\perp(1)}\, \tilde{D}_1\big]$$

which allows us to access the worm-gear T function $\tilde{g}_{1T}^{\perp}$

- *Note that the worm-gear L function $\tilde{h}_{1L}$ drops out, and transversity and pretzelosity are degenerate: $\epsilon S_T (\tilde{h}_1 + \tilde{h}_{1T}^{\perp}/4)$

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Improved Resolution

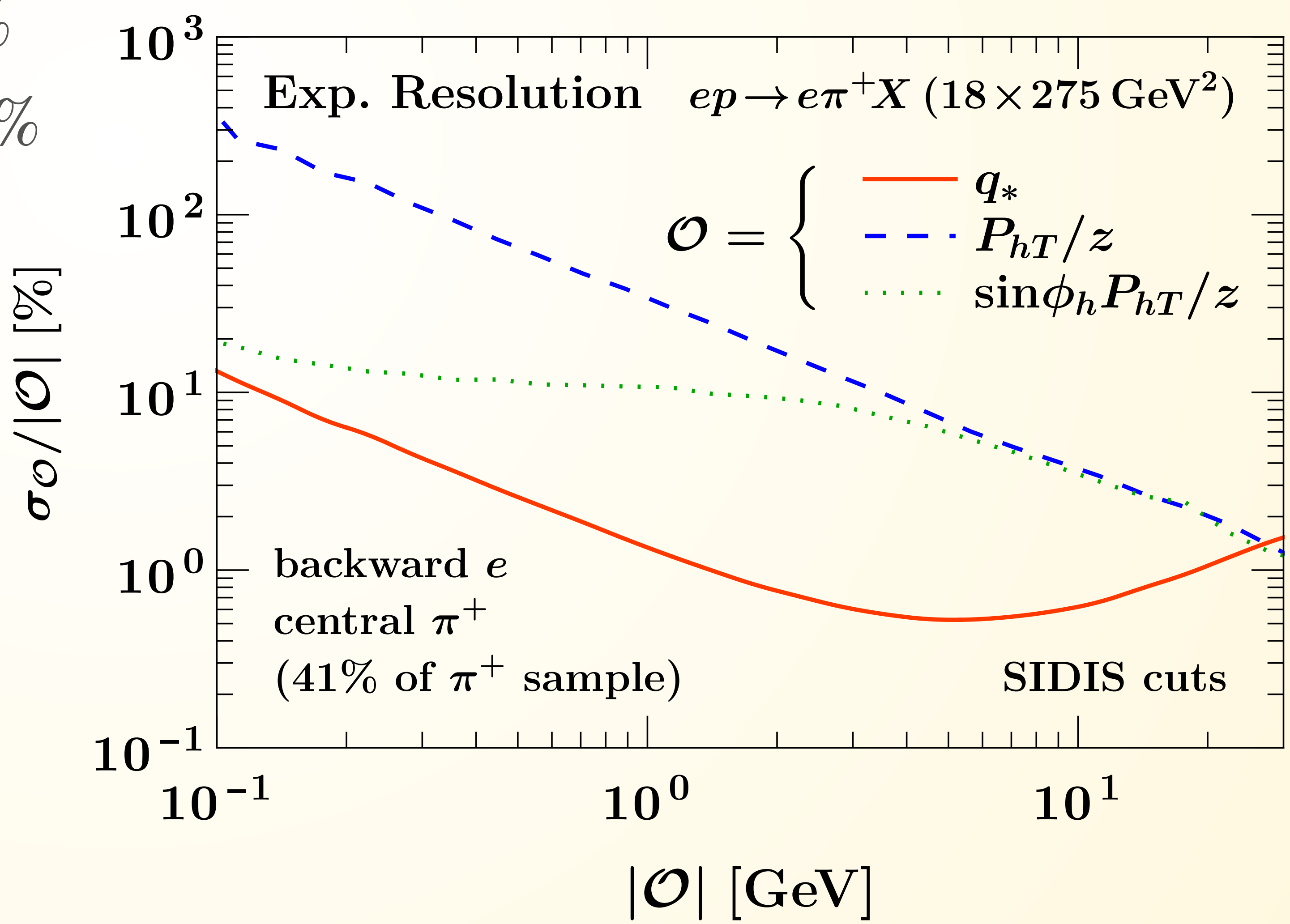
SIDIS cuts:
$x > 0.001$
$0.01 < y < 0.95$
$z > 0.05$
$Q^2 > 16\,\mathrm{GeV}^2$
$W^2 = (P+q)^2 > 100\,\mathrm{GeV}^2$

- Simulated in Pythia, $\mathcal{O}(10^8)$ events, Gaussian smeared

EIC Yellow Report Design Requirements, arXiv:2103.05419

- Momentum resolution:
  c: $\sigma_p/p = 0.05\,\%\,p/\mathrm{GeV} \oplus 0.5\,\%$
  f/b: $0.05 - 0.1\,\%\,p/\mathrm{GeV} \oplus 1 - 2\,\%$
  Angular resolution
  $\sigma_\theta = \sigma_\phi = 0.001$

- **Order of magnitude resolution improvement** in the TMD region ($\lesssim 2\,\mathrm{GeV}$) from using q_*



STATISTICAL SENSITIVITY OF q_* VS P_{hT}

- We test how well our observable can **extract parameters** of a given nonperturbative model for unpolarized cross section

$$\sigma_{\text{unpol}} \sim f_1(z, k_T) D_1(z, k_T)$$

- Recall that the nonperturbative model enter TMDs through:

perturbative result & evolution using SCETlib

$$\tilde{f}_{1f}(x, b_T, \mu, \zeta) = \tilde{f}_{1f}(x, b^*(b_T), \mu, \zeta)\, \tilde{f}_1^{\text{NP}}(x, b_T)$$
$$\tilde{D}_{1f}(z, b_T, \mu, \zeta) = \tilde{D}_{1f}(z, b^*(b_T), \mu, \zeta)\, \tilde{D}_1^{\text{NP}}(z, b_T)$$

M. Ebert, J. Michel, F. Tackmann et. al, DESY-17-099

- We use a simplified version of the MAPTMD 22 global fits

$$\tilde{f}_1^{\text{NP}} = e^{-\omega_1 b_T^2}$$
$$\tilde{D}_1^{\text{NP}} = \alpha\, e^{-\omega_2 b_T^2} + (1-\alpha)(1-\omega_3 b_T^2)\, e^{-\omega_3 b_T^2}$$

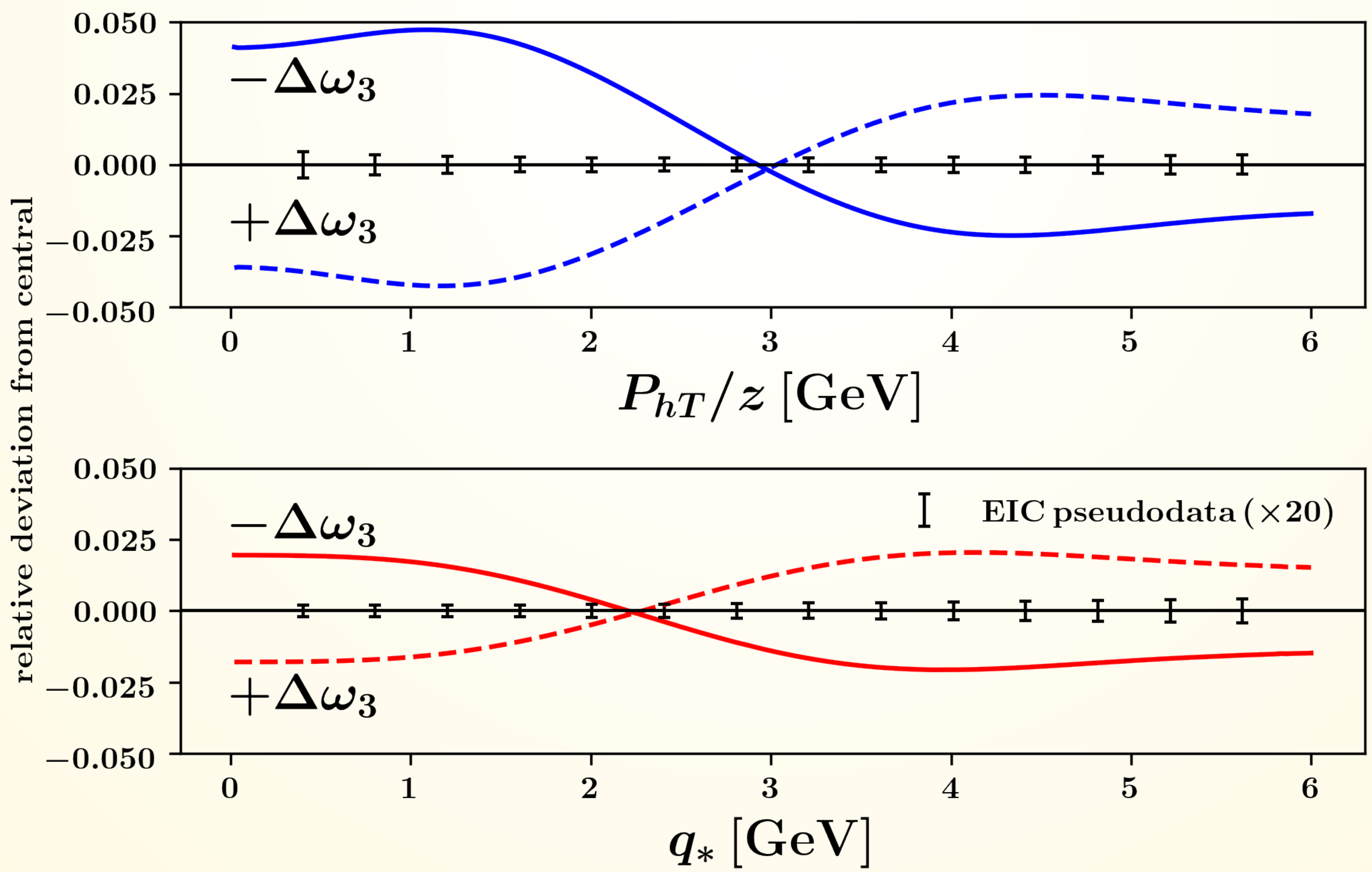
(Fixing x, z, Q^2) A. Bacchetta et. al, MAPTMD 22, arXiv:2206.07598

- Parameters ω_i encodes the shape of TMDs at large distances

STATISTICAL SENSITIVITY OF q_* VS P_{hT}

- Use the central value and standard deviation of the global fits in MAPTMD 22 as our Gaussian prior
- Use Bayesian analysis to test the posterior distribution

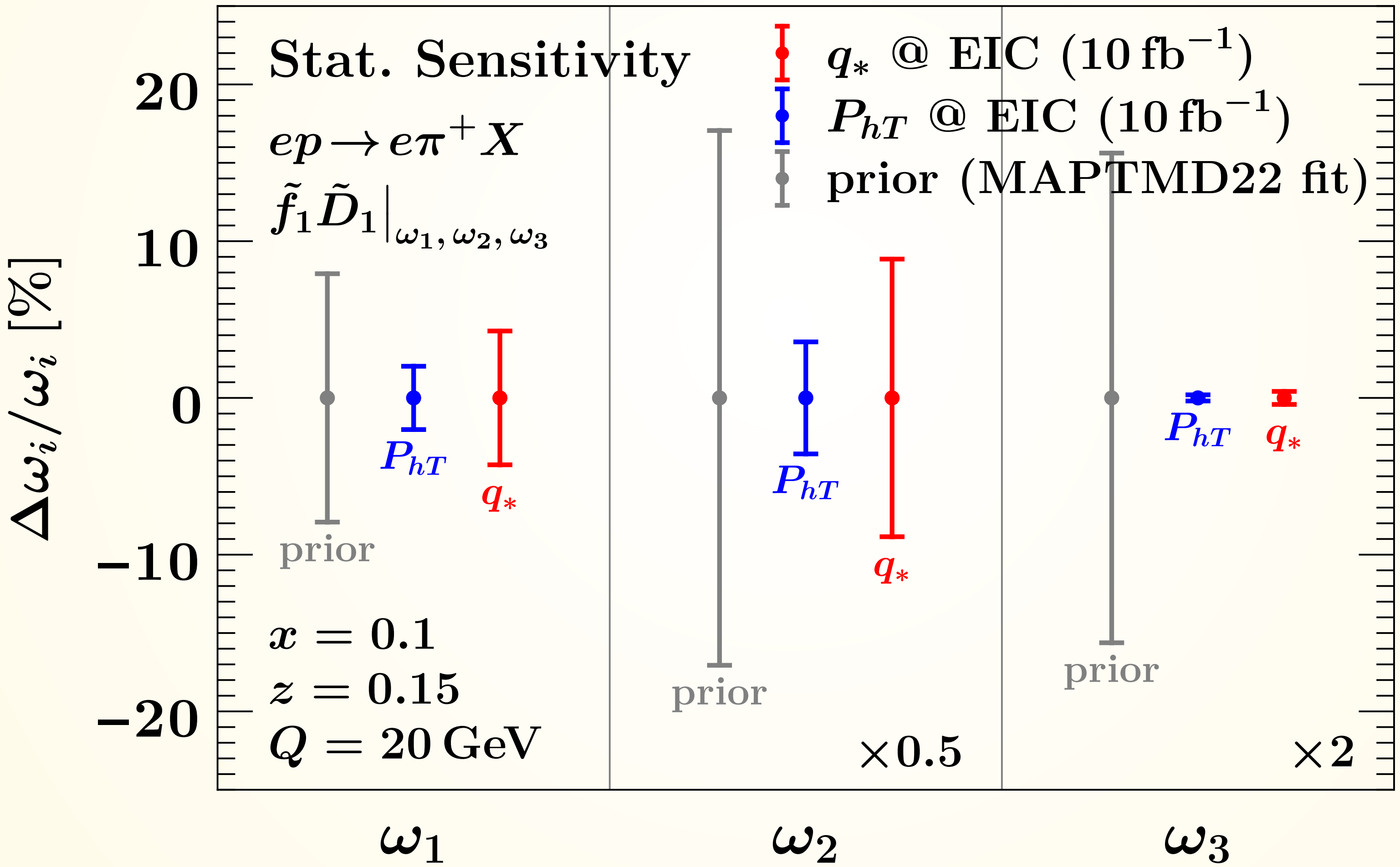
$$\pi(\omega_i \,|\, d_n) \propto \exp\left[-\sum_n \left(\frac{d_n - t_n(\omega_i)}{\sigma_n}\right)^2\right] \pi(\omega_i)$$



impact of ω_3 on P_{hT} vs q_* spectrum

STATISTICAL SENSITIVITY OF q_* VS P_{hT}

- Compare between using our new observable q_* and using transverse momentum P_{hT} directly:



- q_* has **comparable statistical sensitivity** to P_{hT} , while having a much superior resolution

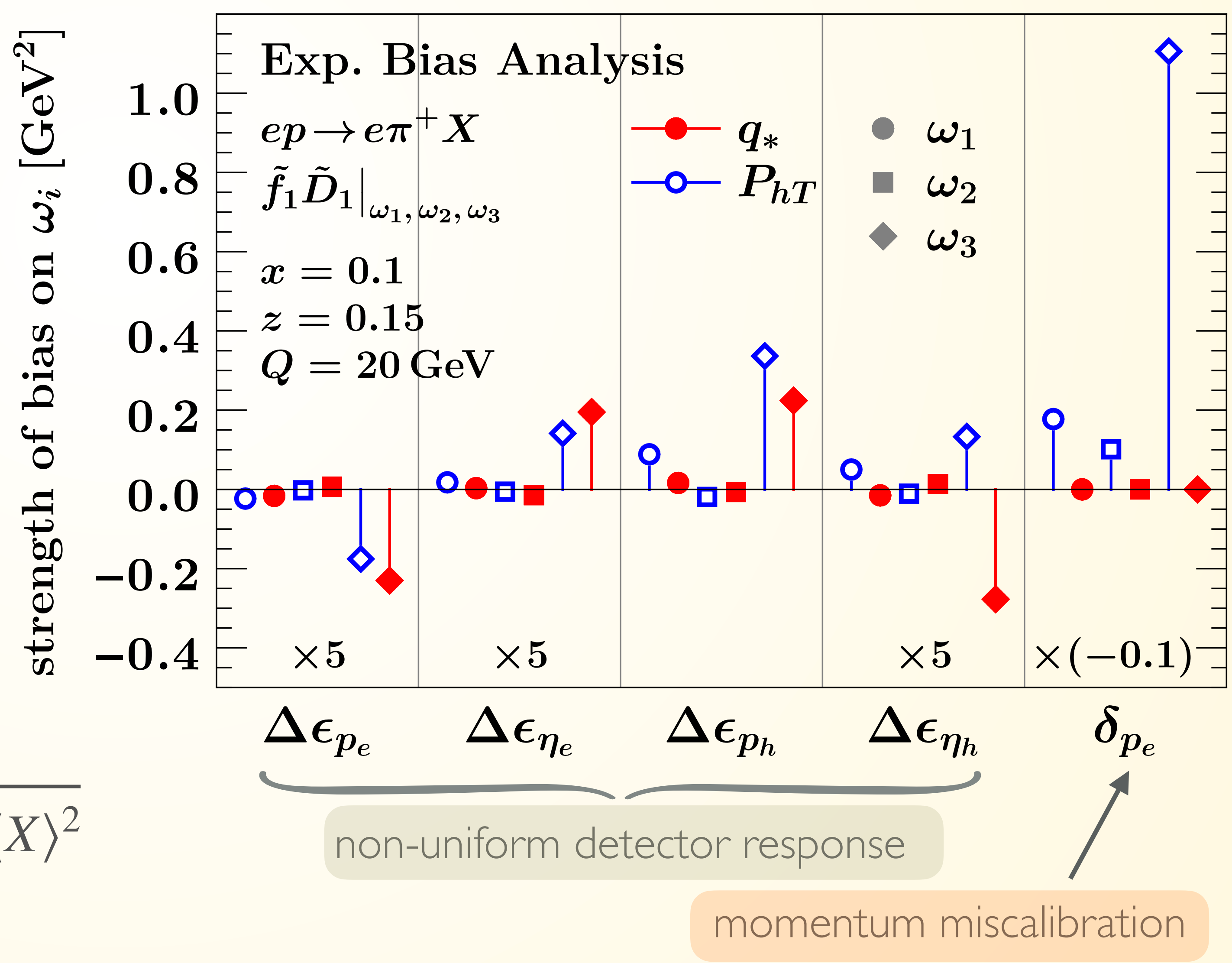
Robustness against Bias

- Assess the **robustness** of the observables against systematic uncertainties using the same setup

- P_{hT} is highly susceptible to momentum mis-calibration, $p_e \to (1 + \delta p_e) p_e$ while q_* is **unaffected**

- **Comparable robustness** against non-uniform detector response

$$\varepsilon(X) = 1 + \Delta\varepsilon_X \frac{X - \langle X \rangle}{\Delta X}, \Delta X = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$$



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Power corrections

(In the light target limit $M \ll Q$)

- Consider leading corrections in $\lambda \sim P_{hT}/(zQ)$ to y, Q^2

$$y_* \equiv \frac{1}{1+e^{\Delta\eta}} = y + \mathcal{O}(\lambda^2),$$

$$Q_*^2 \equiv \left(2E_N\right)^2 \frac{e^{\eta_e+\eta_h}}{1+e^{\Delta\eta}} = Q^2\left(1 - \cos\phi_h \sqrt{\frac{1}{1-y}}\frac{P_{hT}}{zQ}\right) + \mathcal{O}(\lambda^2)$$

- For definition of q_* we can equivalently write

$$q_* \equiv Q_*\sqrt{1-y_*}\,\tan\phi_{\rm acop}^{\rm EIC} = 2E_N \frac{e^{\eta_h}}{1+e^{\Delta\eta}} \tan\phi_{\rm acop}^{\rm EIC}$$

- Measuring cross section in q_* experimentally allows **easy reinterpretation** and power correction on the theory front

Finite Mass effects

- Useful when the target mass is on the same order as Q
- To leading order in $\lambda \sim P_{hT}/(zQ)$ we have

$$\gamma = \frac{2xM}{Q}$$

$$\tan\phi^{\rm rest}_{\rm acop} = \frac{\sin\phi_h\, P_{hT}}{zQ}\sqrt{\frac{1+\gamma^2}{1-\gamma^2 y^2/4-y}} + \mathcal{O}(\lambda^2)$$

$$q_*^M \equiv -E_\ell^{\rm rest}\frac{[\cos(\theta_e+\theta_h)+\cos\theta_h]\tan(\frac{\theta_e}{2})\sin\theta_h}{\sin(\theta_e+\theta_h)} \qquad \tan\phi^{\rm rest}_{\rm acop} \overset{\rm LP}{=} -\sin\phi_h\frac{P_{hT}}{z}$$

- Can easily boost to EIC frame and use EIC frame angles

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Summary

- TMDs are interesting subjects to study and can tell us a lot about hadron structures

$$q_* \equiv 2E_N \frac{e^{\eta_h}}{1+e^{\eta_h-\eta_e}} \tan \phi_{\mathrm{acop}}^{\mathrm{EIC}}$$

- We have proposed an angular observable q_* in SIDIS that is sensitive to TMDs and has superior experimental resolution
- We have proven factorization for the q_* cross section and shown that we may access individual polarized TMDs
- Studying q_* at EIC will allow us to push forward the understanding of hadronization and confinement

Thank you!