Classical computation using tensor networks and quantum computation with qubits and qumodes.

Raghav Govind Jha

223 Jefferson Lab

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Outline

- Introduction to tensor RG (renormalization group)
- Applications to Ising model with magnetic field, 2d O(2) model, 2d SU(2)m gauge theory, and 3d O(2) model with chemical potential
- Another facet of tensors: Real-time evolution in Ising Field Theory (IFT) using Matrix Product States (MPS)
- Moving to quantum computing: Qubits (d=2 Hilbert space), Qumodes (continuous variables, infinite dimensional HS) ✕
- Application of qubit method to understand O(3) model recently by other groups and our ongoing work on CV formulation. ✕

Unequal parts definition - "First three/Last two >> 1"

Different RG methods

Various renormalization group (RG) schemes (list not exhaustive) have been introduced over the past 5-6 decades:

- Kadanoff's spin blocking RG [1966] & Wilson's RG [1975]
- Density Matrix Renormalization Group (DMRG) [White, 1992]
  (DMRG is an extension to Wilson RG and is well-suited to all 1d systems not only restricted to impurity problems)
- Tensor Renormalization Group [Levin and Nave, 2007] + HOTRG [Xie et al., 2012]
- Tensor Network Renormalization (TNR) [Vidal et al., 2015] [related to MERA]

But why tensors?

- Provides an arena to study lower-dimensional (critical and gapped) systems faster than any other known method available today! [2d Ising model in 15 seconds on laptop]
- Formulation in terms of tensors can help us study models where the usual Monte Carlo (MC) methods fail (such as finite-density, θ-term). In addition, the thermodynamic limit can be approached faster and partition function can be computed unlike MC.
- Description of a quantum state in terms of tensors (MPS) can be useful to study real-time dynamics
- Known to play a key role in emergence of space-time via proposals like AdS/MERA etc.

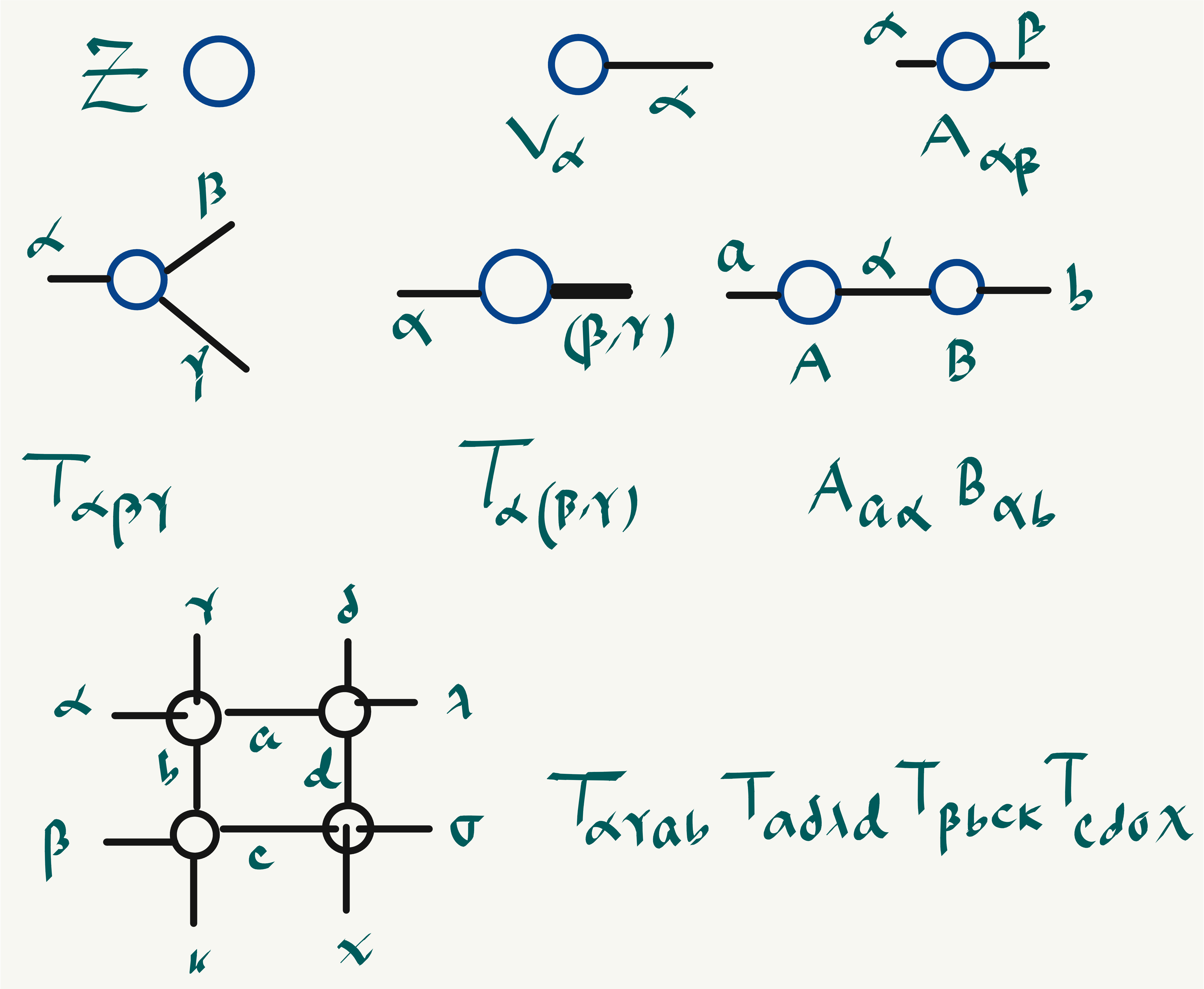
Formulation of tensors

Tensor methods have both Lagrangian and Hamiltonian applications.

- State approach: We can approximate the ground state i.e., $|\psi\rangle = \sum_{i_1,\cdots,i_N} C_{i_1,\cdots,i_N} | i_1, \cdots, i_N \rangle$ of a model with local Hamiltonian of N spins in fewer coefficients than 2^N, O(N).

- Action approach: We approximate the partition function using tensor networks considering decomposition of Boltzmann weight (truncate if necessary) and then coarse-graining by performing successive iterations.

Notation



Z
Vα
α
α
β
Aαβ
α
β
γ
α
(β,γ)
a
α
b
A
B
Tαβγ
Tα(β,γ)
Aaα Bαb
γ
δ
α
λ
a
b
d
β
c
σ
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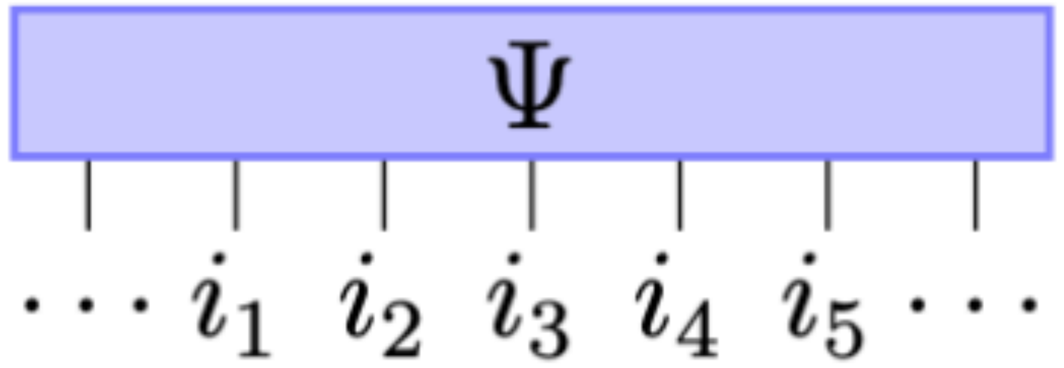
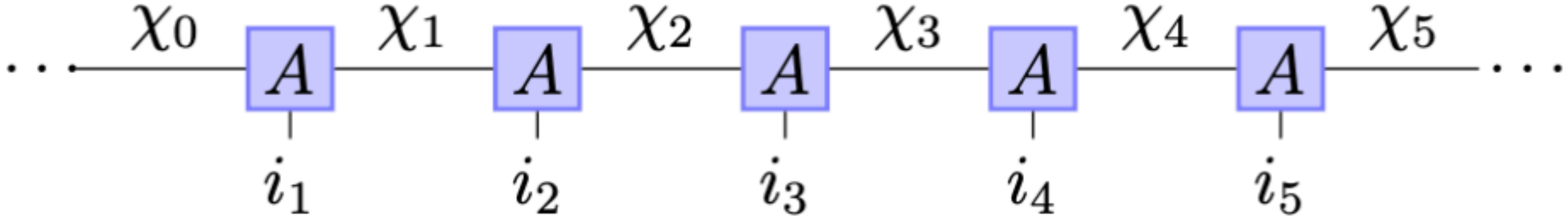
χ
Tαγab Tadλd Tβbcκ Tcdσχ

Matrix Product States (MPS)

$$\Psi_{\ldots i_1 i_2 \ldots i_5 \ldots} =$$



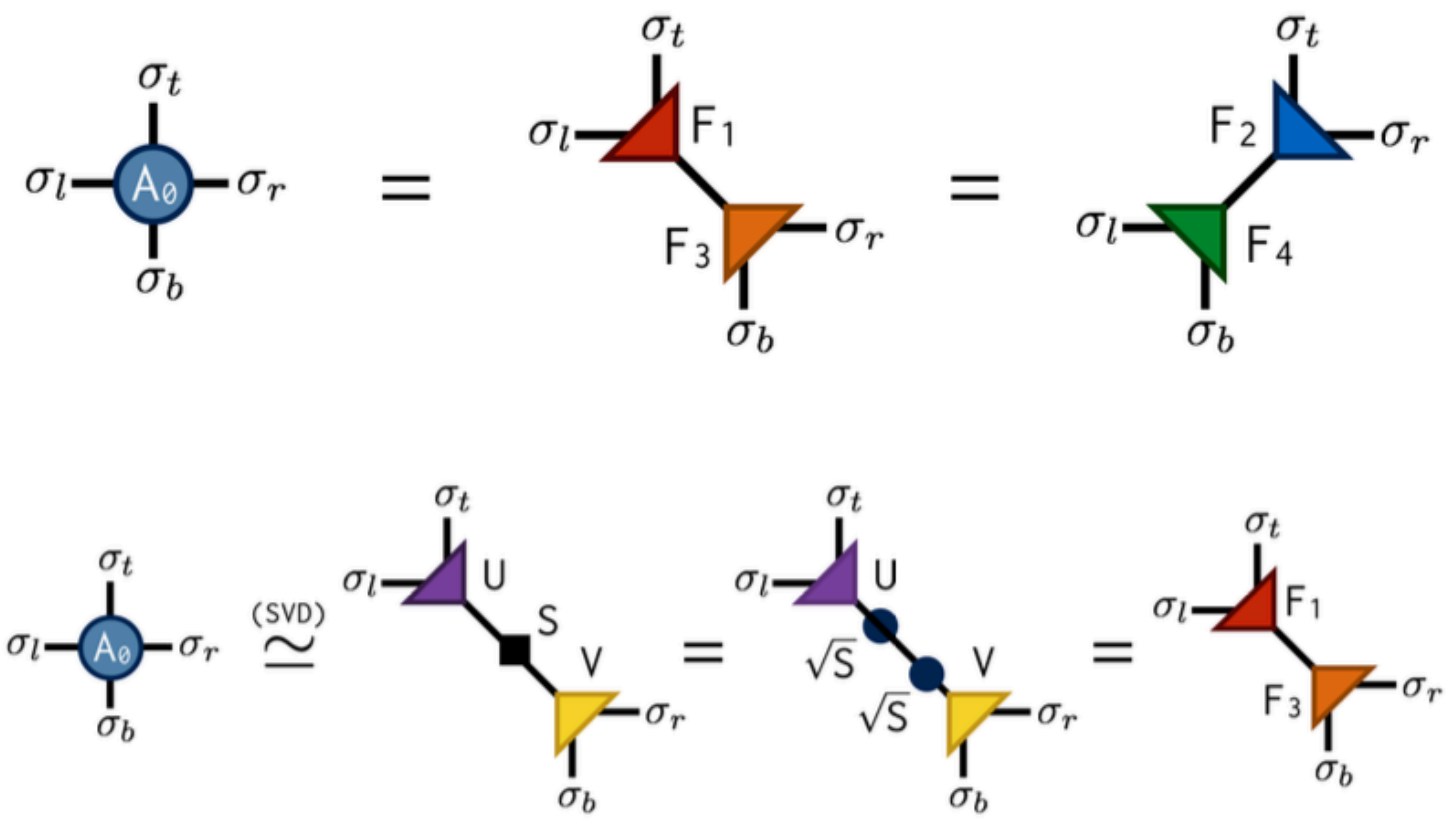
$$\Psi_{\ldots i_1 i_2 \ldots i_5 \ldots} =$$



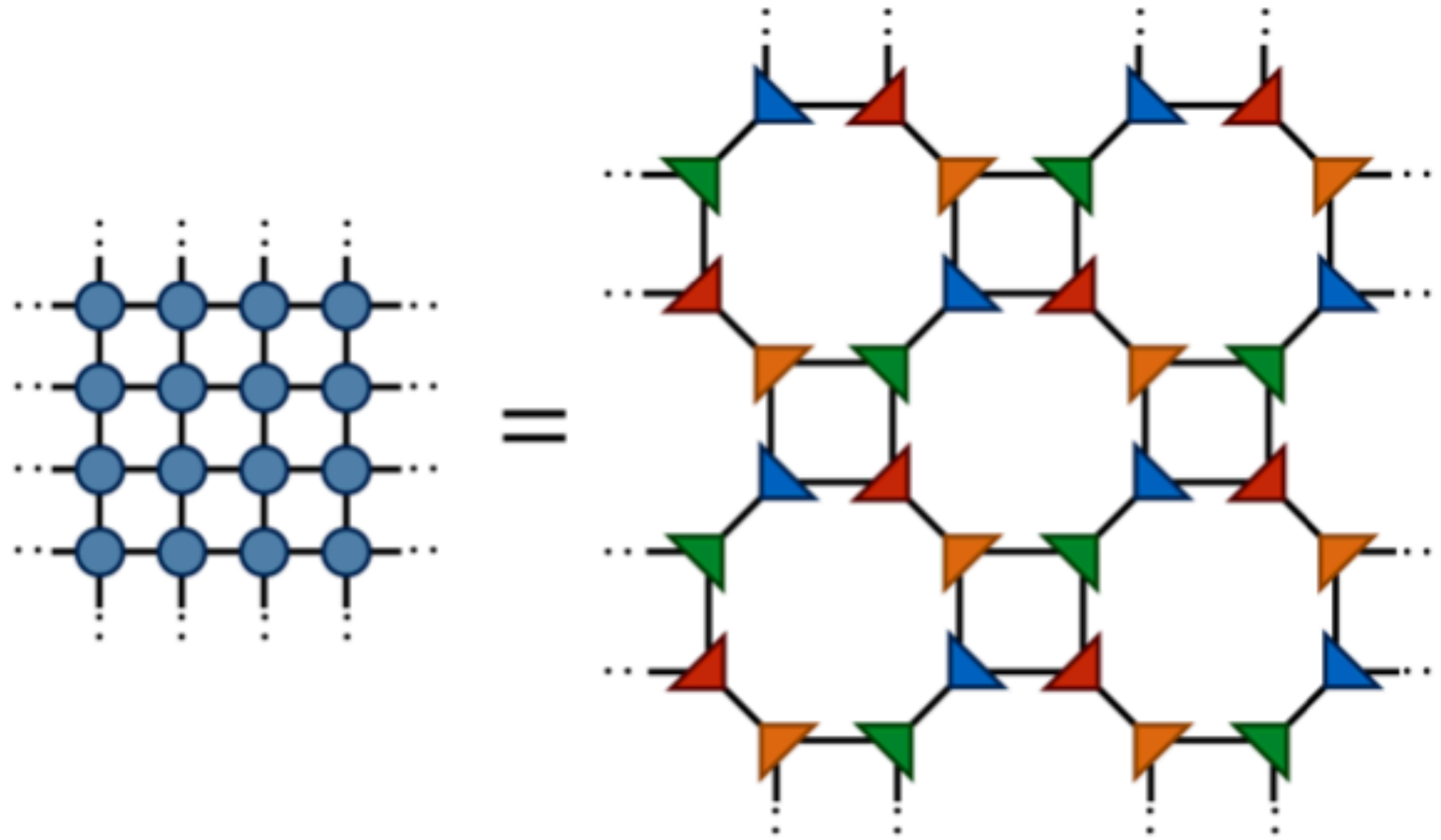
1811.11027

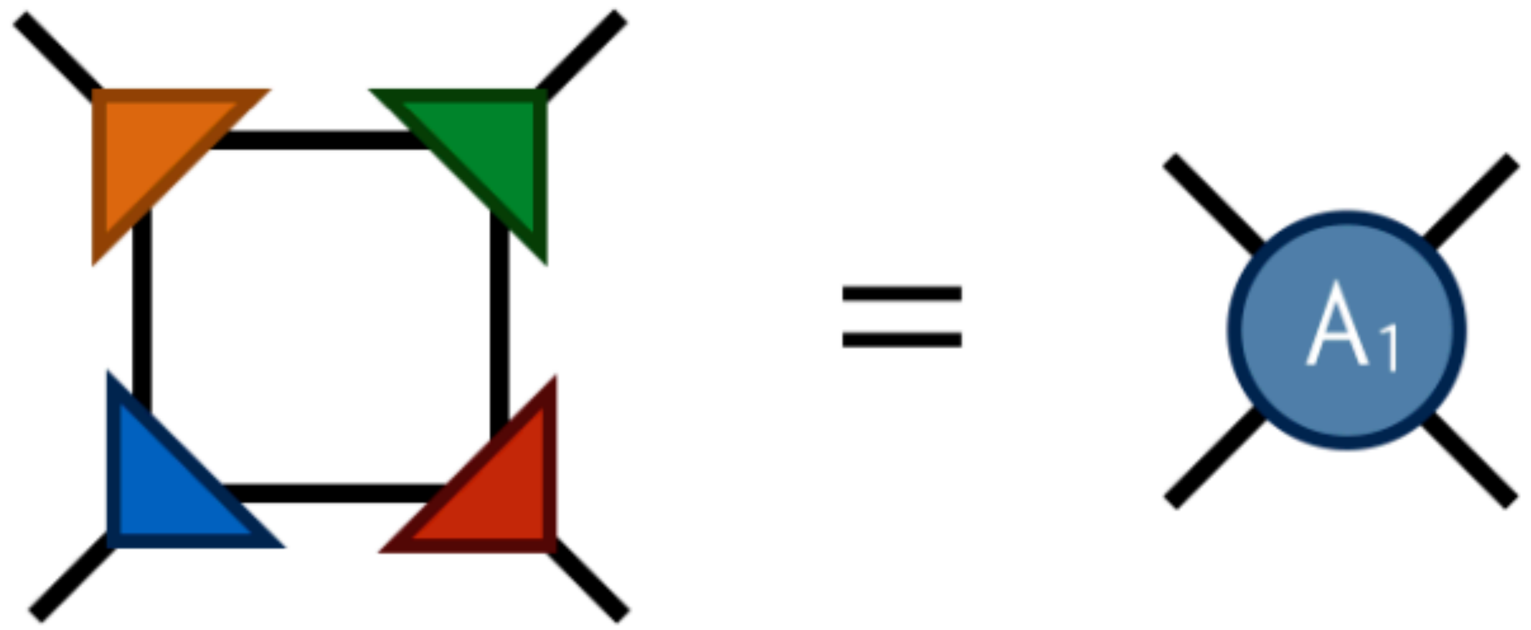
This talk [TRG]!

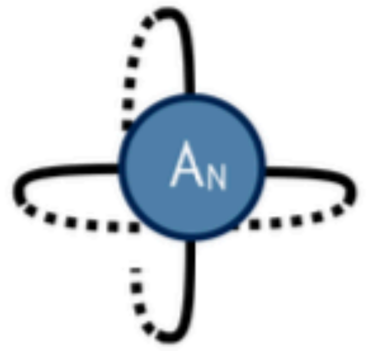
For this talk we will restrict to the application of tensor networks when dealing with statistical systems in Euclidean dimensions. This amounts to evaluating Z to best possible accuracy. This is usually a NP (non-polynomial) hard problem. We will start with an initial network and then perform coarse-graining to approach the correct target theory. For example, the schematic representation of TRG can be shown as:

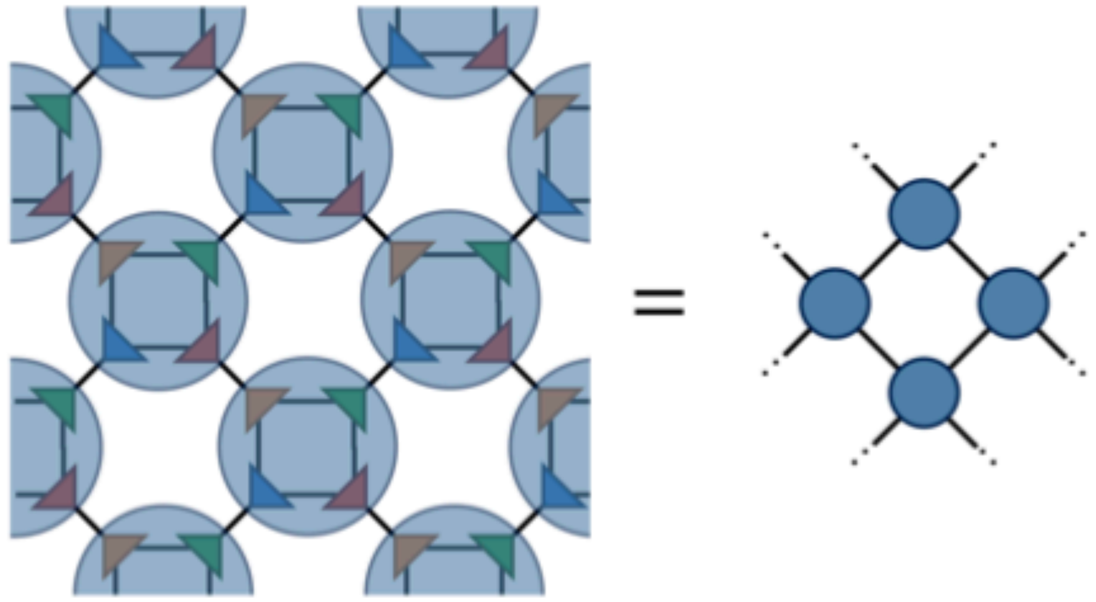


TRG continued!



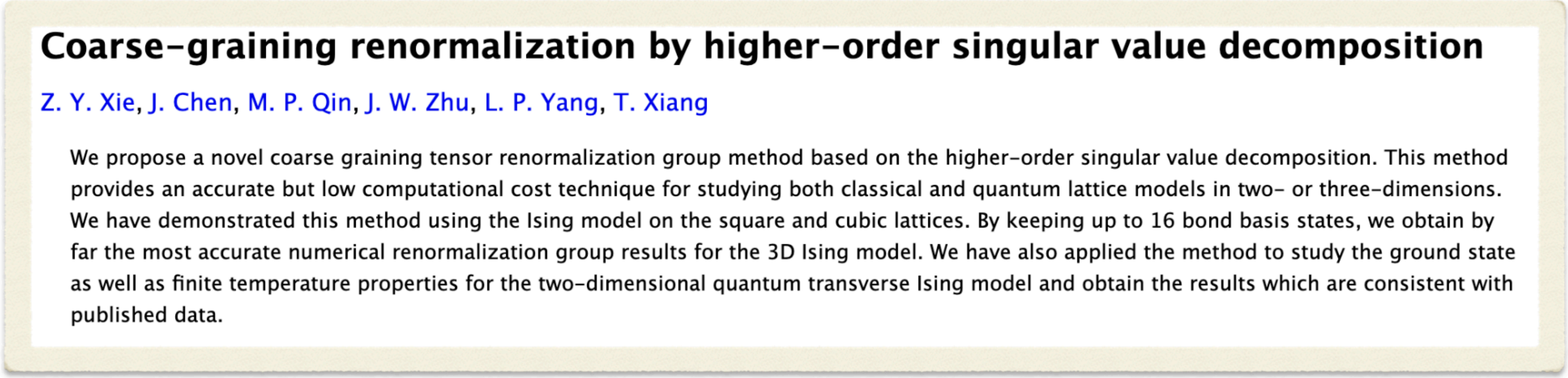






Improved TRG

In its crude form as developed by Levin and Nave in 2007, this method cannot deal with higher dimensional systems. For that, after about five years, HOTRG [higher-order] TRG was developed based on higher-order SVD (HOSVD) to reduce the errors due to truncation. First introduced in arXiv: 1201.1144 and has been successfully applied to statistical systems in $d = 3{,}4$.


Coarse-graining renormalization by higher-order singular value decomposition

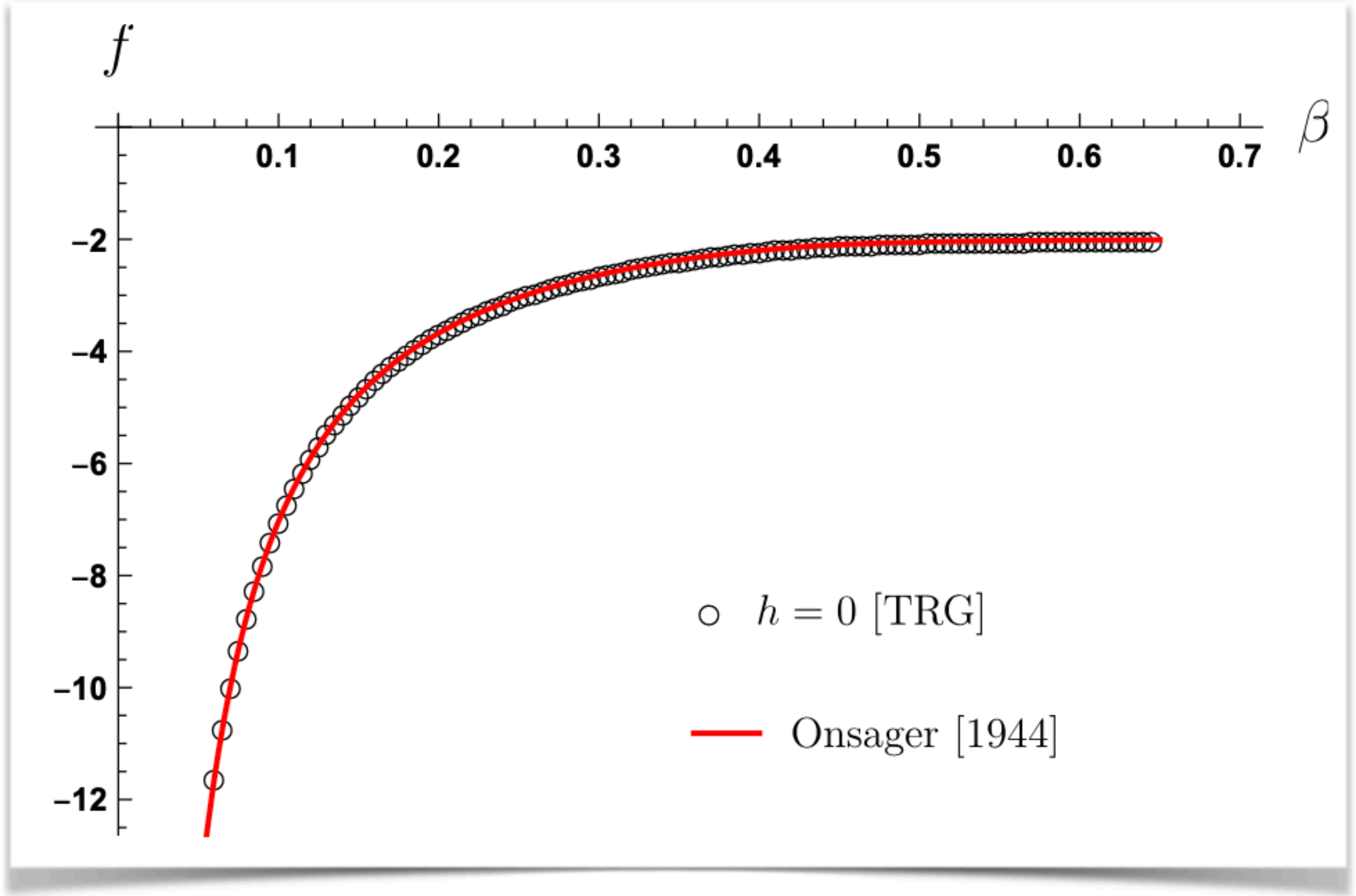
Z. Y. Xie, J. Chen, M. P. Qin, J. W. Zhu, L. P. Yang, T. Xiang

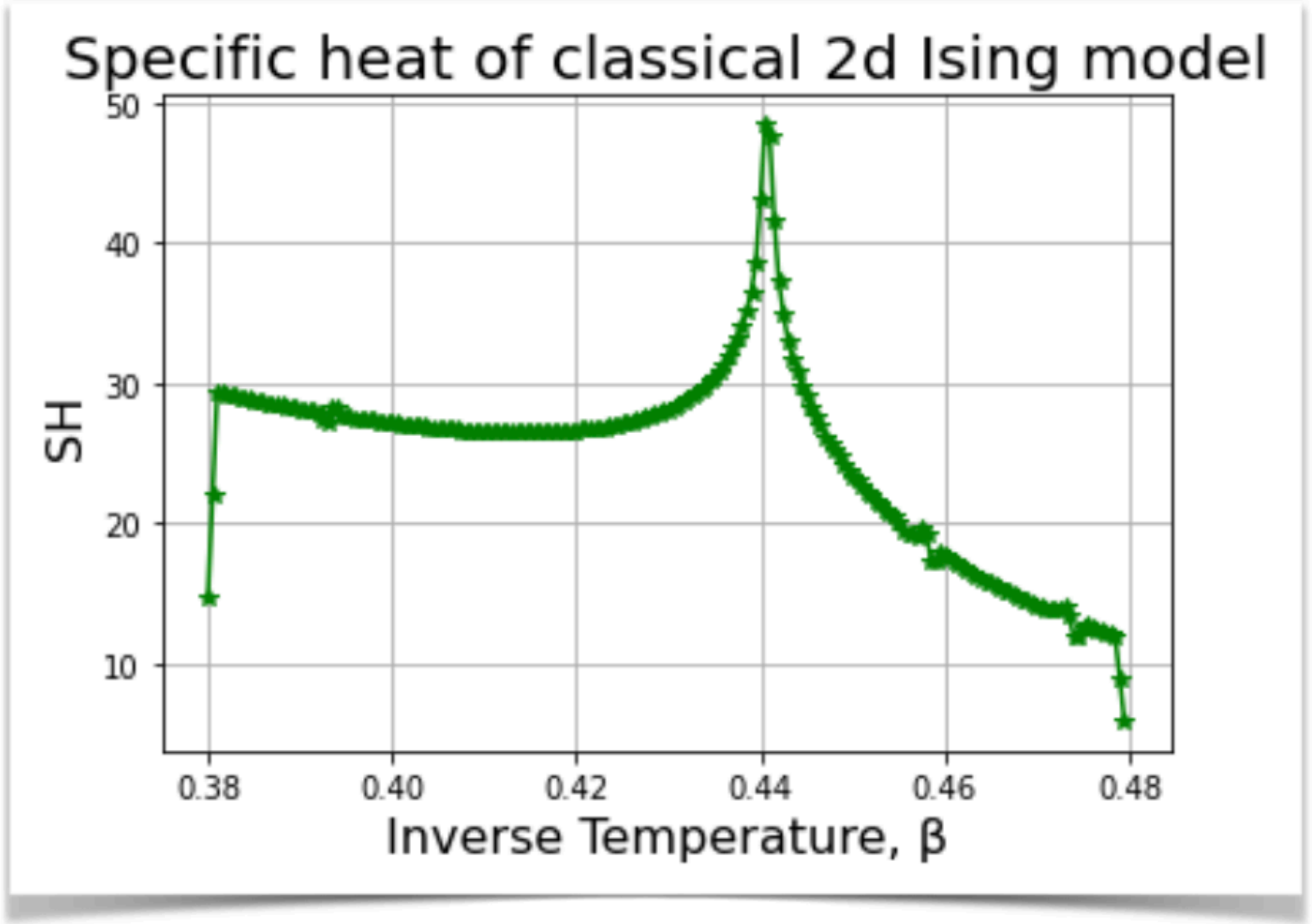
We propose a novel coarse graining tensor renormalization group method based on the higher-order singular value decomposition. This method provides an accurate but low computational cost technique for studying both classical and quantum lattice models in two- or three-dimensions. We have demonstrated this method using the Ising model on the square and cubic lattices. By keeping up to 16 bond basis states, we obtain by far the most accurate numerical renormalization group results for the 3D Ising model. We have also applied the method to study the ground state as well as finite temperature properties for the two-dimensional quantum transverse Ising model and obtain the results which are consistent with published data.

Simple demonstration

We have motivated this idea of TRG but it is best if we apply it to some simple system with known solution. Ising model is the perfect playground for this! The exact solution is given by [Onsager, 1944] with critical inverse temperature $\beta \approx 0.440687$

$$f(\beta) = -\frac{1}{\beta}\left(\ln(2) + \frac{1}{8\pi^2}\int_0^{2\pi}\int_0^{2\pi} \ln\left[2\cosh^2(2\beta) - \sinh(2\beta)\cos(\phi_1) - \sinh(2\beta)\cos(\phi_2)\right] d\phi_1 d\phi_2 \right)$$





15 seconds on modern laptop!

Ising with magnetic field

But, if we introduce magentic field, the model becomes unsolvable. It is an outstanding open problem for more close to 80 years! Some cases for imaginary magentic field values are solvable due to Yang-Lee [1952] and Merlini [1974] but for general real h, not much is known on a regular lattice. For random graph, it was solved by Kazakov and Boulatov in 1986 by a map to Hermitian two-matrix model. If we define $z = e^{-2\beta h}$, then Onsager case is $z = 1$ while Merlini solution is for $z = -1$

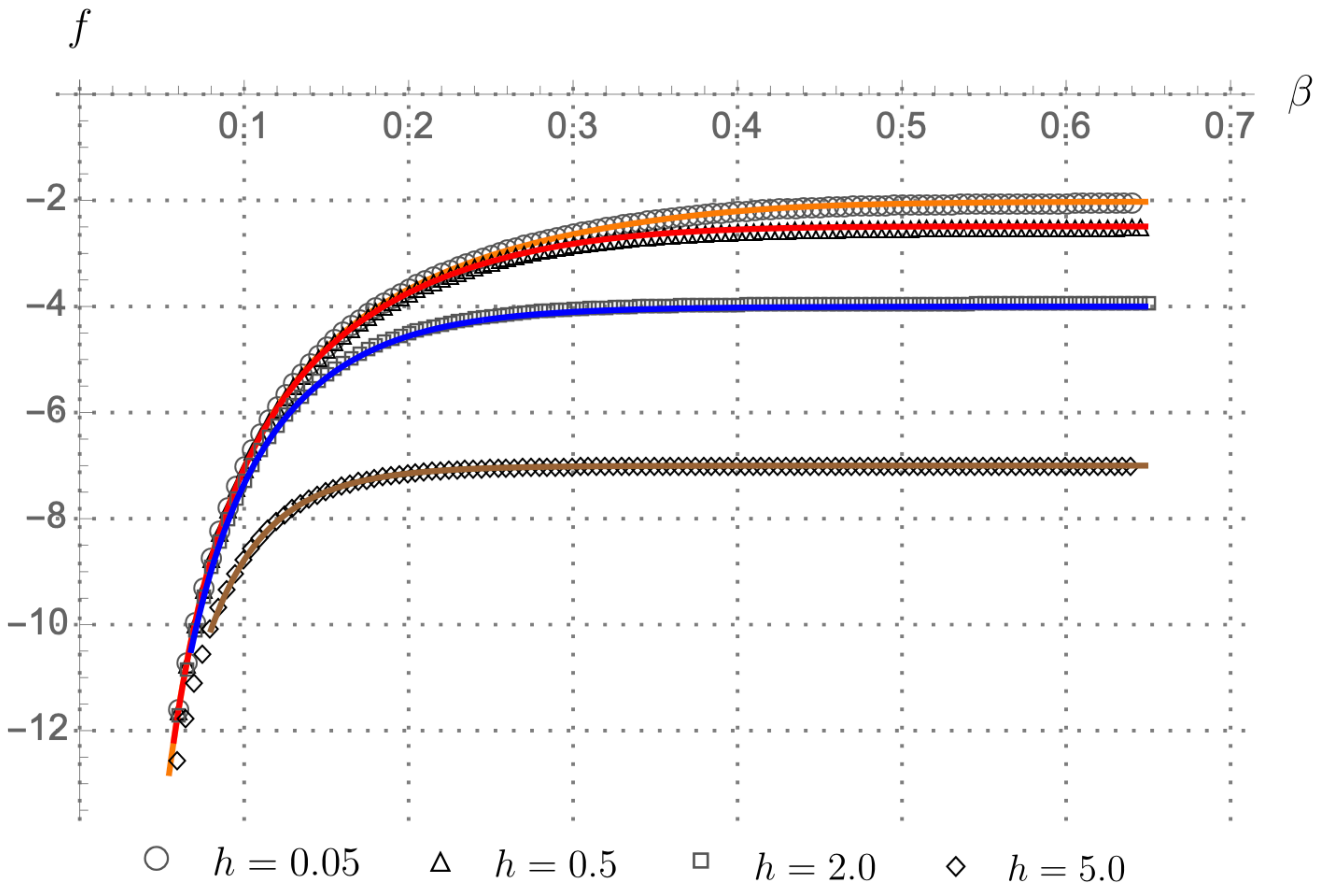
$$f\left(\beta, \frac{i\pi}{2\beta}\right) = -i\frac{\pi}{2} - \frac{1}{\beta}\left(\ln 2 + \frac{1}{16\pi^2}\int_0^{2\pi}\int_0^{2\pi} \ln\left[\sinh^2(2\beta)\left(1 + \sinh^2(2\beta) + \frac{\cos(\phi_1+\phi_2) - \cos(\phi_1-\phi_2)}{2}\right)\right] d\phi_1 d\phi_2\right).$$

On the solution of the two-dimensional ising model with an imaginary magnetic field βH=h=iπ/2

D. Merlini

Lettere al Nuovo Cimento (1971-1985) **9**, 100–104 (1974) | Cite this article

Ising with (real) magnetic field - Numerics



O(2) model

RGJ, arXiv: 2004.06314

We can study the simplest spin model with continous O(2) global symmetry using these methods. It was studied first in 2013 by Yu et al. [1309.4963] and by Vanderstraeten et al. [1907.04576]. We revisit this work and improved the results by few digits of precision for determination of the BKT phase trasnsition. The Hamiltonian is given by:

$$\mathscr{H} = -J\sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) - h\sum_i \cos\theta_i$$

In order to construct the tensor representation, we decompose the Boltzmann weight using Jacobi-Anger expansion and integrate over θ-variables:

$$\exp\left(\beta\cos(\theta_i - \theta_j)\right) = I_0(\beta) + \sum_{\nu=-\infty,\neq 0}^{\infty} I_\nu(\beta) e^{i\nu(\theta_i - \theta_j)}$$

$$T_{ijkl} = \sqrt{I_i(\beta)I_j(\beta)I_k(\beta)I_l(\beta)}\, I_{i+k-j-l}(\beta h)$$

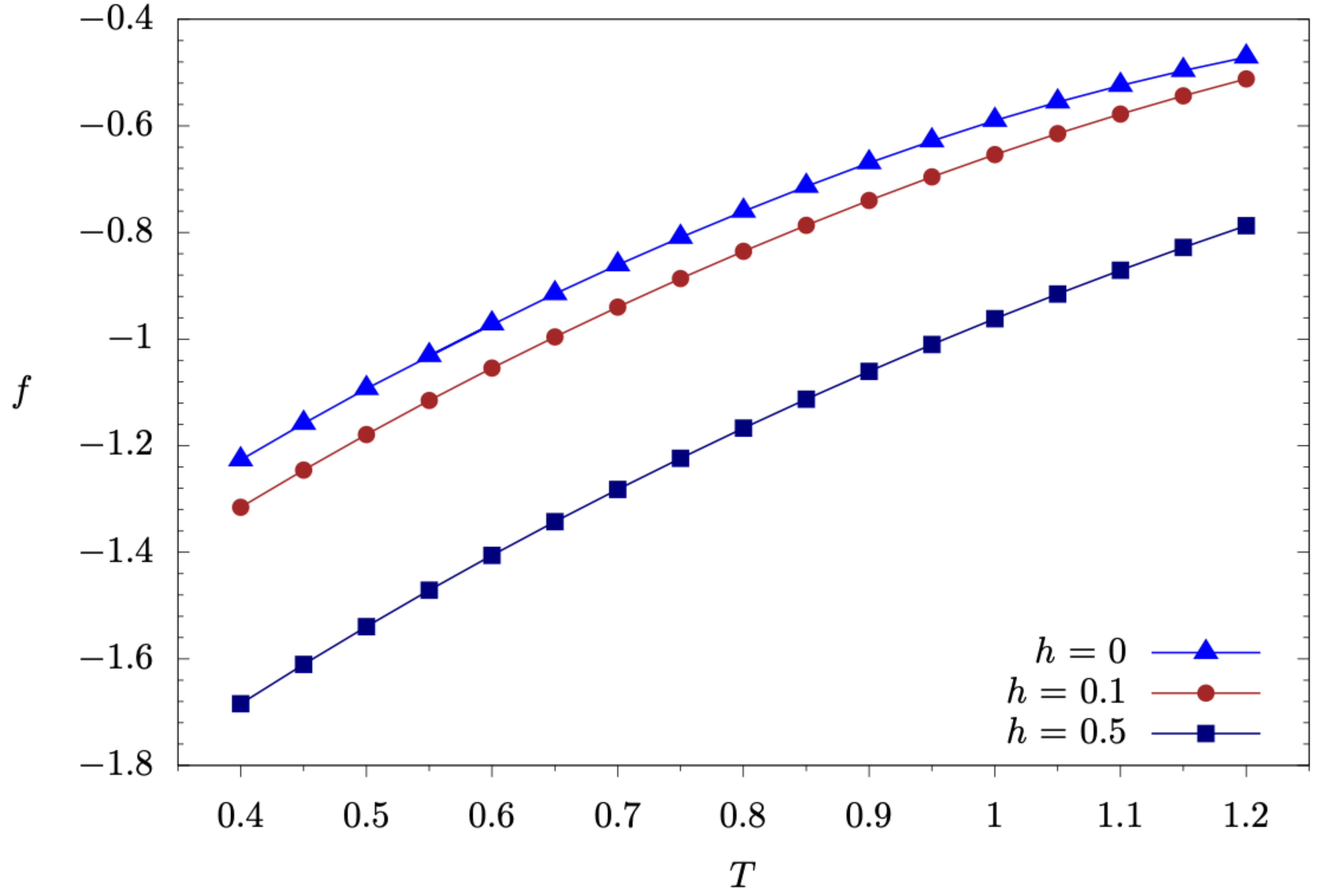
O(2) model

RGJ, arXiv: 2004.06314

The δ-function for $h = 0$ ensures the conservation of U(1) charges. This model has a famous BKT transition corresponding to unbinding of vortex pairs. Note that in two dimensions, continuous symmetry cannot break due to the famous Mermin-Wagner -Hohenberg-Coleman theorem and hence one might expect no phase transition but the BKT transition is special case. The transition is from a quasi-long range ordered (QLRO) to a disordered phase. At some temperature, all the vortices and anti-vortices are free to move, which destroys the correlations between distant spins and breaks QLRO. It was the first example of a topological phase transition. It is of infinite order in Ehrenfest classification sense - “none of the derivatives of free energy is discontinuous”.

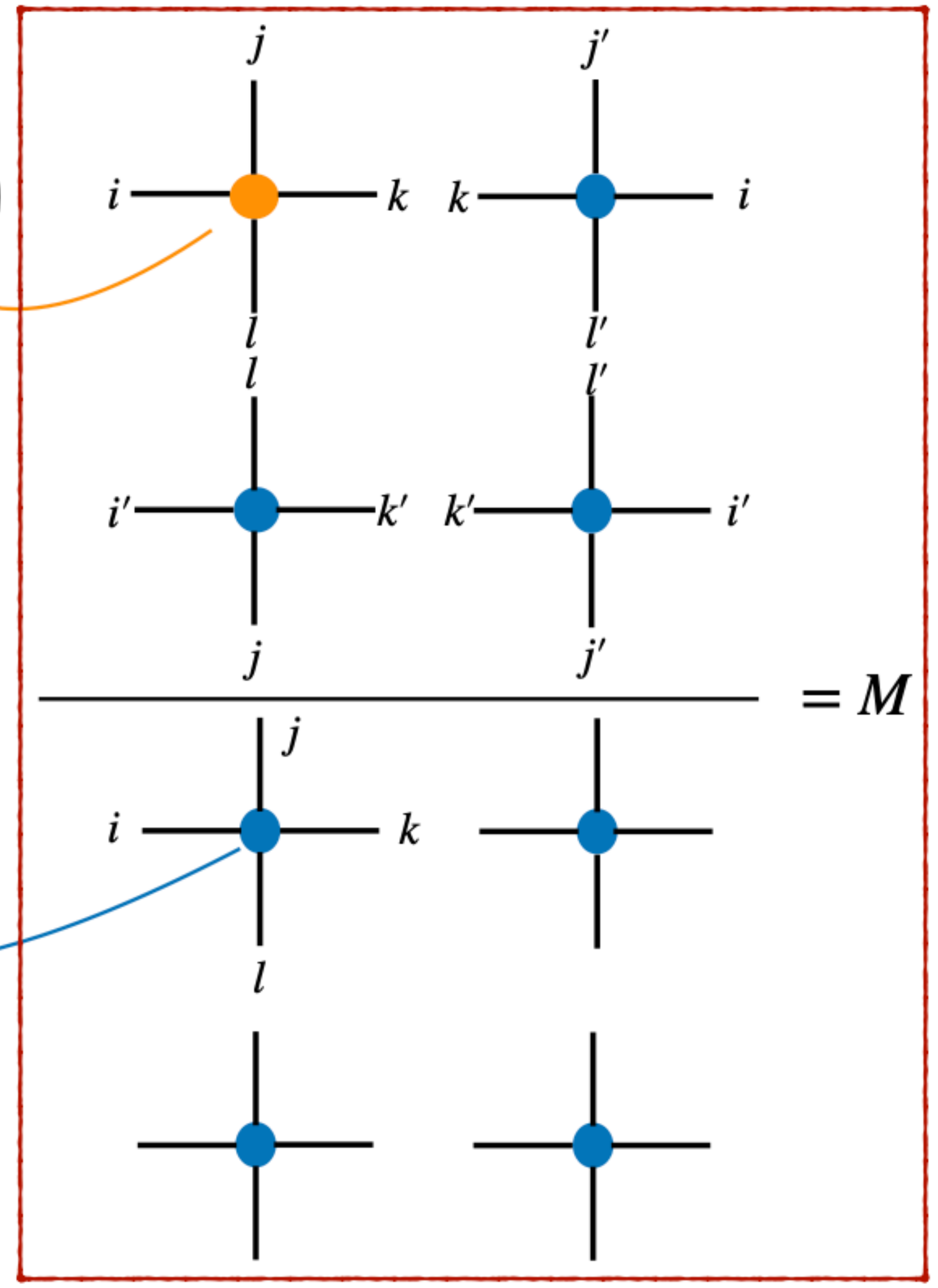
Results - O(2) model

RGJ, arXiv: 2004.06314



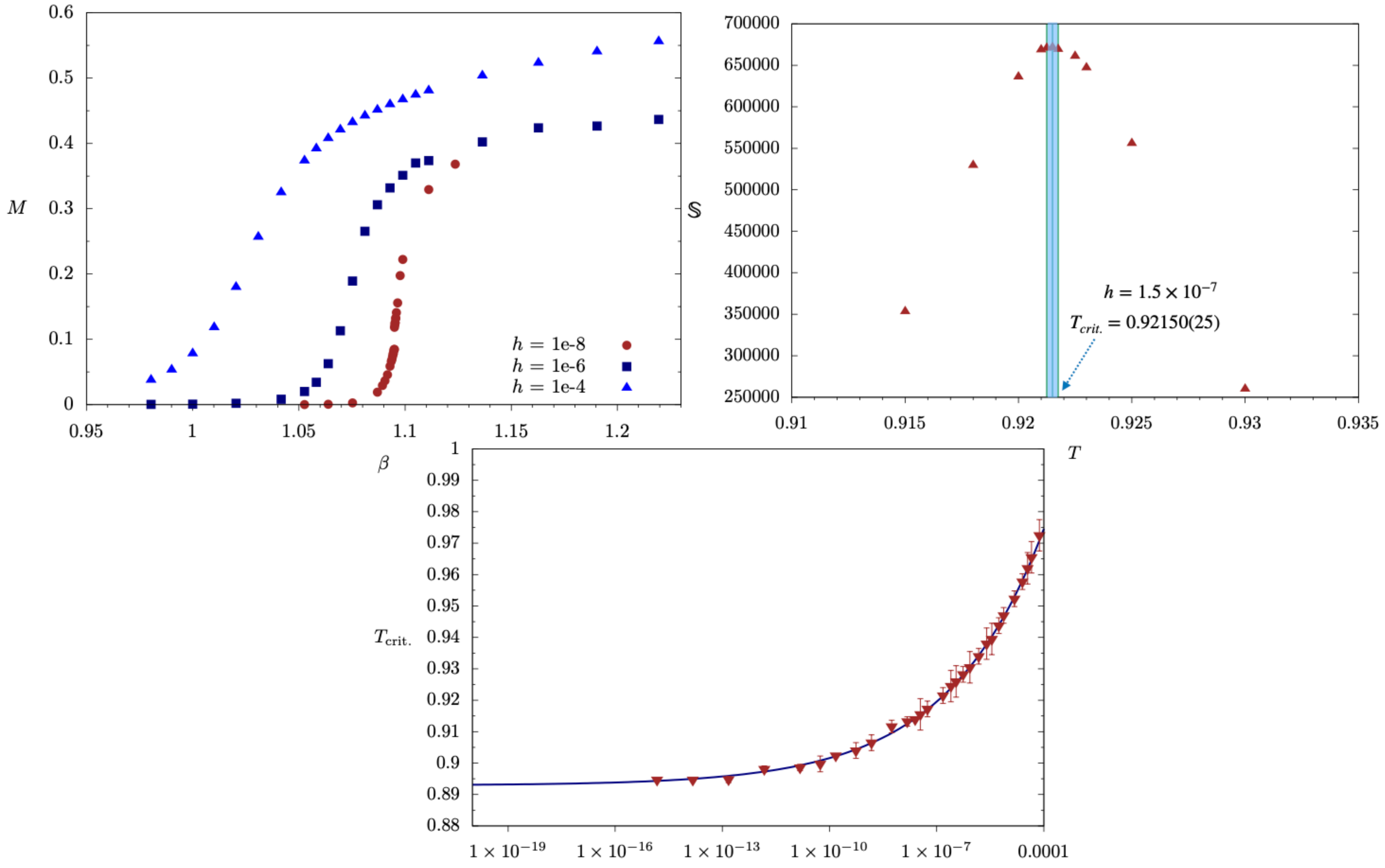
$$\tilde{T}_{ikjl} = \sqrt{I_i(\beta)I_k(\beta)I_j(\beta)I_l(\beta)}\left(\frac{I_{i+j-k-l-1}(\beta h) + I_{i+j-k-l+1}(\beta h)}{2}\right)$$

$$T_{ikjl} = \sqrt{I_i(\beta)I_k(\beta)I_j(\beta)I_l(\beta)}\,I_{i+j-k-l}(\beta h)$$



Results - O(2) model

RGJ, arXiv: 2004.06314



SU(2) gauge/Higgs

Bazavov, Catterall, RGJ, U-Yockey, arXiv: 1901.11443

One of the interesting applications of tensor networks have been to study some simple gauge theories in two and higher dimensions. Suppose we consider the Wilson's SU(2) lattice action in unitary gauge given by:

$$S = -\beta \mathrm{Tr} \square - \kappa \mathrm{Tr U}$$

where β is the gauge coupling and κ is the matter coupling. We fix to unitary gauge as done in earlier works by Greensite et al. [also Fradkin-Shenker-Osterwalder-Seiler (FSOS)]. Once we do the character (strong coupling) expansion, we can construct the initial tensors. However, we have to truncate over the irreps. of the SU(2) group. This truncation is not very well-understood and this is a major area of research. These truncations are also required when we want to explore quantum simulation of gauge theories using qubits.

SU(2) gauge/Higgs

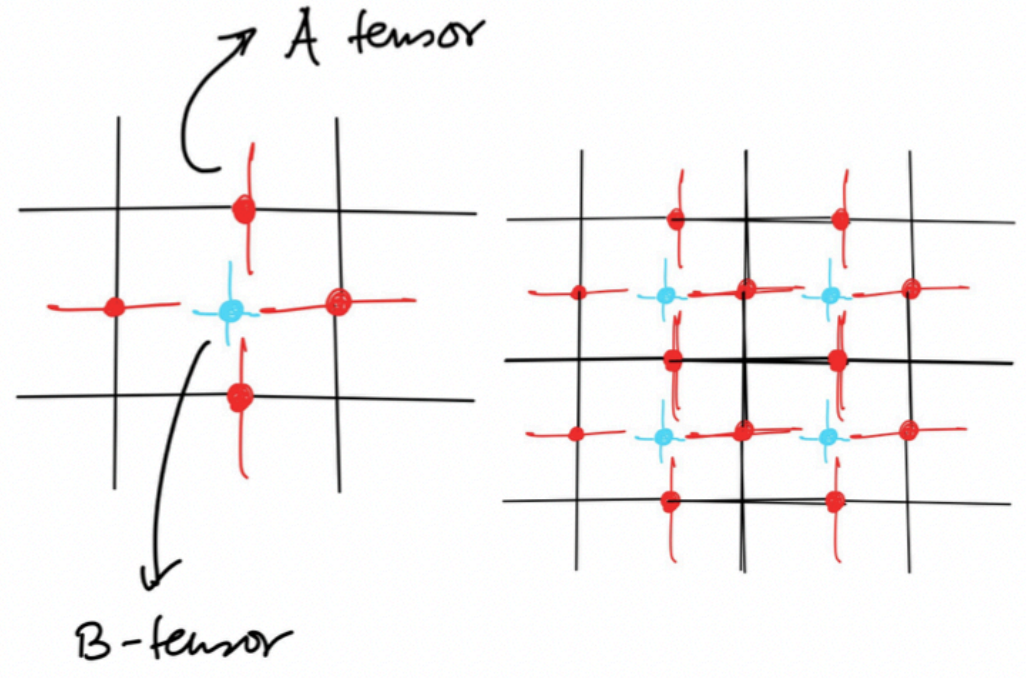
Bazavov, Catterall, RGJ, U-Yockey, arXiv: 1901.11443

The rank-4 initial tensor (T) can be decomposed in terms of link (A) and plaquette operators (B) (very similar in spirit to Kitaev's toric code which is often prototype of QEC) as:

$$A_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb})}(\kappa) = \frac{1}{d_{r_r}} \sum_{\sigma=|r_r - r_l|}^{r_r + r_l} F_\sigma(\kappa) C^{r_r m_{rb}}_{r_l m_{lb} \sigma (m_{rb} - m_{lb})} \times C^{r_r m_{ra}}_{r_l m_{la} \sigma (m_{rb} - m_{lb})} .$$

$$B_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb})(r_a m_{al} m_{ar})(r_b m_{bl} m_{br})} = \begin{cases} F_r(\beta) \quad \delta_{m_{la}, m_{al}} \delta_{m_{ar}, m_{ra}} \delta_{m_{rb}, m_{br}} \delta_{m_{bl}, m_{lb}} & \text{if } r_l = r_r = r_a = r_b = r \\ 0 \quad \text{else.} \end{cases}$$

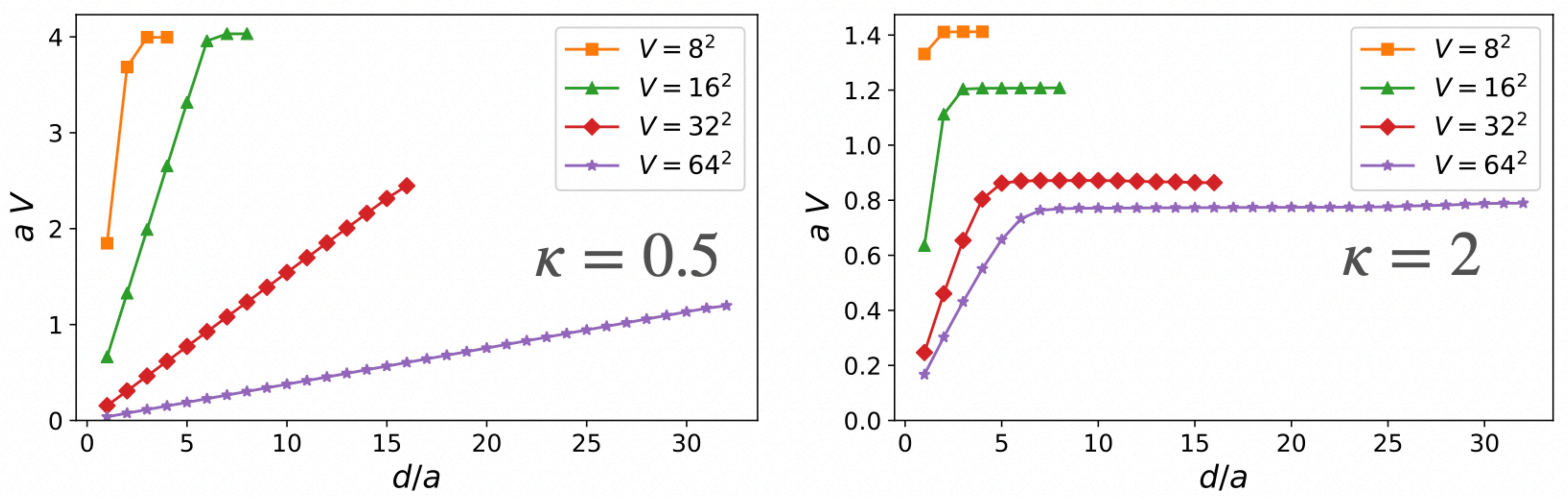
$$T_{ijkl}(\beta, \kappa) = \sum_{\alpha,\beta,\gamma,\delta} B_{\alpha\beta\gamma\delta}(\beta) L_{\alpha i} L_{\beta j} L_{\gamma k} L_{\delta l}(\kappa), \quad \text{where} \quad A_{ij} = \sum_k L_{ik} L^T_{kj}$$



Results

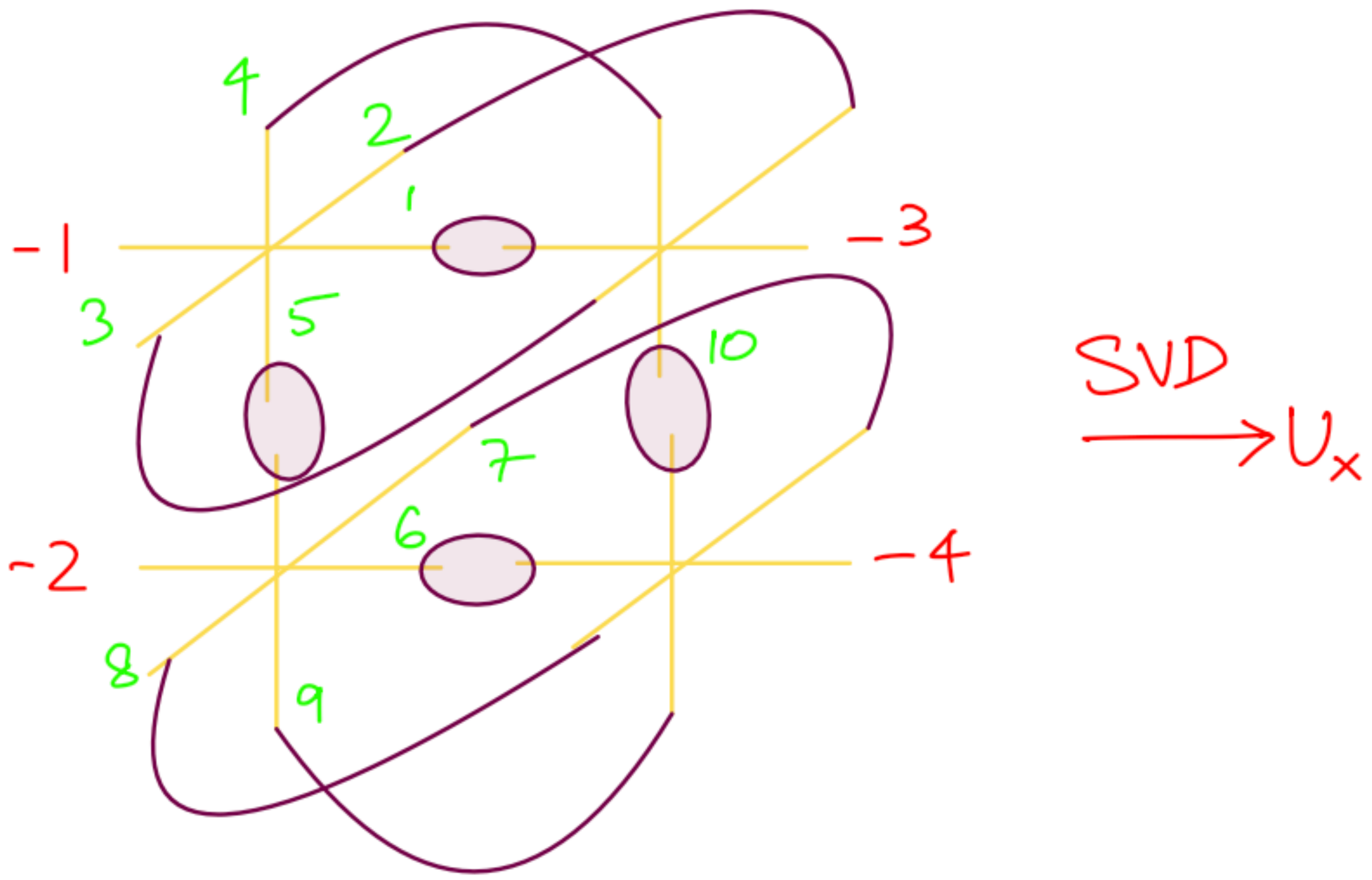
Bazavov, Catterall, RGJ, U-Yockey, arXiv: 1901.11443

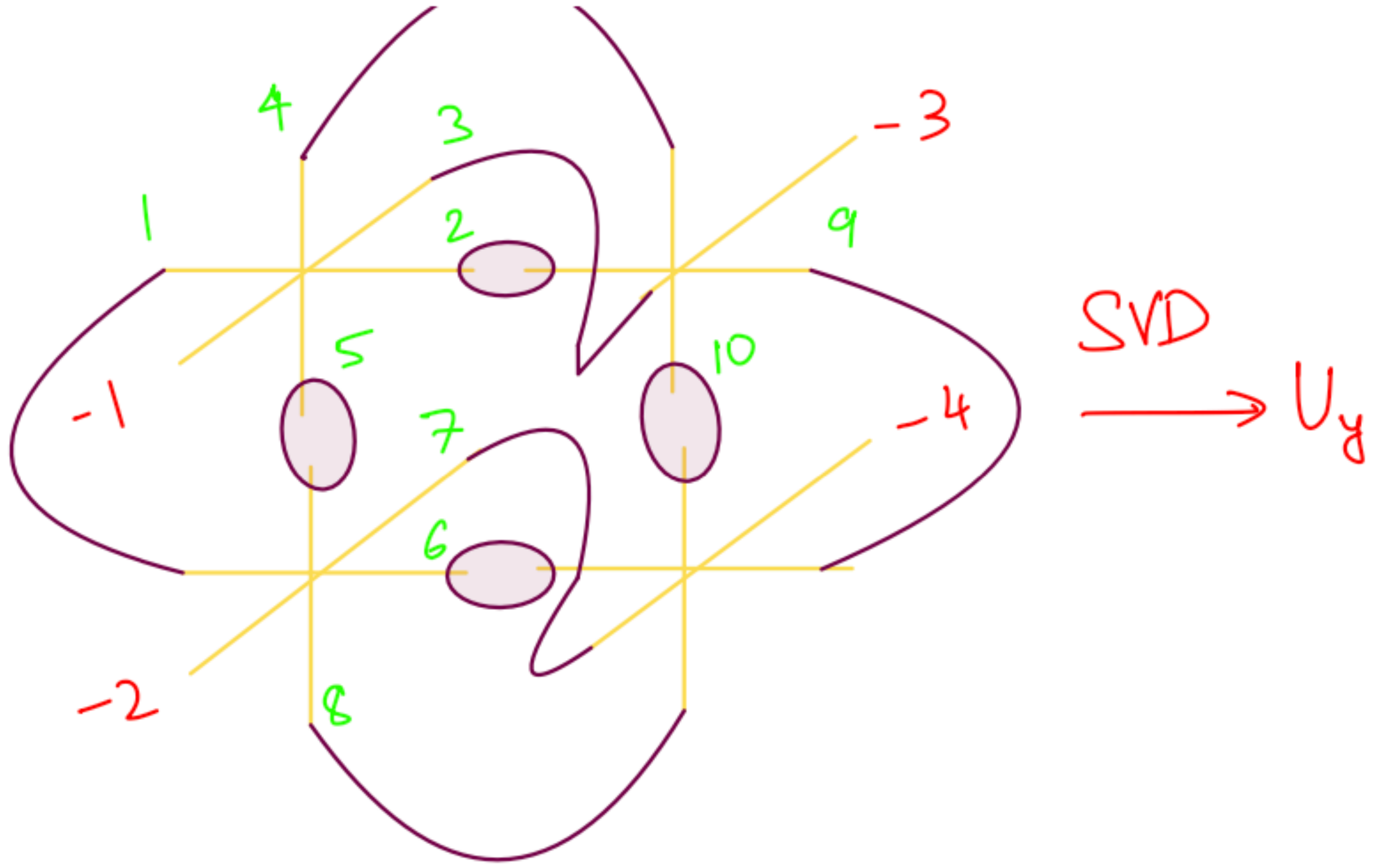
We studied several observables but the most important was to compute Polyakov loop correlator given by: $C(d) \sim \exp(-\beta V(d))$. This observable is often used in lattice QCD to monitor confinement (area law) when $V(d) \propto d$. In Higgs phase, it is expected to be independent of d. Our results are consistent with earlier MC studies done in arXiv: 1402.7124 with hints of crossover around $\kappa \sim 1.4$



Moving to 3d

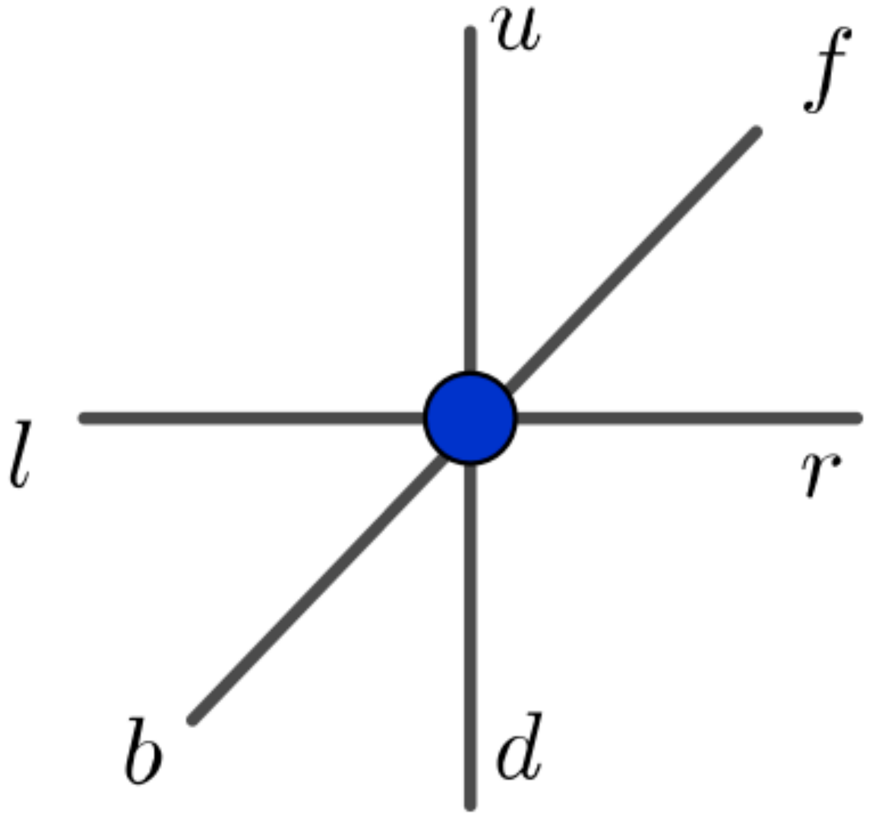
Though tensor methods works very well for lower-dimensional systems, it was not explored much for $d \geq 3$ because of several problems involved (computer time which scales like $O(D^{4d-1})$, memory requirements, effects of truncation etc.).



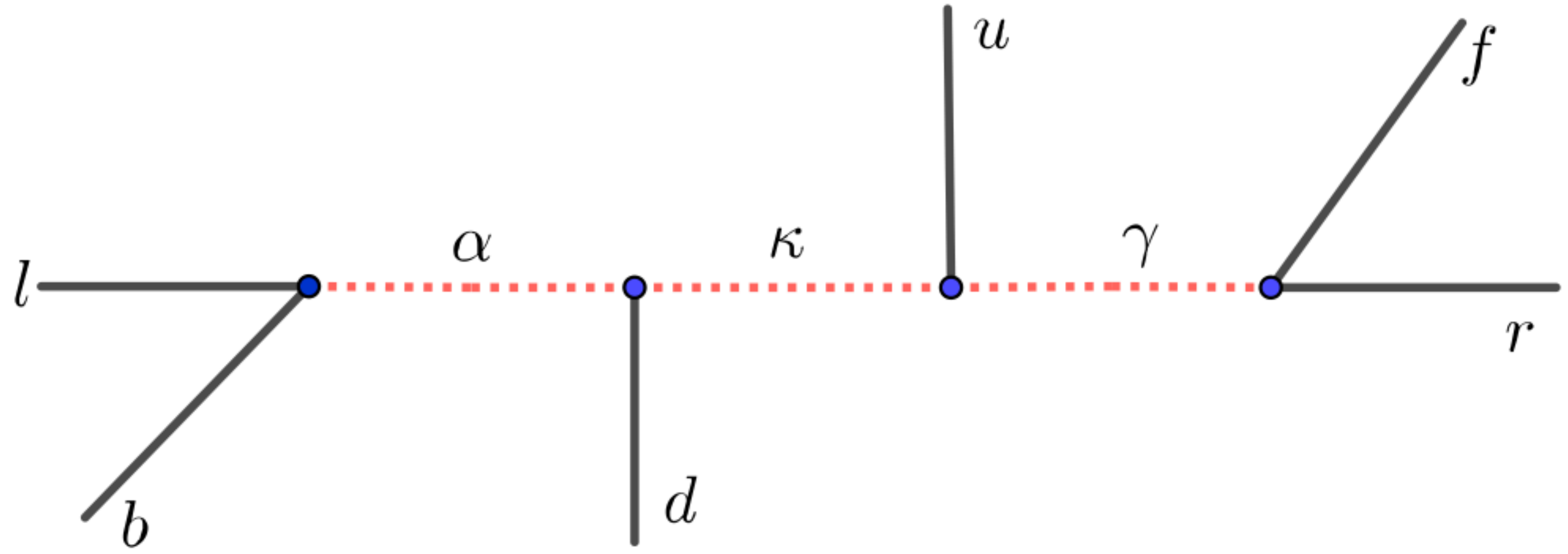


Recent development: Triad method

In 2019, Kadoh et al. found that it is often faster to deal with not the rank-six tensor in 3d directly, but decompose it in terms of several rank-three tensor known as triads. This reduces the cost drastically and we can study some statistical systems which were more difficult before.

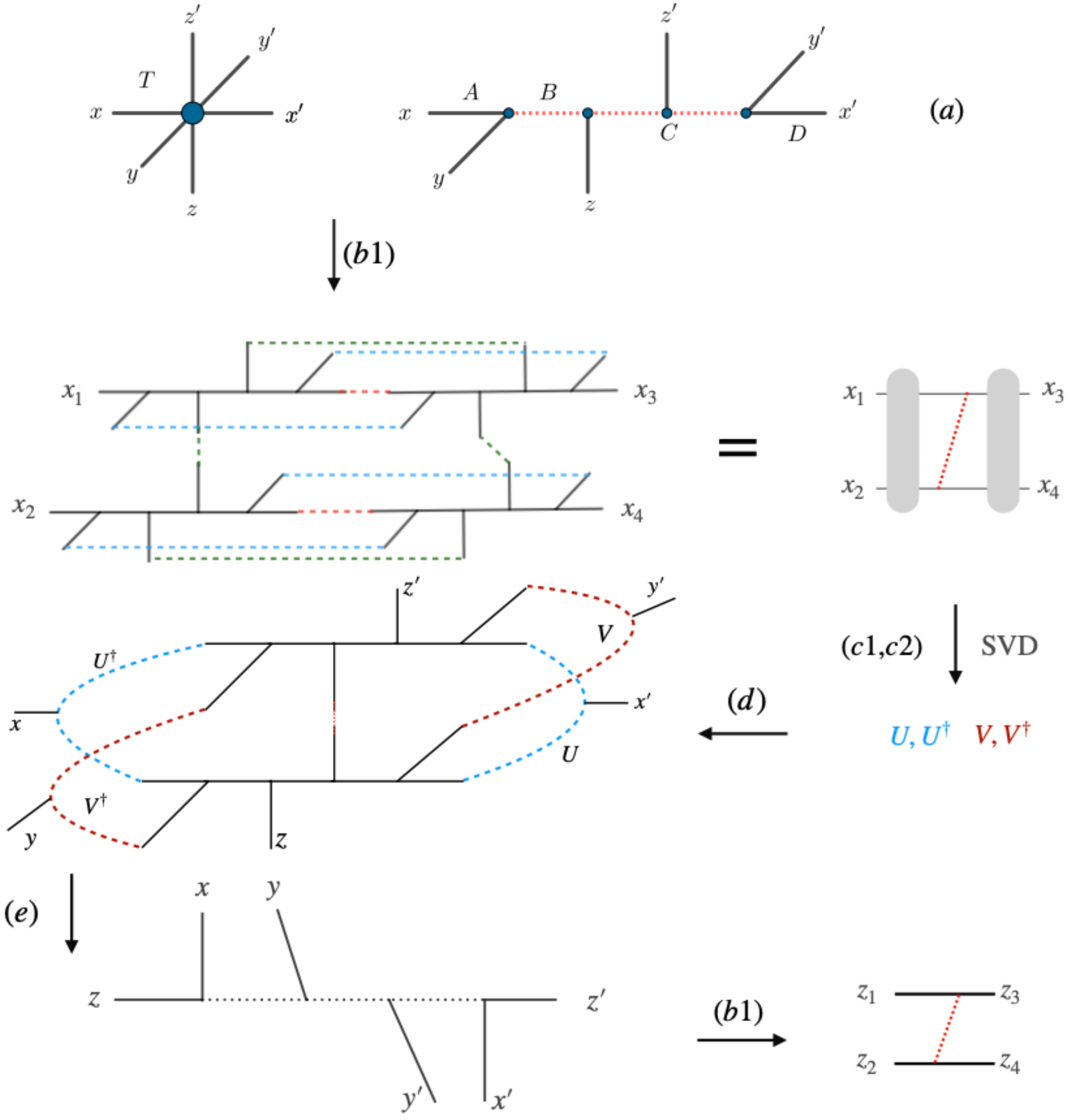


T_{lrudfb}



$A_{lb\alpha}B_{\alpha d\kappa}C_{\kappa u\gamma}D_{\gamma fr}$

Basic step in Triad TRG



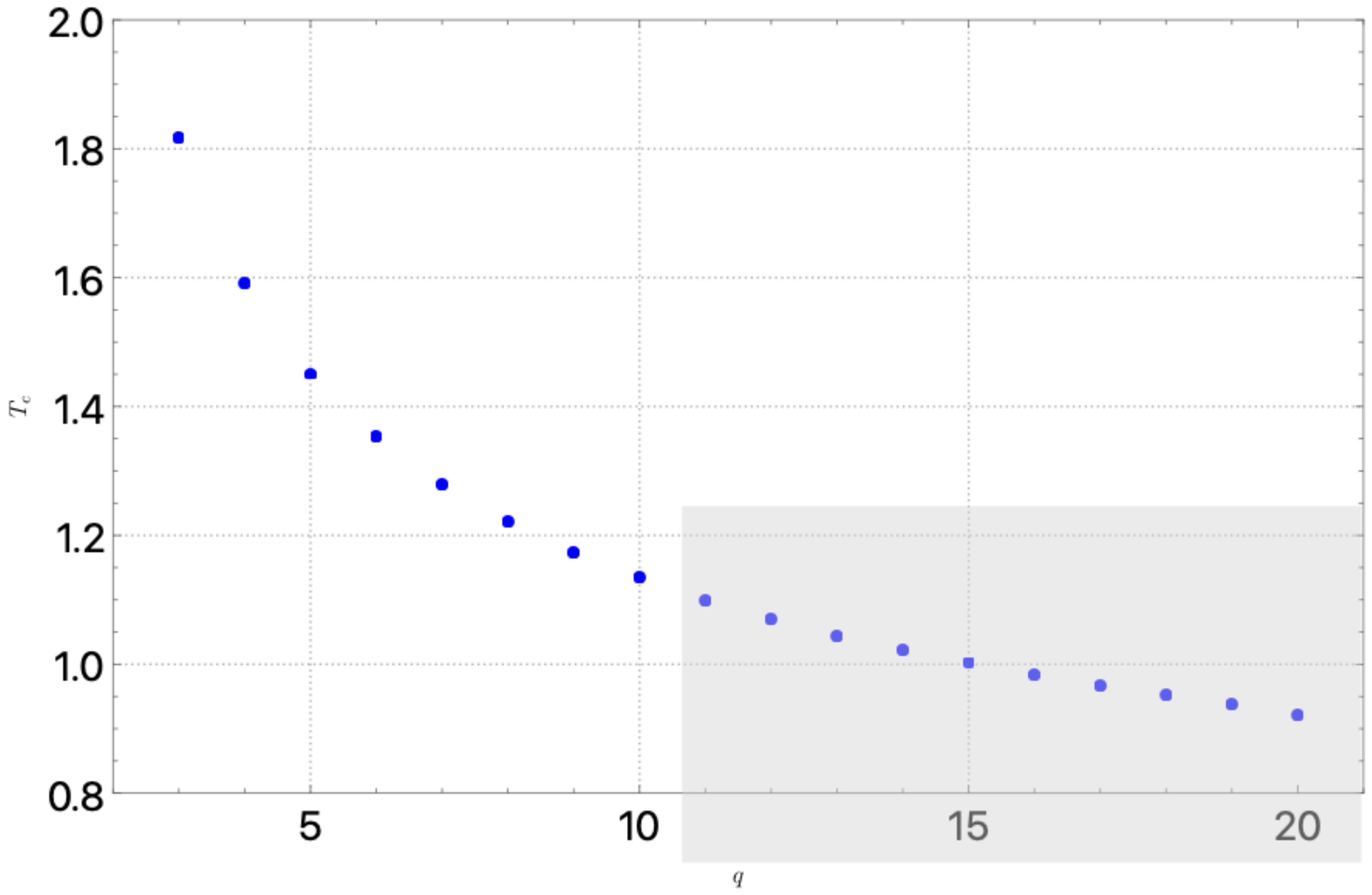
Status of 3d spin systems [w tensors]

Apart from Ising model on cubic lattice, not much had been done for Potts model or even the O(2) model.

- Ising model studied but critical exponents not yet computed!
- q-state Potts model in the large q limit [RGJ, arXiv: 2201.01789]
- $O(2)$ model at finite density [First study: RGJ, Bloch, Lohmayer, Meister, arXiv: 2105.08066]

Potts model

We can generalize the local Hilbert dimension of Ising model by allowing for local $\dim(\mathscr{H}) = \mathrm{q}$ with large q. This problem had been considered using Monte Carlo for $3 \leq q \leq 10$ however it soon becomes diffi cult. We explore this using tensor methods and could locate the phase transition for $10 \leq q \leq 20$. Note that unlike in two dimensions, there is no known analytic expression for $T_c(q)$. Find an expression, long outstanding problem. See Baxter (1982)



New results, arXiv: 2201.01789

3d O(2) model

The model is well-studied by various methods such as Monte-Carlo and conformal bootstrap. In fact, the motivation of using tensor methods to this model is to use a third approach. In bootstrap, one studies the scaling dimension of charge zero scalar operator which is related to critical exponent as ν = 1/(3 - Δ_s) while α = 2 - $d\nu$. The tensor methods we used was good enough to study thermodynamical observables but were not accurate enough to compute these coefficients yet! So, matching to Monte Carlo and CB for scaling dimensions of operators at critical point seems several years away and would need some way of optimizing the triad algorithm we used or finding a much more efficient way of doing tensor RG in three dimensions! Very much an open problem.

3d O(2) model - general formulation

The action is given by:

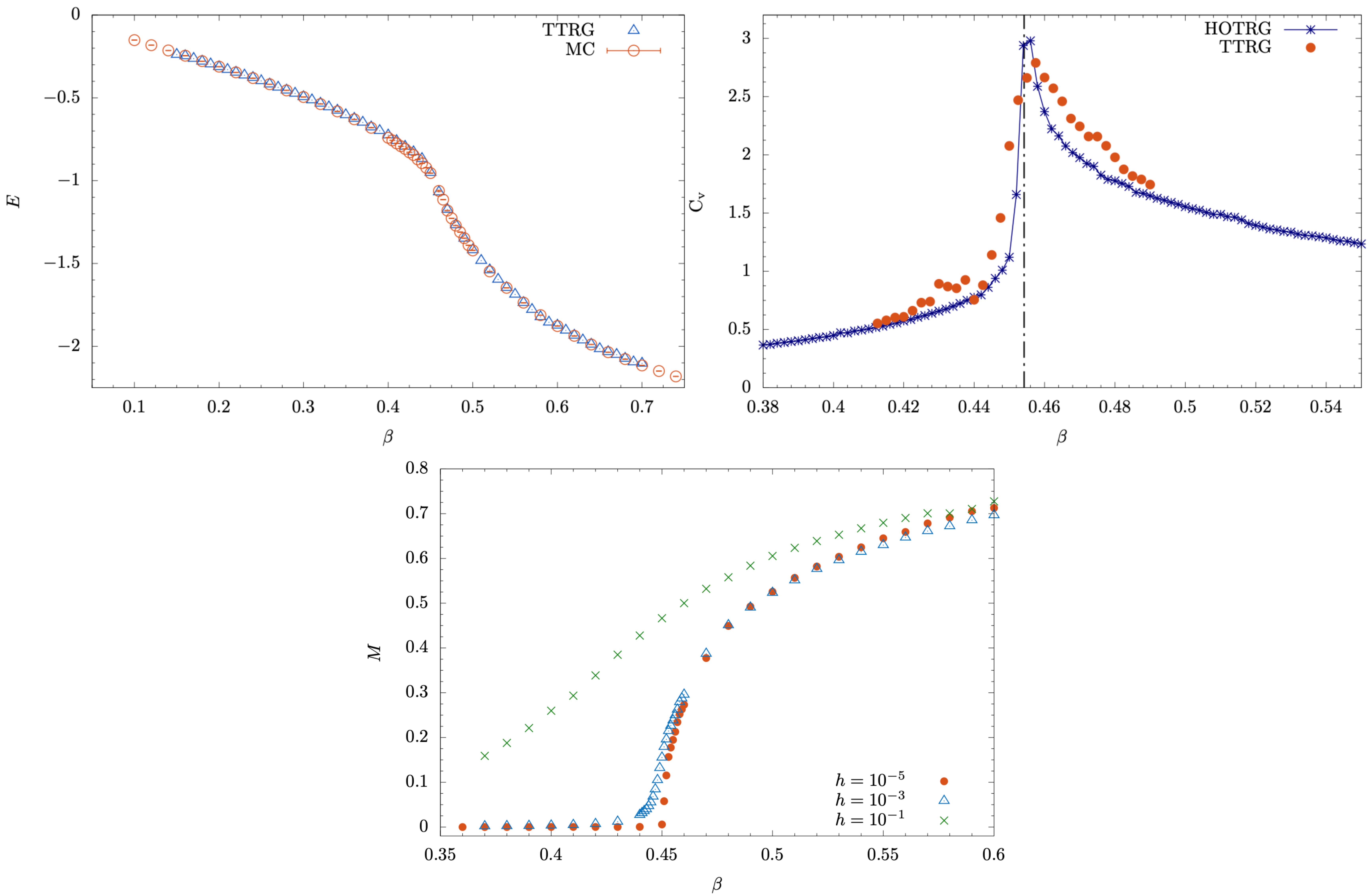
$$S = -\beta \sum_j \sum_{\nu=0}^{2} \cos(\theta_j - \theta_{j+\hat{\nu}} - i\mu\delta_{\nu,0}) - \beta h \sum_{i=1}^{V} \cos(\theta_i)$$

and the initial tensor can be written (similar to 2d case) as:

$$T_{lrudfb} = \sqrt{IIIIII(\beta) e^{\mu(u+d)}} I_{l+u+f-r-d-b}(\beta h)$$

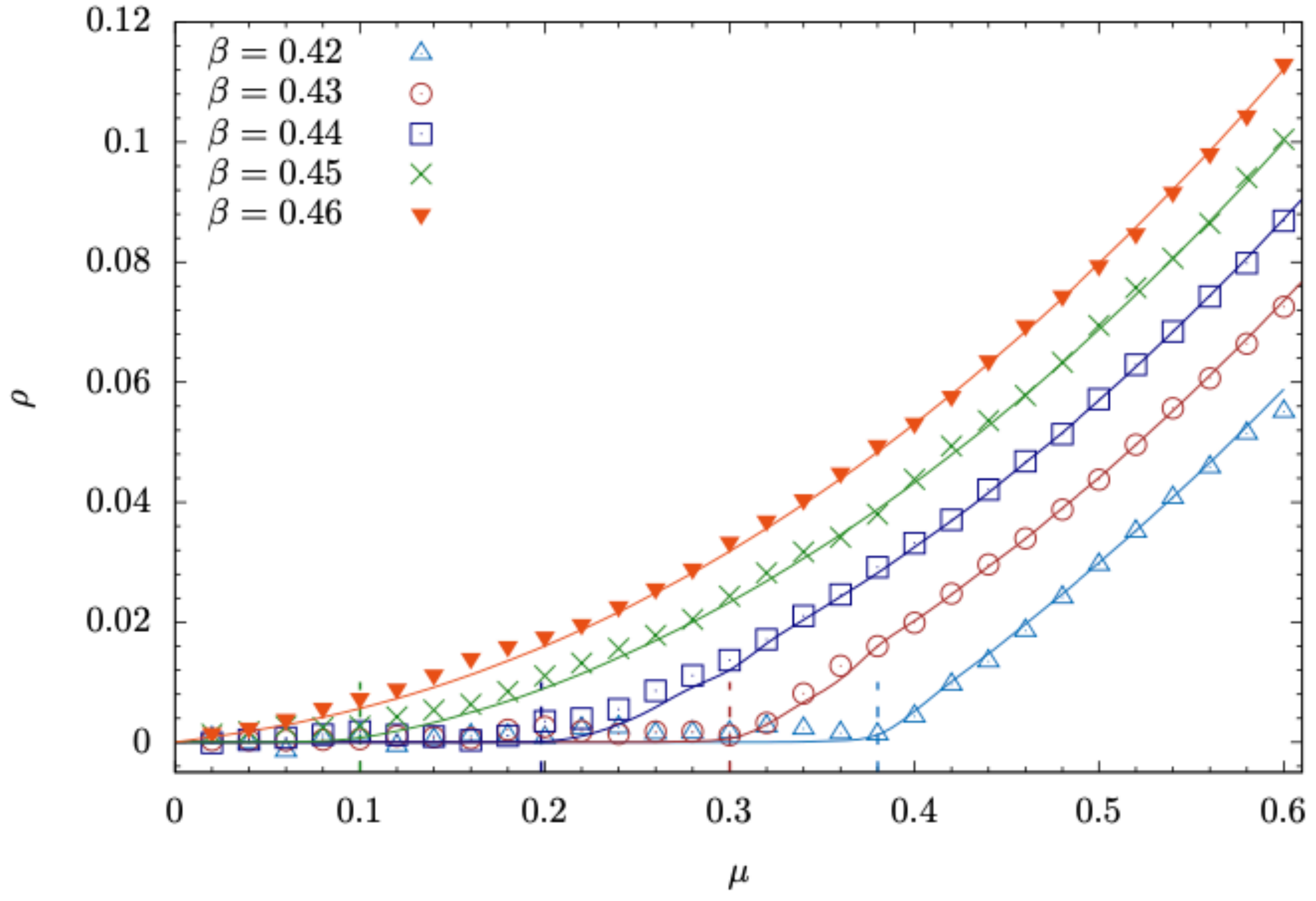
Note that the presence of studied by usual Monte Carlo methods since the action is complex. Tensor networks are promising for studying these systems and those with topological θ-term. A very long-term goal is to study finite-density QCD!

3d O(2) model - Results



3d O(2)- Finite μ and Silver Blaze

If we consider finite-μ, there is a very interesting phenomenon well-studied in QCD literature called `silver-blaze'. For this we need to compute the particle number density as: $\rho = \partial_\mu \ln Z$. This quantity remains zero until some critical μ_c (proportional to mass gap) if the theory is gapped as is 3d O(2) for $\beta < \beta_C$ and even though μ is part of partition function, it does nothing.



Real-time scattering in IFT

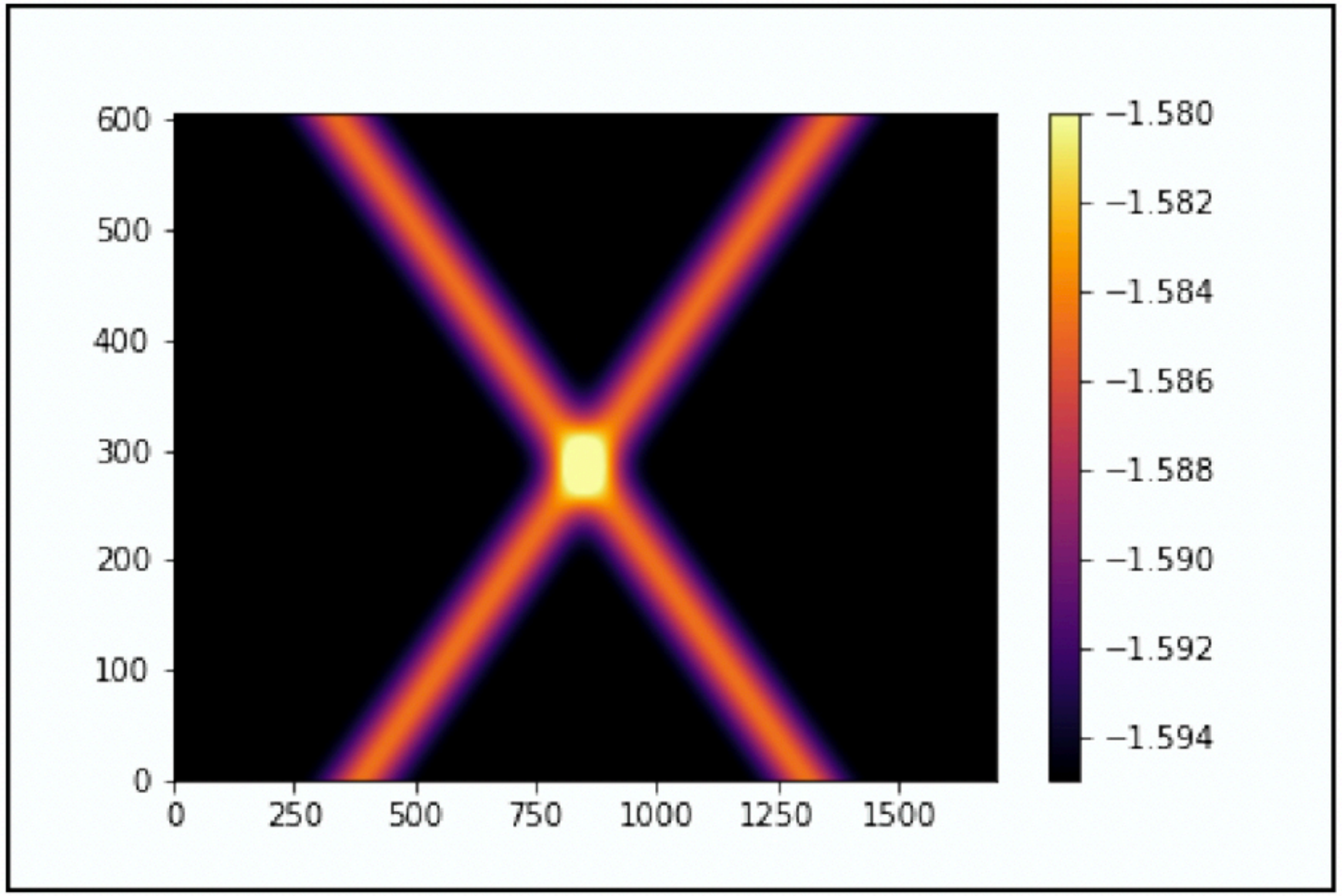


Figure Credits: Ashley Milsted, Dominik Neuenfeld

MPS approach to scattering in IFT

in progress, Milsted et al.

With tensor network methods, we can approximate the ground state of quantum Ising chain with local Hamiltonian $H = -\sum Z_i Z_{i+1} - hZ_i + gX_i$ where we take the double scaling limit $h \to 0,\ g \to 1$ (corresponding to critical temp in 2d classical case). If we define $\tau = T/T_C - 1$, then the RG parameter $\eta = \tau/|h|^{8/15}$ determines the behaviour of the model. Zamolodchikov found that the model is integrable for $\eta = 0$ and he computed the mass spectrum which consists of eight particles (three below the threshold of $2m$) and five owing their stability to integrability. This model also has another integrable limit of $\eta \to \infty$ for all τ. We first start with a random MPS and through imaginary time evolution, find the ground state of the H given above.

Stable particles in $h - \tau$ plane

in progress, Milsted et al.

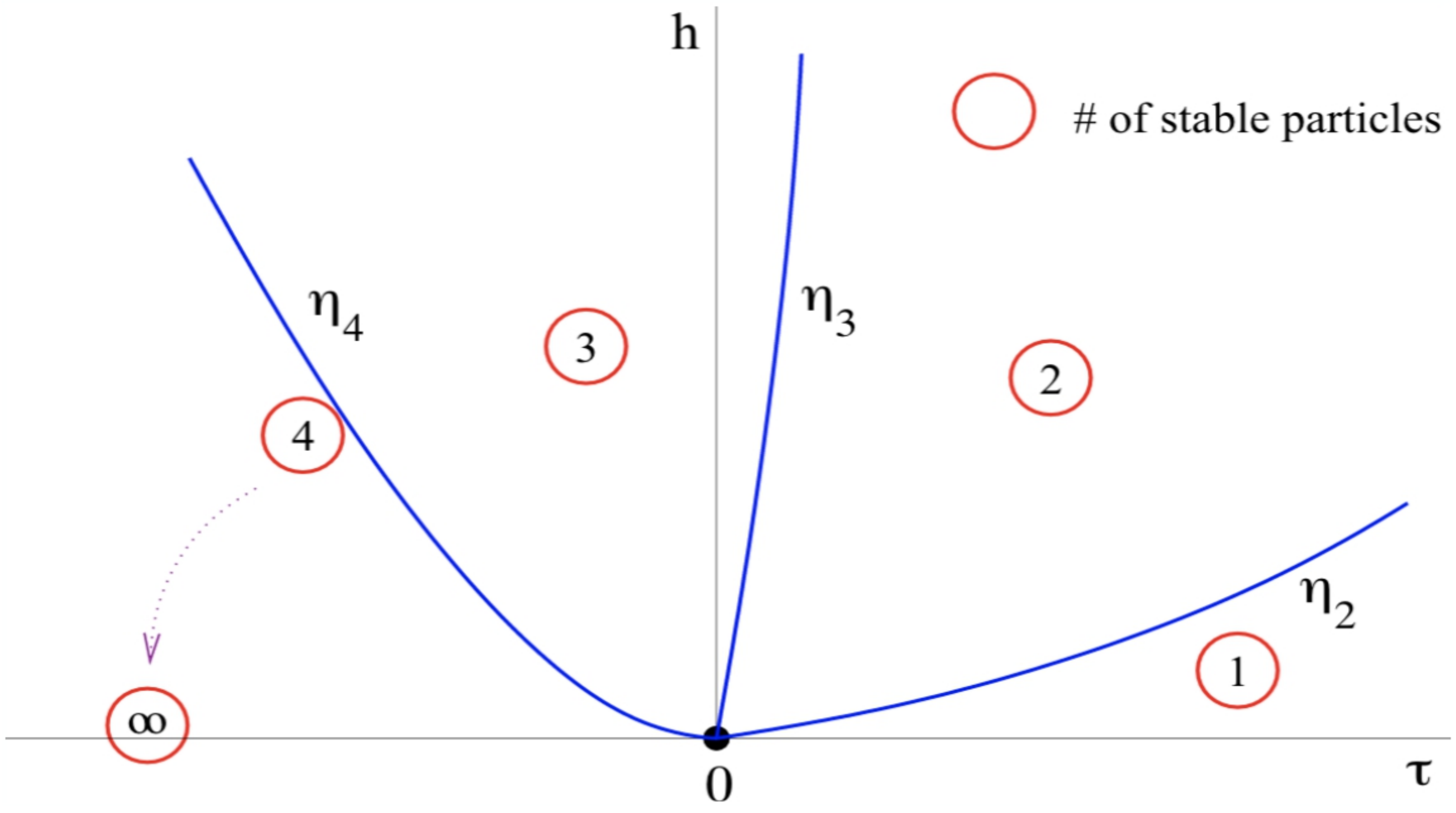


Figure Credits: G. Delfino
Delfino, Mussardo et al, arXiv: hep-th/0507133

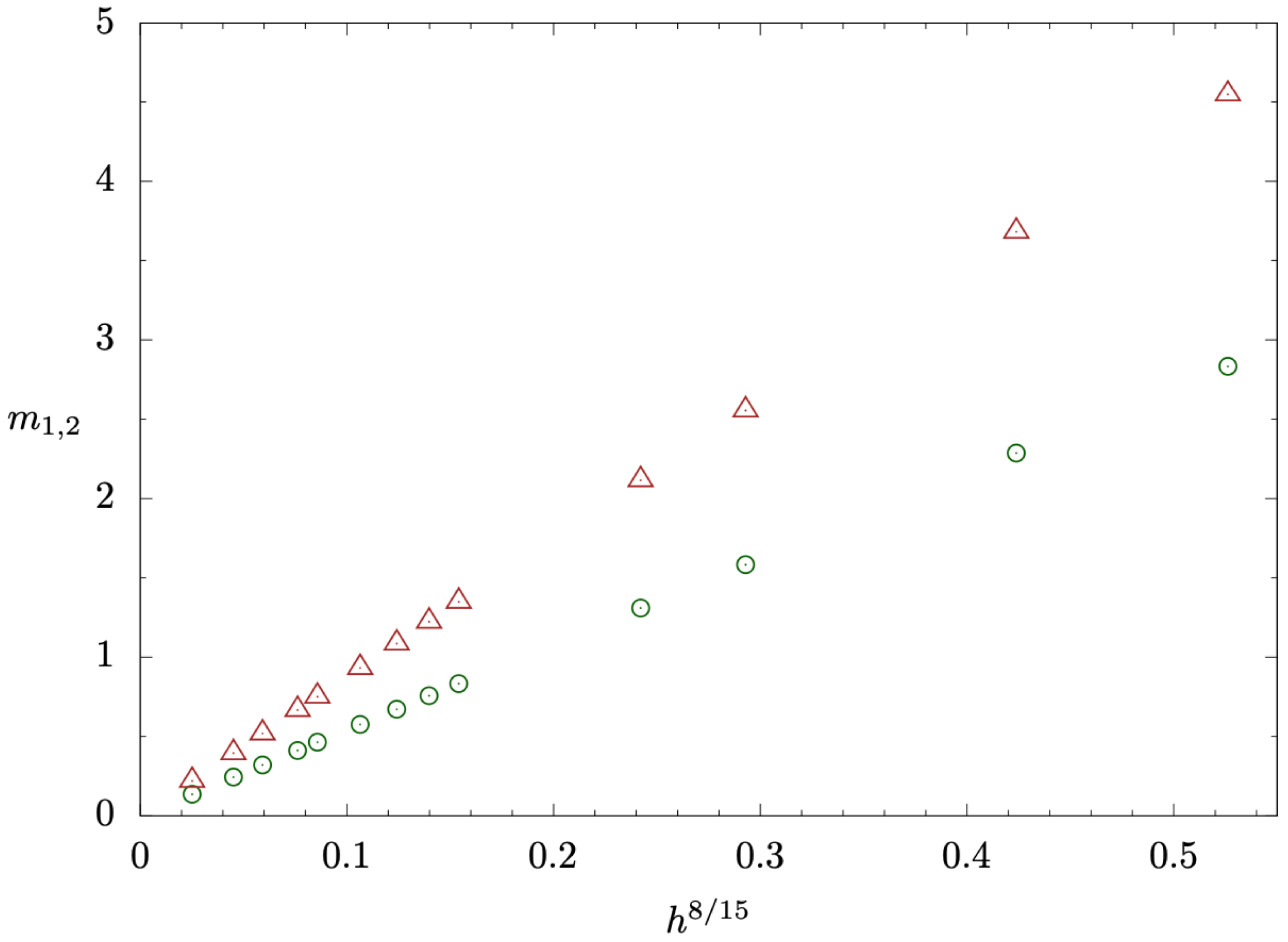
Spectrum at E_8 point

$$
\begin{aligned}
m_2 &= 2\cos\tfrac{\pi}{5} m_1 &&\approx 1.618 m_1 \\
m_3 &= 2\cos\tfrac{\pi}{30} m_1 &&\approx 1.989 m_1 \\
m_4 &= 2\cos\tfrac{\pi}{5}\cos\tfrac{7\pi}{30} m_1 &&\approx 2.405 m_1 \\
m_5 &= 4\cos\tfrac{\pi}{5}\cos\tfrac{2\pi}{15} m_1 &&\approx 2.956 m_1 \\
m_6 &= 4\cos\tfrac{\pi}{5}\cos\tfrac{\pi}{30} m_1 &&\approx 3.218 m_1 \\
m_7 &= 8(\cos\tfrac{\pi}{5})^2\cos\tfrac{7\pi}{30} m_1 &&\approx 3.891 m_1 \\
m_8 &= 8(\cos\tfrac{\pi}{5})^2\cos\tfrac{2\pi}{15} m_1 &&\approx 4.783 m_1
\end{aligned}
$$

Zamolodchikov's solution is the most complicated integrable model known in Physics" — Subir Sachdev

Spectrum close to E_8 point

All stable particles have very specific dependence on h and we checked this using MPS calculations. One can also compute η_2 and η_3 i.e., where the particle 2 and particle 3 becomes unstable. The data is at T=Tc.



Real-time evolution and scattering

Once we have created a MPS which is faithful representation of the ground state of quantum spin chain, we construct excitations on top [quasi particles] and then evolve them in real-time using TDVP methods [time-dependent variational principle]. TDVP is a very popular alternative to Trotterization with several advantages and was introduced in the seminal paper:

Time-Dependent Variational Principle for Quantum Lattices

Jutho Haegeman, J. Ignacio Cirac, Tobias J. Osborne, Iztok Pižorn, Henri Verschelde, and Frank Verstraete
Phys. Rev. Lett. **107**, 070601 – Published 10 August 2011

Article | References | Citing Articles (346) | PDF | HTML | Export Citation

Open questions!

- We know that $|S| = 1$ at two integrable points. However, we do not know how it behaves (interpolates) between these two limits. We can compute these close to the integrable points by doing perturbation theory (known as FFPT) but general regime needs support from numerical results. We know that close to FF, $P_{11} = 1 - P_{\text{prod.}} = 0$ till $E = 3m$, and $P_{\text{prod.}} > 0$ after that incoming energy. We see this in our results and also the agreement to FFPT.
- What is the high-energy behaviour between this integrable points. There is a conjecture that $P_{11} = 1 - P_{\text{prod.}} \to 0$ as $E \to \infty$ close to FF and $P_{11} = 1 - P_{\text{prod.}} \to 1$ as $E \to \infty$ close to E8 with a transition between. We see some signs for this behaviour. There are other major 'complex' issues close to E8 (resonance etc.).

Note that Note that in d = 2 we can have scattering without particle production and hence $P_{\text{prod.}} = 0$ but it is prohibited in $d > 2$ by Aks theorem [S. Aks, "Proof that scattering implies production in quantum field theory," Journal of Mathematical Physics (1965), 516-532.]

Quantum computing methods

- **Digital quantum computing**: Use qubits to perform computations. There are three steps in general: 1) Initial state-preparation, 2) Implementing unitary evolution using quantum gates, 3) Measurements.

- **Analog quantum computing**: Use of continuous variables (local Hilbert space is strictly infinite-dimensional) to carry out state preparation, time evolution, and measurements

Quantum gates

$$-\boxed{H}- = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \quad , \quad |0\rangle -\boxed{H}- |+\rangle$$

$$-\boxed{X}- = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \quad , \quad |0\rangle -\boxed{X}- |1\rangle$$

$$-\boxed{Z}- = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} \quad , \quad |1\rangle -\boxed{Z}- \quad -|1\rangle$$

$$-\boxed{Y}- = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$$

$$-\boxed{P}- = \begin{bmatrix}1 & 0\\ 0 & e^{i\phi}\end{bmatrix}$$

$$-\boxed{S}- = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

$$-\boxed{T}- = \begin{bmatrix}1 & 0\\ 0 & e^{\frac{i\pi}{4}}\end{bmatrix} = e^{\frac{i\pi}{8}}\begin{bmatrix}e^{\frac{-i\pi}{8}} & 0\\ 0 & e^{\frac{i\pi}{8}}\end{bmatrix}$$

Physics Applications

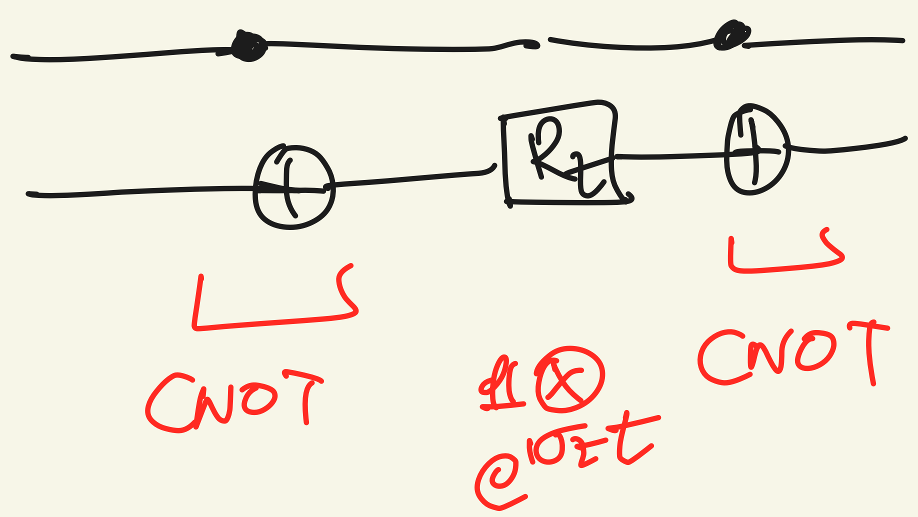
Want to implement $\hat{U} = e^{-i\hat{H}t}$

Suppose $H = \sigma_z \otimes \sigma_z$, then

$$e^{i(\sigma_z \otimes \sigma_z)t} = \cos(t)\,\mathbb{1} + i\sin(t)\,\sigma_z \otimes \sigma_z$$

can be written as $CNOT(\mathbb{1} \otimes e^{i\sigma_z t})CNOT$

Therefore,



Say $H = \sigma_y \otimes \sigma_y \otimes \sigma_z$

Then we can still write

$e^{-i\hat{H}t}$ noting that

$$Y = S^{\dagger} H Z H S$$

Now suppose that similar to example with qubits where we had

$$H = \sigma_z \otimes \sigma_z + \sigma_x \otimes \sigma_x + \mathbb{I}_2$$

could have been represented $e^{-i\hat{H}t}$ by quantum circuit.

Suppose $$\hat{q}_k = \frac{\hat{a}_k + \hat{a}_k^+}{\sqrt{2}}$$

$$\hat{p}_k = \frac{\hat{a}_k - \hat{a}_k^+}{i\sqrt{2}}$$

are the quadrature operators for mode 'k' satisfying $[\hat{q}_k, \hat{p}_l] = i\delta_{kl}$

Then

Displacement $\longrightarrow e^{i\hat{q}_k s_1}$ $\quad s_i \in \mathbb{R}$ for all 'i'

Squeezing $\rightarrow e^{i(\hat{q}_k \hat{p}_k + \hat{p}_k \hat{q}_k) s_2}$

Beam-Splitter $\rightarrow e^{i(\hat{p}_k \hat{q}_l - \hat{q}_k \hat{p}_l)}$

$\hookrightarrow$ quadratic Hamiltonian

O(3) model in 1+1-dimensions using CVs

The Hamiltonian is given by:

$$\hat{H} = \frac{1}{2\beta} \sum_i \mathsf{L}_i^2 - \beta \sum_{\langle i,j \rangle} \mathsf{n}_i \cdot \mathsf{n}_j,$$

A recent paper has studied this model using digital quantum computing [arXiv: 2210.03679]. They found that while for $\beta \ll 1$, the results for ground-state energy is reasonable using SDKs [PennyLane], it is much harder for $\beta \gg 1$.

CV gates

Displacement $\quad D_i(\alpha) = e^{(\alpha \hat{a}_i^\dagger - \alpha^* \hat{a}_i)}$

Rotation $\quad R_i(\phi) = e^{i\phi \hat{n}_i}$

Squeezing $\quad S_i(z) = e^{\frac{1}{2}(z^* \hat{a}_i^2 - z \hat{a}_i^{\dagger 2})}$

Beam Splitter $\quad BS_{i,j}(\theta, \phi) = e^{\theta(e^{i\phi} a_i^\dagger a_j - e^{-i\phi} a_i a_j^\dagger)}$

Cubic Phase $\quad V_i(\gamma) = e^{\frac{i\gamma}{6} \hat{x}_i^3}$

Matrix rep. of these gates.

$$D_i(\alpha) = \begin{pmatrix} b_i \\ b_i^\dagger \\ \mathbb{I} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 1 & \alpha^* \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_i \\ a_i^\dagger \\ \mathbb{I} \end{pmatrix}$$

$$R_i(\phi) = \begin{pmatrix} b_i \\ b_i^\dagger \end{pmatrix} = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix} \begin{pmatrix} a_j \\ a_j^\dagger \end{pmatrix}$$

$$S_i(re^{i\theta}) = \begin{pmatrix} b_i \\ b_i^\dagger \end{pmatrix} = \begin{pmatrix} \cosh r & -e^{-i\theta} \sinh r \\ -e^{-i\theta} \sinh r & \cosh r \end{pmatrix} \begin{pmatrix} a_j \\ a_j^\dagger \end{pmatrix}$$

Interaction terms

Studying the quadratic part of the Hamiltonian is not very useful, but for example studying $\lambda\phi^3$ or $\lambda\phi^4$ is more important. It seems that $\lambda\phi^3$ is still better than $\lambda\phi^4$ because one has the cubic phase gate in the CV approach. However, there is no direct CV gate corresponding to quartic part [arXiv: 2002.01402] for details. One interesting toy model in Physics is the O(3) model, here also we run into problems of quartic terms as discussed in the next slide!

O(3) model in 1+1-dimensions using CVs

To study the model using CVs, we have to write the Hamiltonian in the oscillator basis. It turns out that we need two oscillators (qumodes) per lattice site. Though, the kinetic term is simple, the nearest-neighbour term is complicated. Using results from [Schwinger, 1962], the second term looks is quartic in combinations of $a, a^\dagger, b, b^\dagger$ at site 1 and site 2. It appears to be difficult for implementation using CVs. Work in progress.

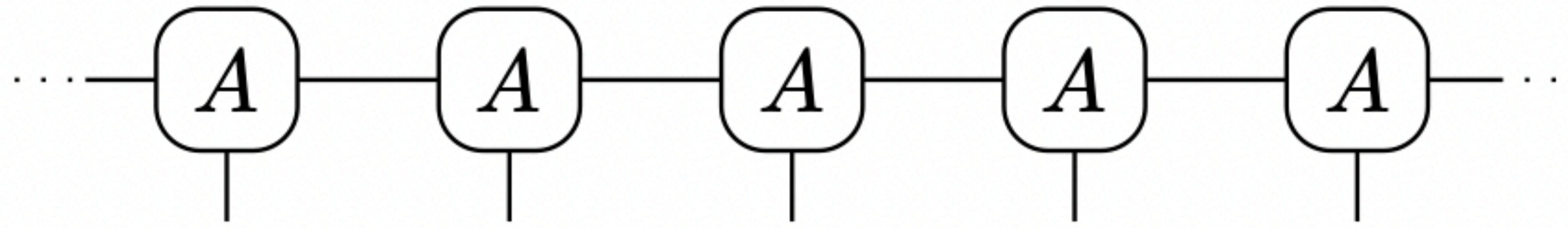
Summary

Tensor network methods have potential to assist in various interesting problems in Physics. On one hand, it can efficiently reproduce the ground state of several quantum systems with MPS and PEPS while on the other hand it can also describe real-space RG in various dimensions and can help us in understanding spin models, complex action systems, gauge theories etc.

Looking quantum mechanically, these models and several others can be studied in future using qubits and qumodes which will hopefully help us understand these problems in a new way.

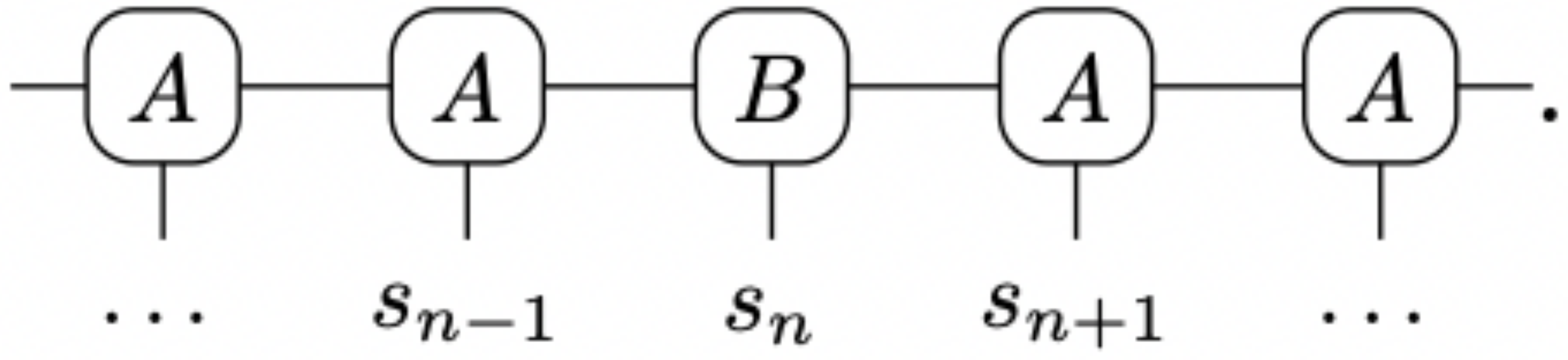
Thank You

$$|\Psi(A)\rangle =$$

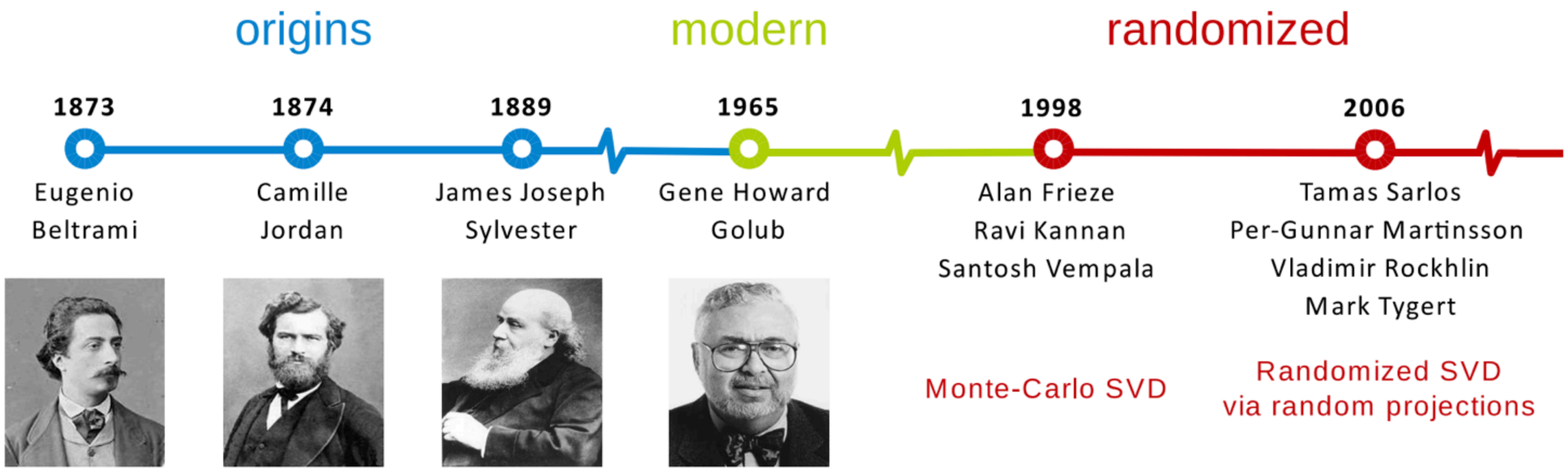


$$d^N \to N d \chi^2$$

$$|\Phi_k(B)\rangle = \sum_n e^{ikn}$$

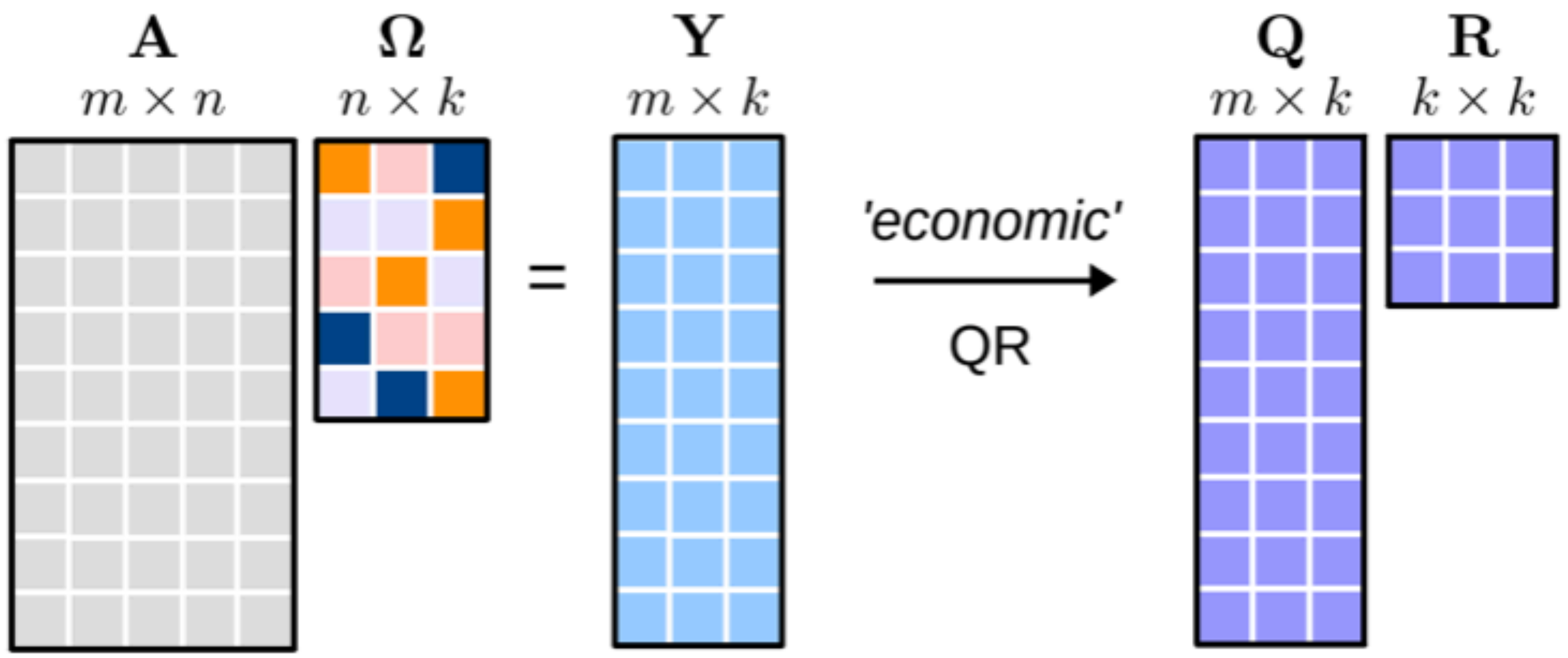


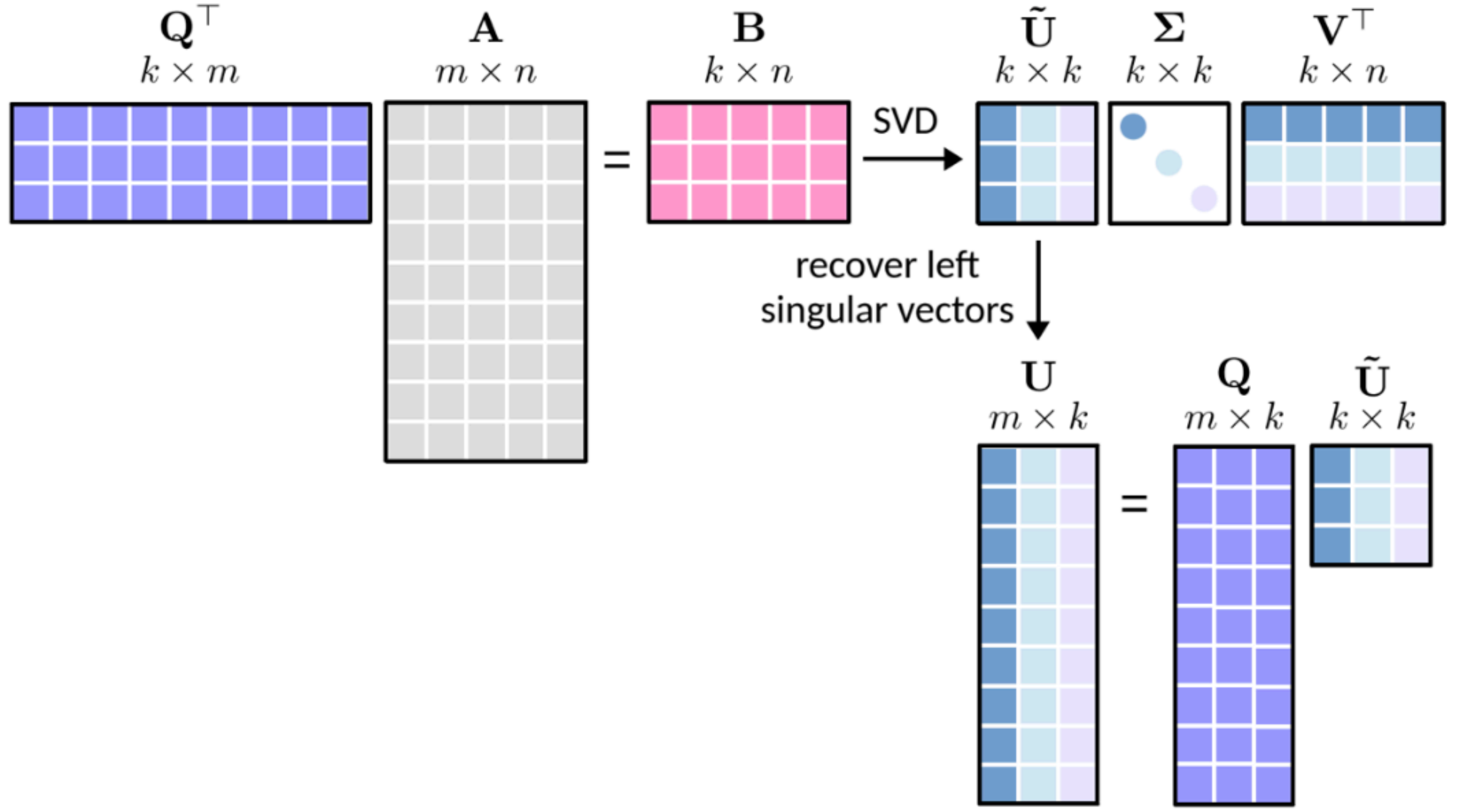
Van Damme et al. 1907.02474



arXiv:1608.02148

Randomised SVD





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