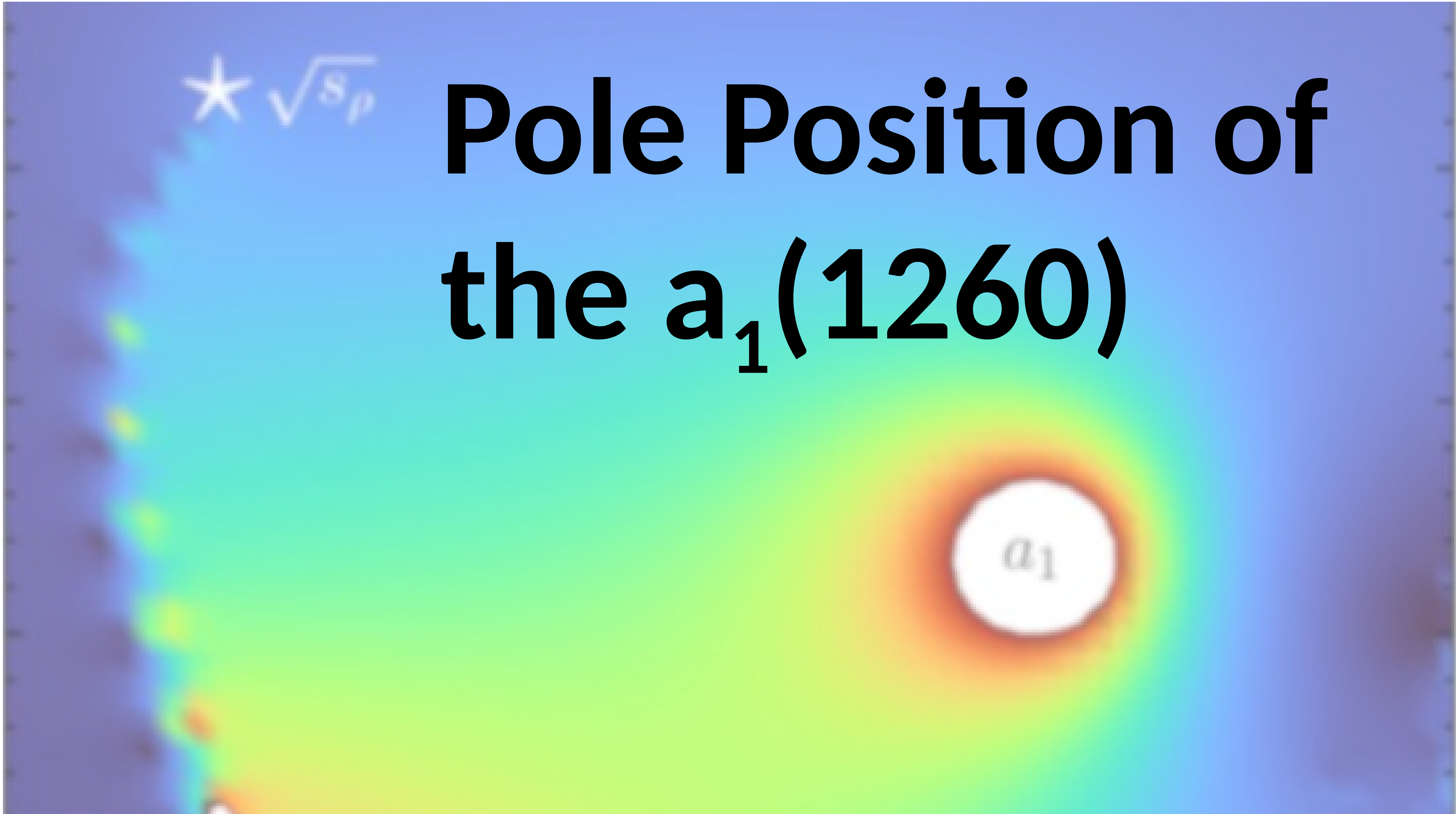
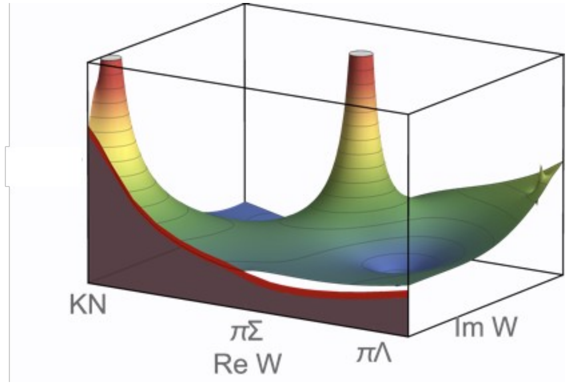
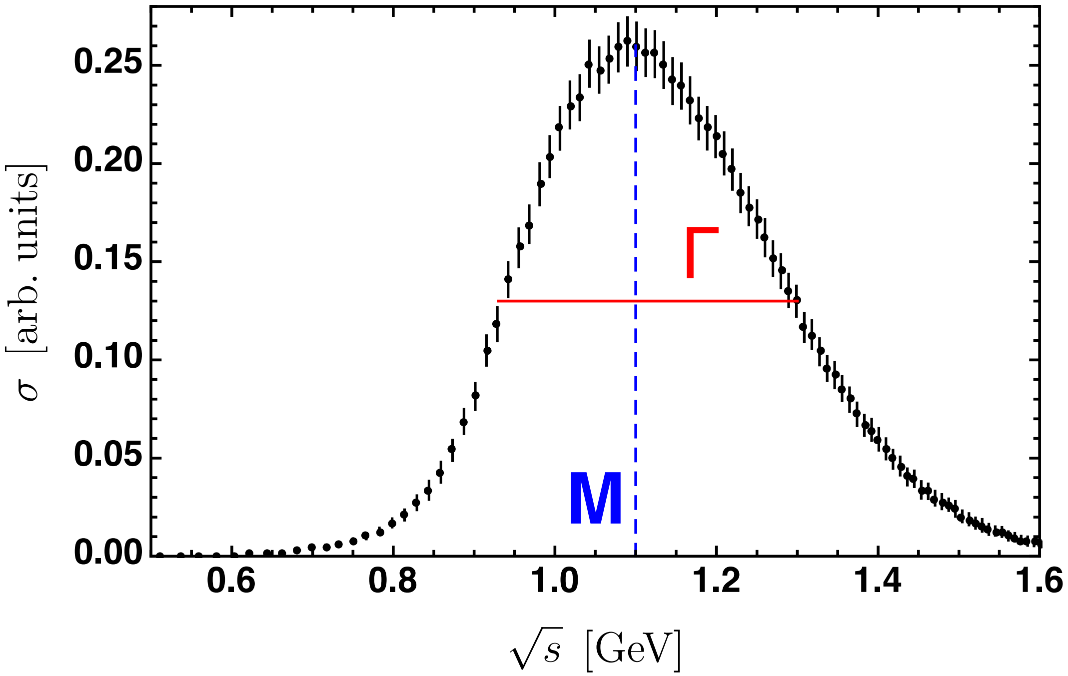
Pole Position of the $a_1(1260)$



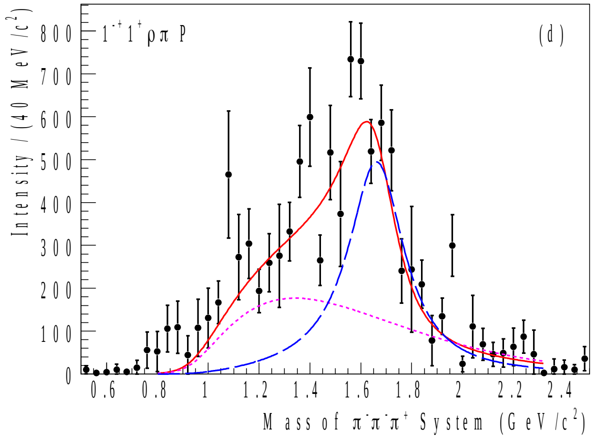
Daniel Sadasivan Andrei Alexandru
Hakan Akdag   Felipe Amorim
Ruari Brett   Chris Culver
Michael Doring   Maxim Mai

Resonances





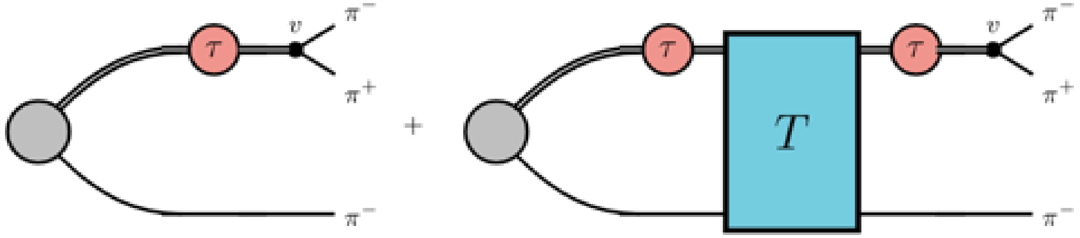
- Situated in the intermediate energy region near the energy threshold of many hadrons..
- Sometimes seen as peak at a certain energy in scattering cross sections.
- Described by certain quantum numbers.
- Can be studied through analytic continuation.
- Useful to relate results to other theories like quark models.

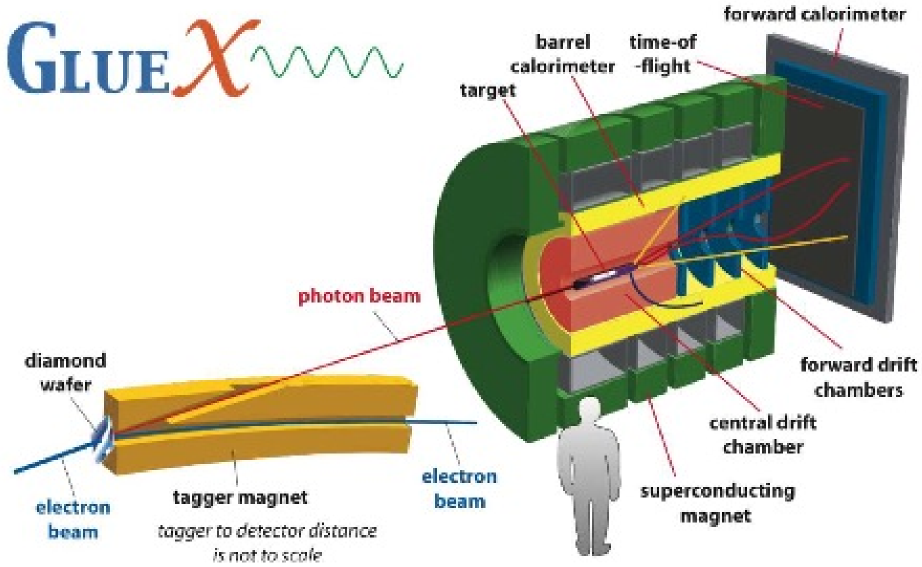


M. Alekseev et al. Phys.Rev.Lett., 104, 2010, 0910.5842.

Applications of Three-body Analysis

Results from the COMPASS experiment produced through three-body analysis. Possibly an exotic state [forbidden quantum numbers] — explicit gluon dynamics[?]

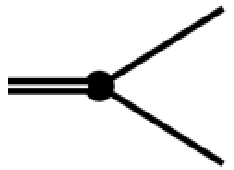




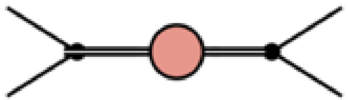
J. Zarling. JPS Conf. Proc., 26:022002, 2019, 1911.11239.

The Isobar Formulation

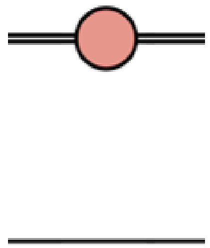
PF Bedaque, HW Griesshammer NUCL PHYS A 671, (2000) 357, arXiv:9907077.



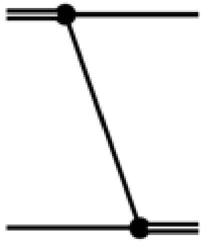
A decay vertex into two particles



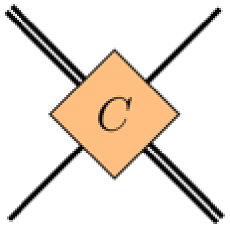
A two-particle scattering system which can be a subsystem in a three-particle system.



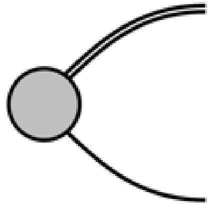
An isobar and a spectator that are disconnected, meaning they do not interact.



An isobar and a spectator with one particle exchange.

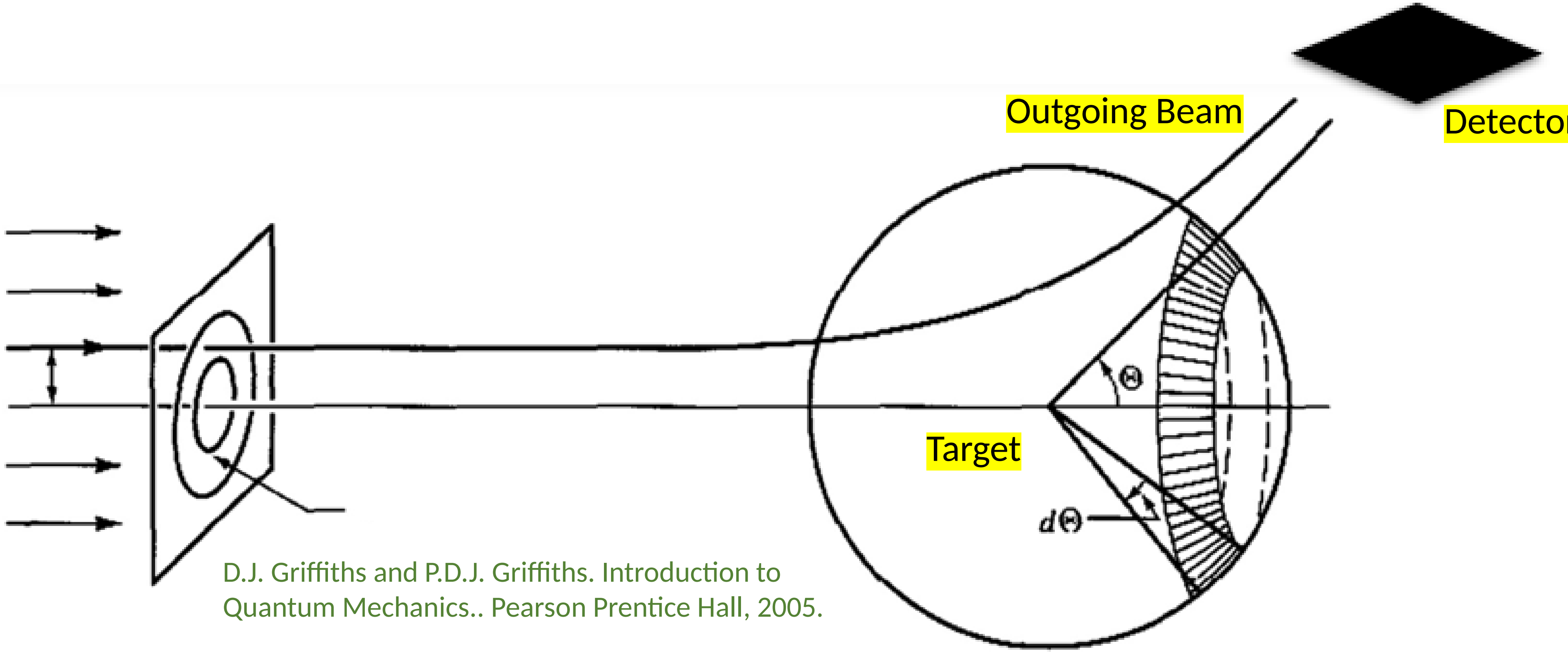


A three-body contact term where all three particles interact.



The decay vertex of a source decaying into an isobar and a spectator.

The S-Matrix and T-Matrix



D.J. Griffiths and P.D.J. Griffiths. Introduction to Quantum Mechanics.. Pearson Prentice Hall, 2005.

$$\Psi_{\text{out}} = S\Psi_{\text{in}}$$

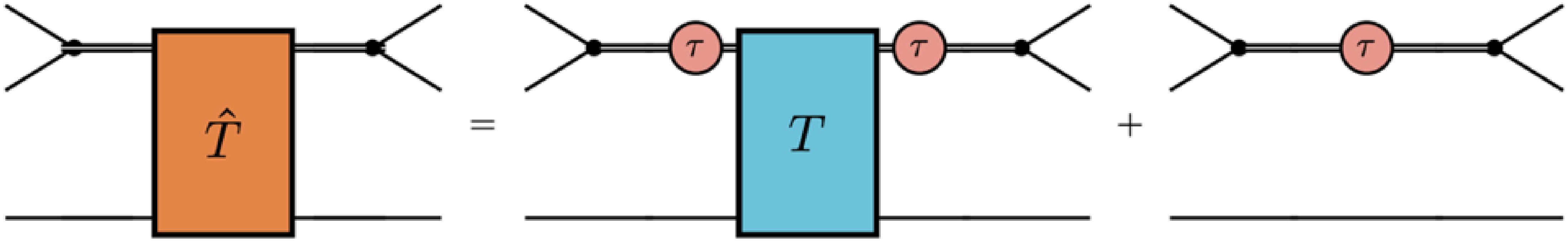
$$\mathbb{S} = \mathbb{1} - 2\pi i \mathbb{T}\,,$$

Unitarity and Decomposition

$$\mathbb{S} = \mathbb{1} - 2\pi i \mathbb{T}\,,$$ Scattering Operator

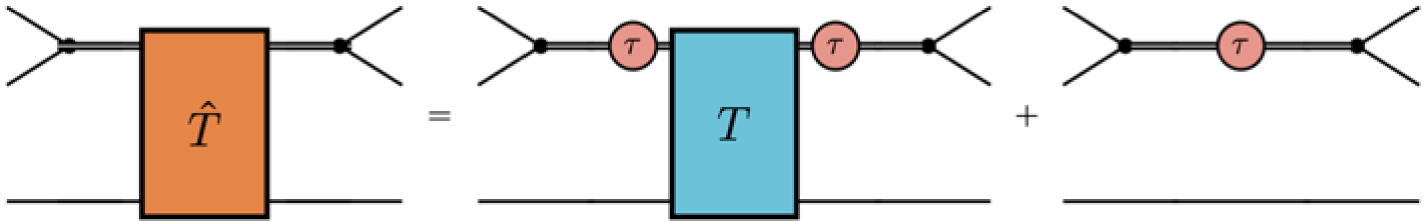
$$\mathbb{S}^\dagger \mathbb{S} = \mathbb{1}\,.$$ Unitarity

$$\mathbb{T} - \mathbb{T}^\dagger = 2\pi i \mathbb{T}^\dagger \mathbb{T}\,.$$ Constraint on T

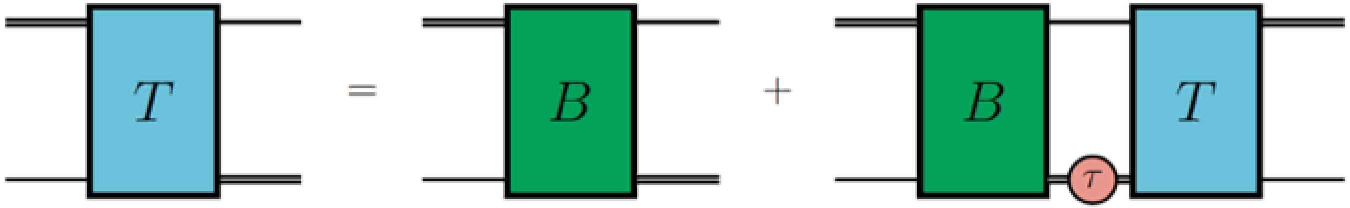


Three-Body Unitarity with Isobars

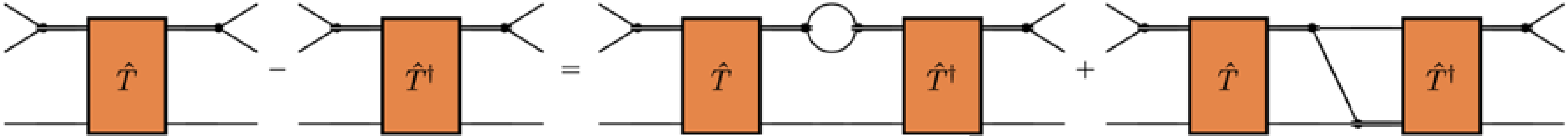
M. Mai, B. Hu, M. Doring, A. Pilloni, and A. Szczepaniak. Eur. Phys. J., A53(9):177, 2017, 1706.06118.



The decomposition of the three-body T-operator



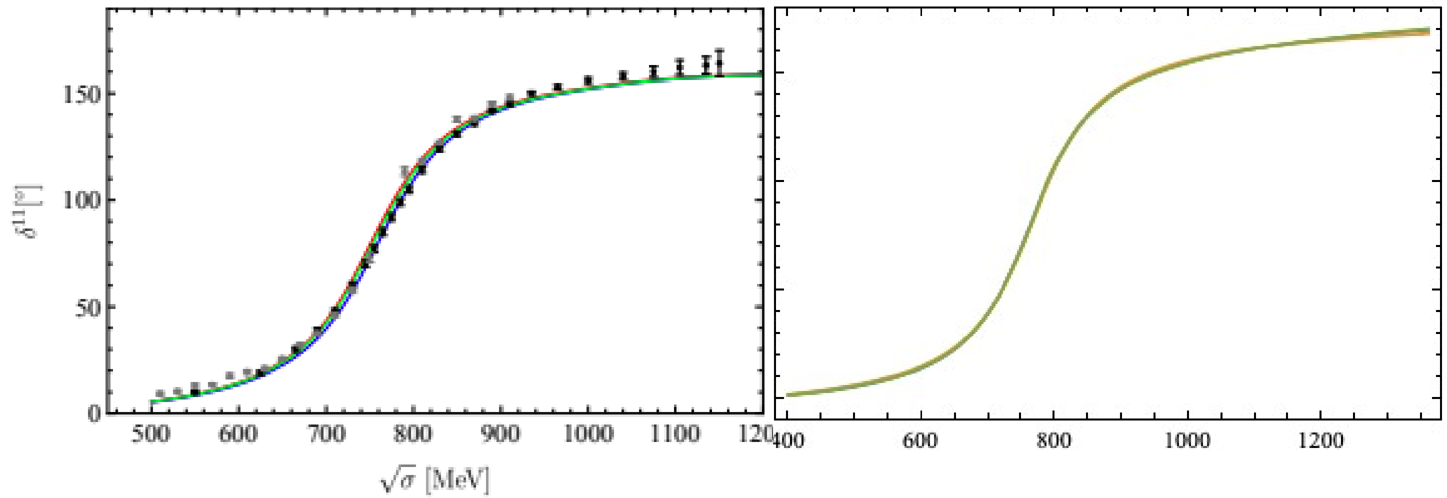
The Bethe-Salpeter Equation



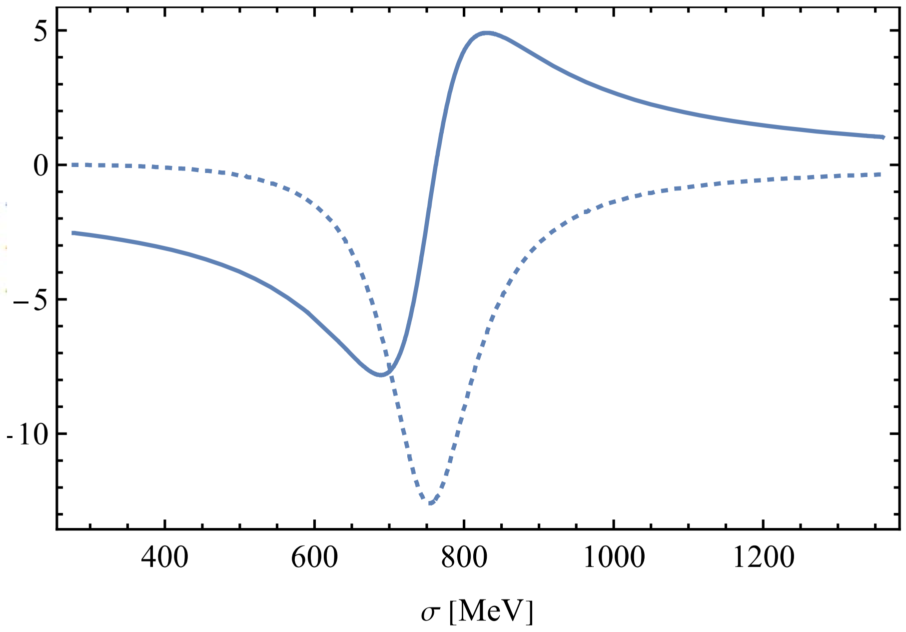
$$\mathbb{T} - \mathbb{T}^\dagger = 2\pi i \mathbb{T}^\dagger \mathbb{T}\,.$$

The constraint of unitarity

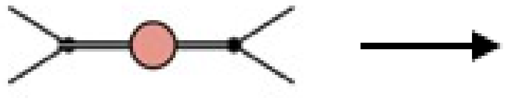
Two-Body Input



A fit to phase shift data for ππ scattering data (left) and a fit to the phase shift calculated from a dispersive solution to the phase shift (left) calculated in J. R. Pelaez, A. Rodas, and J. Ruiz De Elvira. Global parameterization of ππ scattering up to 2 GeV. Eur. Phys. J. C, 79(12):1008, 2019.

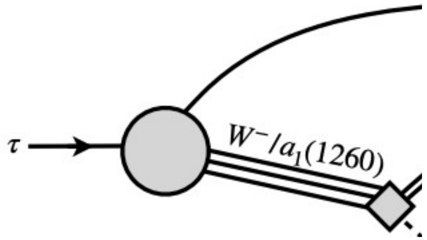


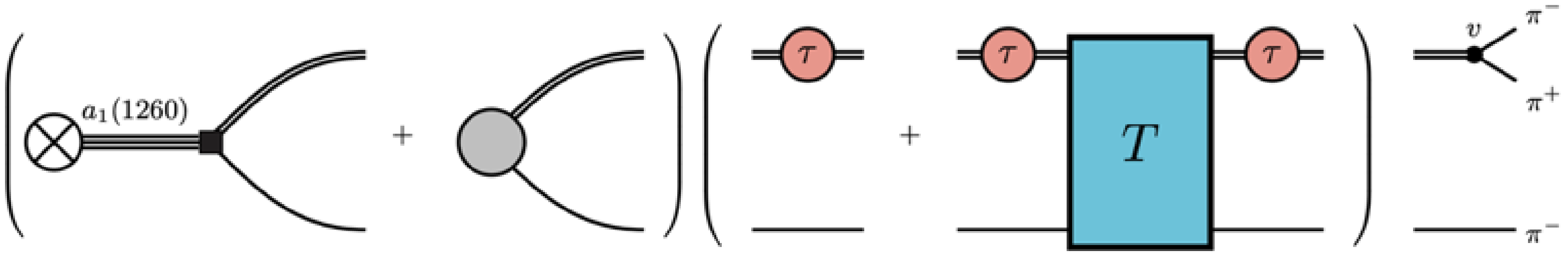
The imaginary (dashed) and real (smooth) parts of the ππ amplitude from the fit. Peaks are associated with the ρ(770) resonance.

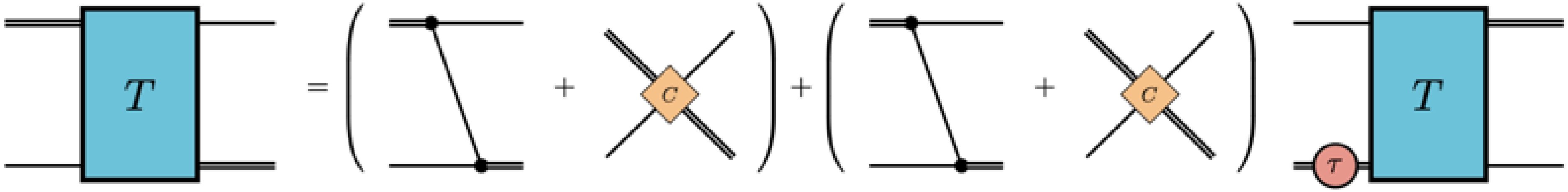


ππ

Formalism for $\tau^- \to \pi^- \pi^- \pi^+ \nu_\tau$

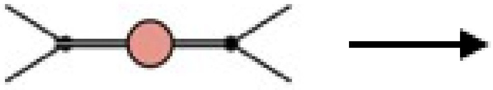






OPE and τ are constrained by unitarity.

Their vertices are not constrained but can be fit to ππ data.



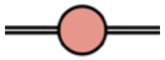
ππ

The contact term and decay terms are not fixed by unitarity and instead are parameterized by a general Laurent expansion.

Both decay terms and the contact term have the correct partial-wave threshold behavior and include free parameters fit to line shape data

Integration Contours

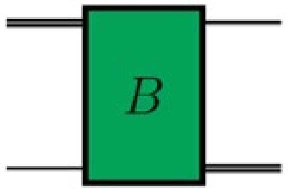
$$B_{\lambda\lambda'}(\boldsymbol{p},\boldsymbol{p}') = \frac{v^*_\lambda(P-p-p',p)v_{\lambda'}(P-p-p',p')}{2E_{p'+p}(\sqrt{s}-E_p-E_{p'}-E_{p'+p}+i\epsilon)},$$



$$\tau^{-1}(\sigma) = K^{-1}(\sigma) - \Sigma(\sigma),$$

$$\Sigma(\sigma) = \int_0^\infty \frac{\mathrm{d}k\,k^2}{(2\pi)^3}\frac{1}{2E_k}\frac{\sigma^2}{\sigma'^2}\frac{\tilde{v}(k)^*\tilde{v}(k)}{\sigma-4E_k^2+i\epsilon},$$

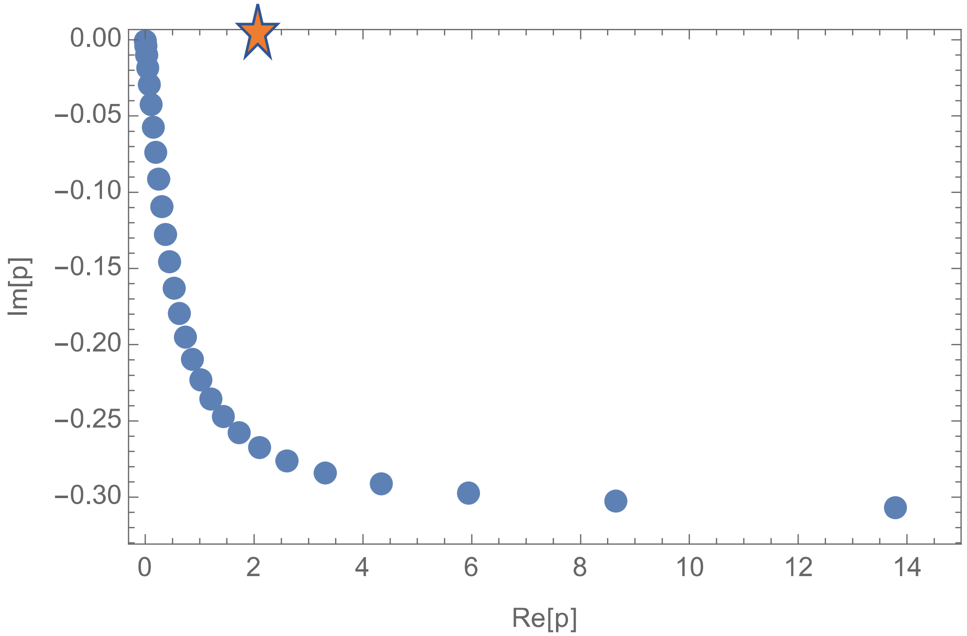
$$\sigma' = (2E_k)^2, \quad \tilde{v}(k) = \sqrt{\frac{16\pi}{3}}g_1 k$$

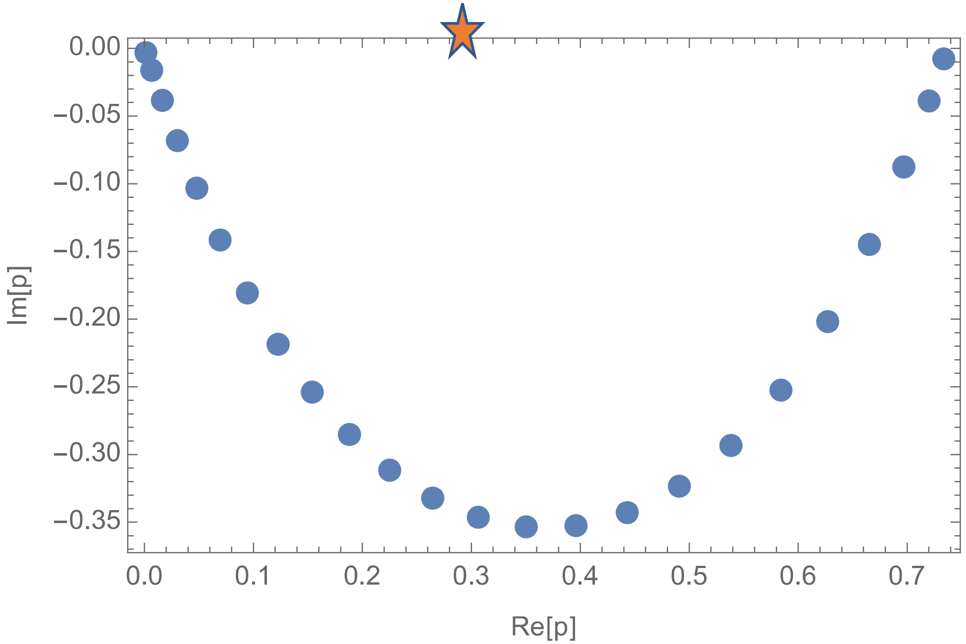


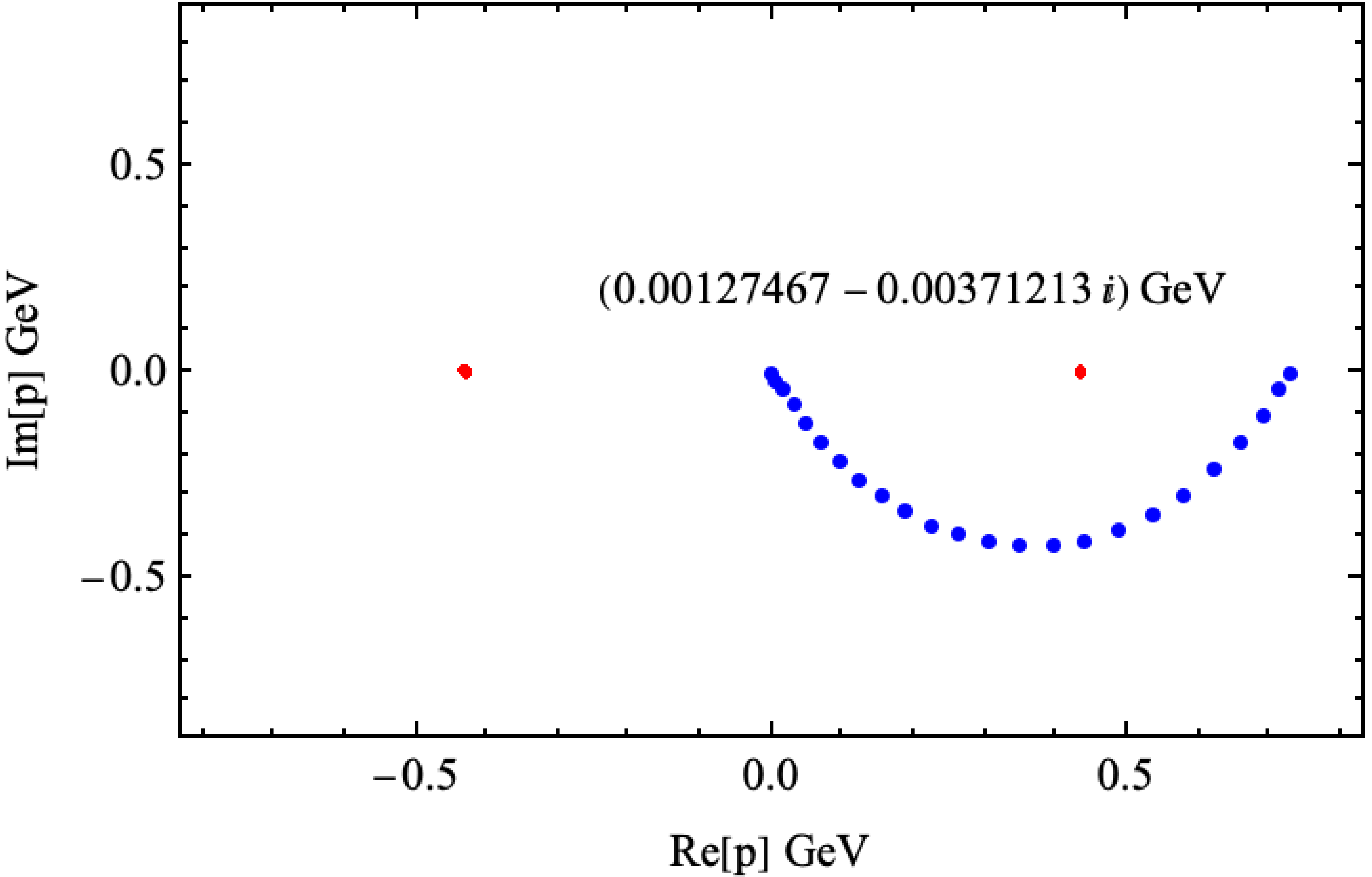
$$B^J_{\lambda\lambda'}(q_1,p) = 2\pi\int_{-1}^{+1}\mathrm{d}x\,d^J_{\lambda\lambda'}(x)B_{\lambda\lambda'}(\boldsymbol{q}_1,\boldsymbol{p}),$$

$$B^J_{LL'}(q_1,p) = U_{L\lambda}B^J_{\lambda\lambda'}(q_1,p)U_{\lambda'L'},$$

$$T^J_{LL'}(q_1,p) = \left(B^J_{LL'}(q_1,p)+C_{LL'}(q_1,p)\right) + \int_0^\Lambda \frac{\mathrm{d}l\,l^2}{(2\pi)^3 2E_l}\left(B^J_{LL''}(q_1,l)+C_{LL''}(q_1,l)\right)\tau(\sigma(l))T^J_{L''L'}(l,p) \quad (10)$$

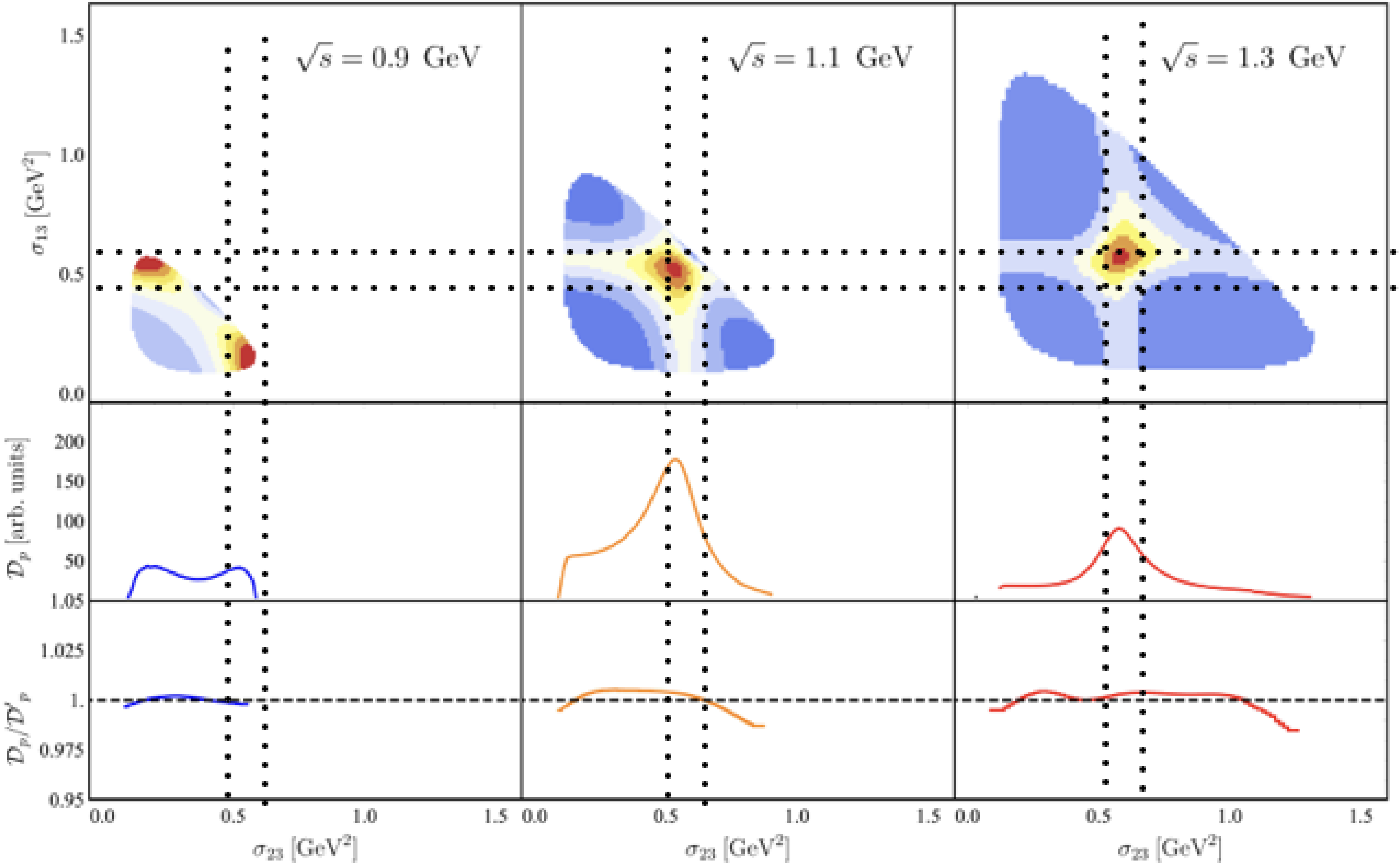






0.5
0.0
−0.5
Im[p] GeV
(0.00127467 − 0.00371213 i) GeV
−0.5
0.0
0.5
Re[p] GeV

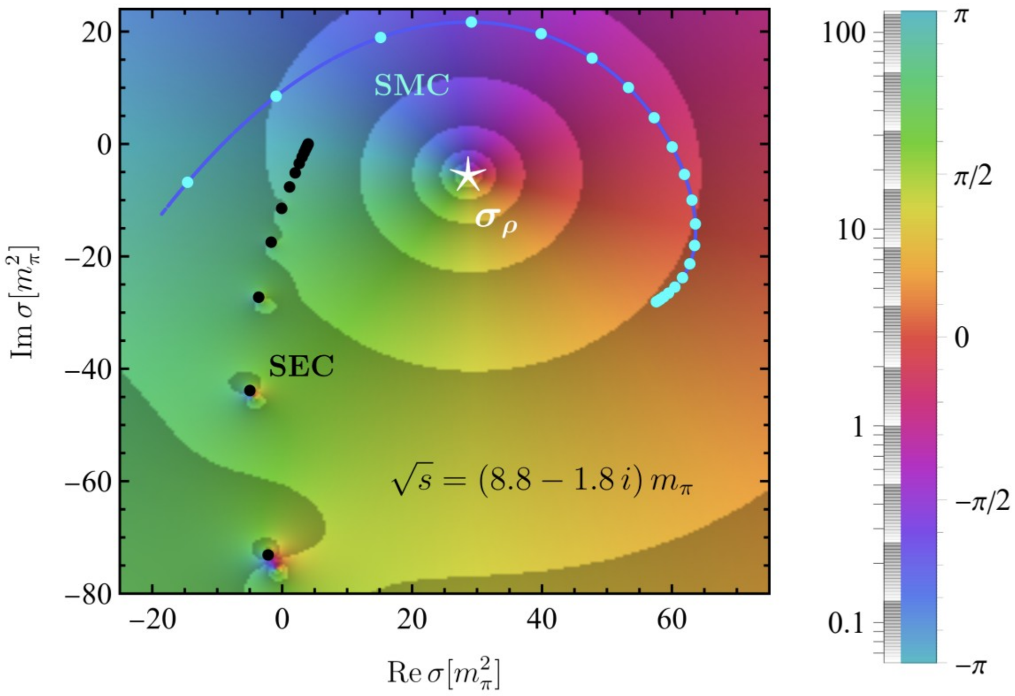
Dalitz Plots



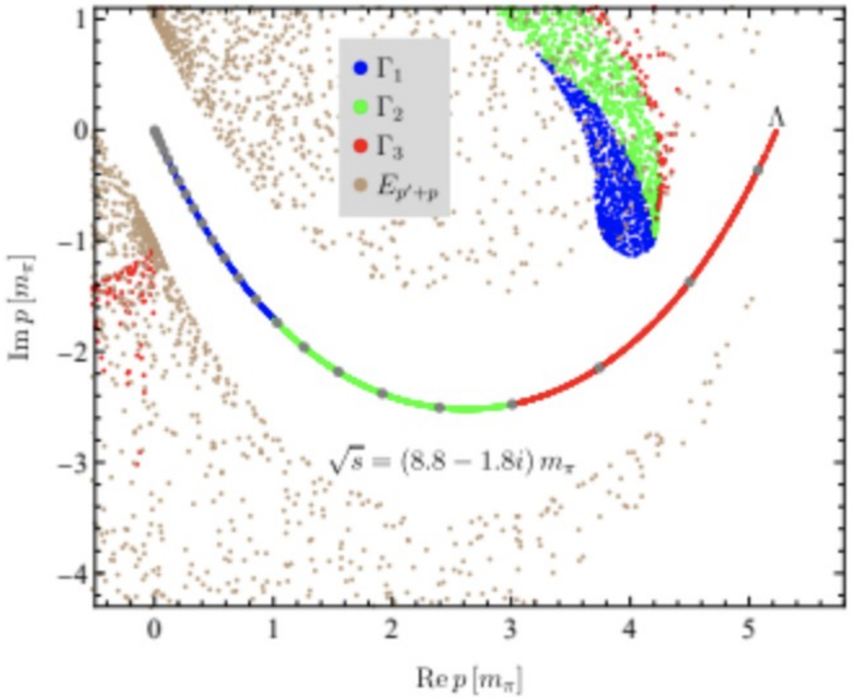
Top Row: Dalitz plots for the decay of the a1(1260).
Middle Row: The projected Dalitz plots
Bottom Row: Effect of rescattering in our 2020 paper.

D.S., M. Mai, H. Akdag and M. Doring, Dalitz plots and lineshape of a1(1260) from a relativistic three-body unitary approach, Phys. Rev. D 101 (2020) 094018 [2002.12431].

Branch Cuts



An illustration of the three-body analytic structure in the two-body invariant mass (σ) plane. Blue dots give the Spectator Momentum Contour (SMC) and black dots give the Self Energy Contour (SEC). The star give the πρ branch point.



An illustration of the Spectator Momentum Contour (SMC) contour in the momentum (p) plane. The dots represent singularities in the One Pion Exchange term.

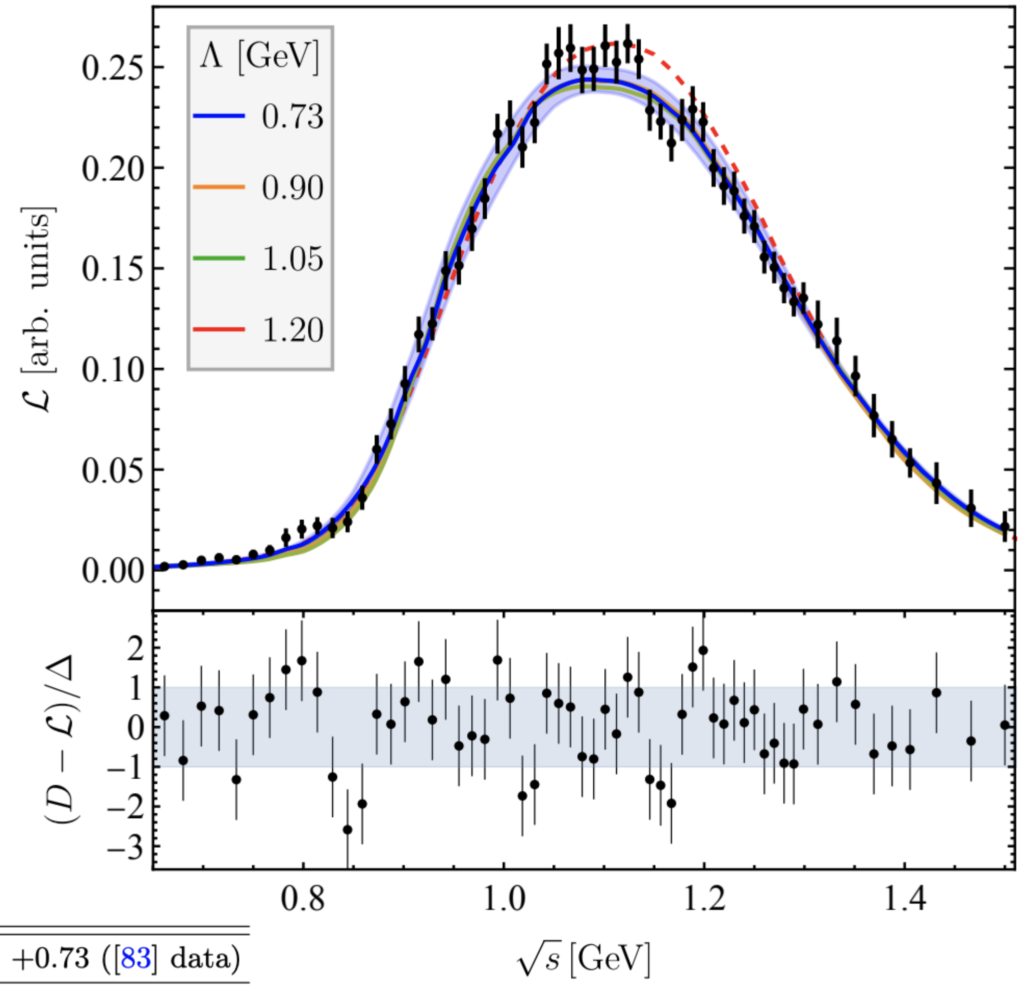
Daniel Sadasivan, Andrei Alexandru, Hakan Akdag, Felipe Amorim, Ruairi Brett, Chris Culver, Michael Doring, Frank X. Lee,2, and Maxim Mai, Pole position of the a1(1260) resonance in a three-body unitary framework, (Under Review at Phys. Rev.) [2112.03355].

Fit

Daniel Sadasivan, Andrei Alexandru, Hakan Akdag, Felipe Amorim, Ruairi Brett, Chris Culver, Michael Doring, Frank X. Lee,2, and Maxim Mai, Pole position of the a1(1260) resonance in a three-body unitary framework, (Under Review at Phys. Rev.) [2112.03355].

Right: Fits for various cutoffs to lineshape data from the ALEPH experiment. The shaded blue area gives the confidence region from a resampling procedure.

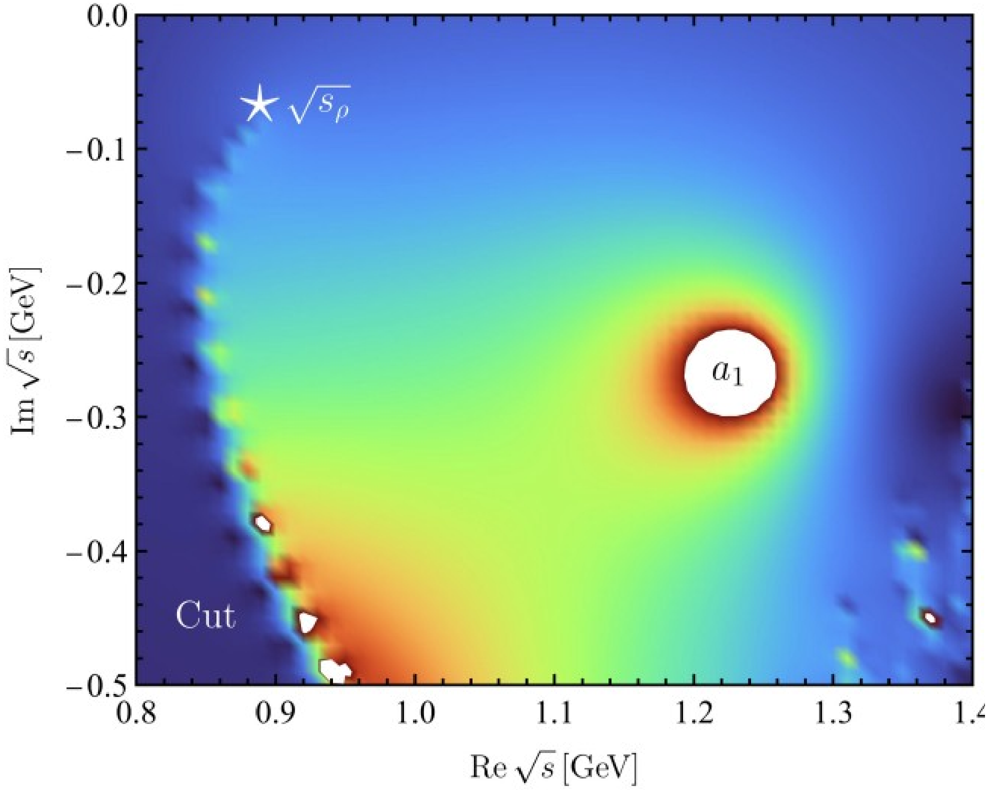
Below: The pole positions and fit parameters for various cutoffs and tests.

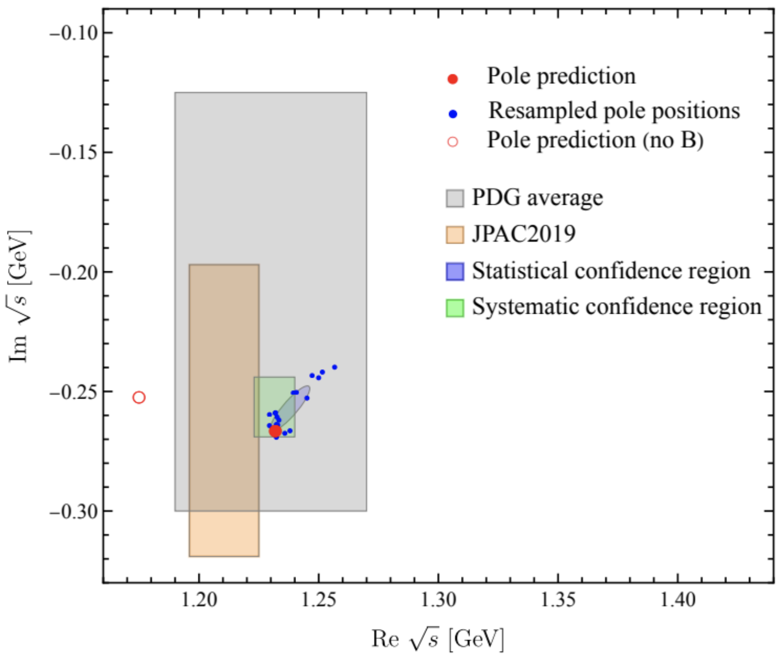


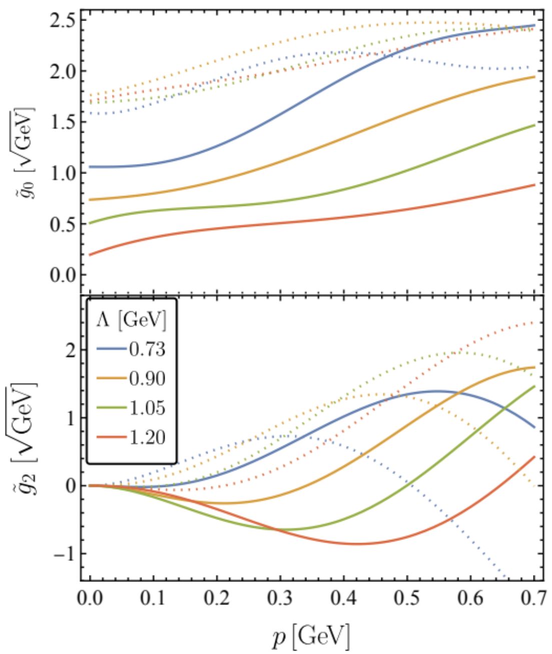
Λ [GeV]	+0.73	+0.90	+1.05	+1.2	+0.73 (no B)	+0.73 ([82] data)	+0.73 ([83] data)
Re $\sqrt{s_0}$ [MeV]	$+1232^{+15}_{-0}$	+1223	+1231	+1240	+1174	+1233	+1230
Im $\sqrt{s_0}$ [MeV]	-266^{+0}_{-22}	−269	−244	−251	−252	−278	−261
$\chi^2/(65-6)$	0.99	1.32	1.60	1.90	2.56	0.99	0.98
c_{00}^{-1}	$+16.48^{+0.005}_{-0.007}$	+14.59	+12.67	+11.53	+20.16	+16.74	+16.49
c_{00}^{0}	$+1.729^{+0.008}_{-0.005}$	+1.750	+1.843	+2.073	+0.019	+1.712	+1.720
m_{a_1} [GeV]	$+1.293^{+0.001}_{-0.000}$	+1.287	+1.281	+1.278	+1.391	+1.296	+1.294
$D_{f0}\times 10^7$ [a.u.]	$-1.841^{+0.049}_{-0.027}$	−2.371	−2.126	−2.250	−0.925	−1.887	−1.829
$D_{f2}\times 10^8$ [a.u.]	$+6.462^{+0.451}_{-0.149}$	+3.094	+1.567	+0.837	−6.824	+6.718	+6.512
$D_{\tilde{f}}\times 10^6$ [a.u.]	$-1.319^{+0.002}_{-0.000}$	−1.358	−1.338	+1.372	−1.235	−1.329	−1.318

Data from Michel Davier, Andreas Hocker, Bogdan Malaescu, Chang-Zheng Yuan, and Zhiqing Zhang, "Update of the ALEPH non-strange spectral functions from hadronic τ decays," Eur. Phys. J. C 74, 2803 (2014), arXiv:1312.1501 [hep-ex].

A1(1260) Pole Parameters







The complex plane with the a_1 pole and $\pi\rho$ branch point and branch cut.

Left: The complex plane with pole position and statistical and systematic confidence regions along with other predictions for the pole position.
Right: The real (smooth) and imaginary (dashed) parts of the couplings for S-wave (lower plot) and D-wave (upper plot).

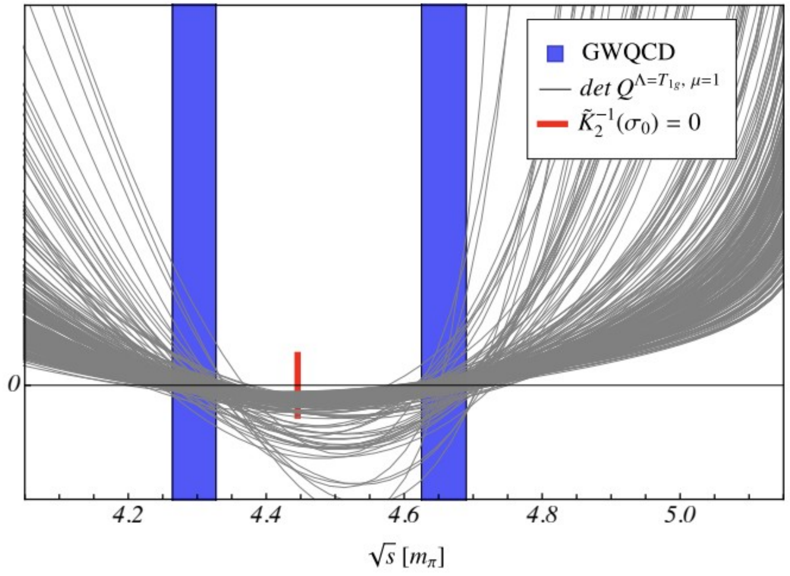
Daniel Sadasivan, Andrei Alexandru, Hakan Akdag, Felipe Amorim, Ruairi Brett, Chris Culver, Michael Doring, Frank X. Lee,2, and Maxim Mai, Pole position of the a1(1260) resonance in a three-body unitary framework, (Under Review at Phys. Rev.) [2112.03355].

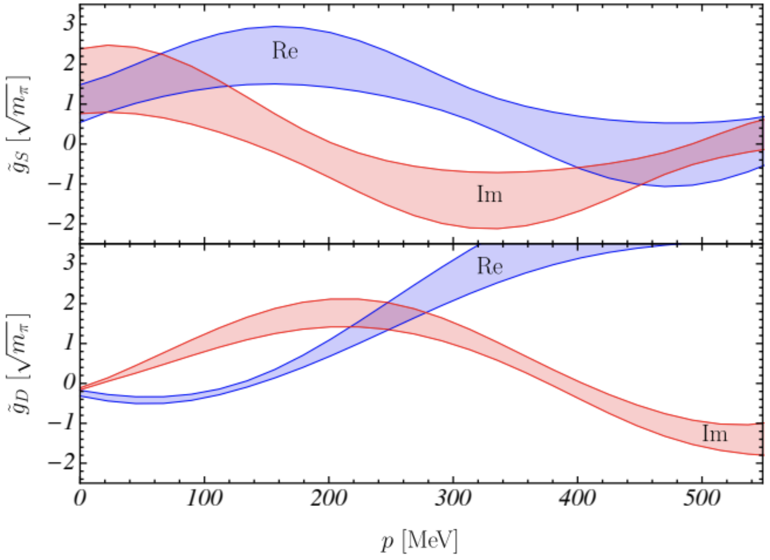
FVU
RFT
NREFT

Lattice Applications

$$0 = \det\left[B(s) + C(s) - E_L\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]_{\substack{(\lambda'\lambda)\\(\boldsymbol{p}'\boldsymbol{p})}}$$

A fit of the quantization condition from the equation above (gray curves) to the eigenvalues (blue bars) calculated from lattice QCD.

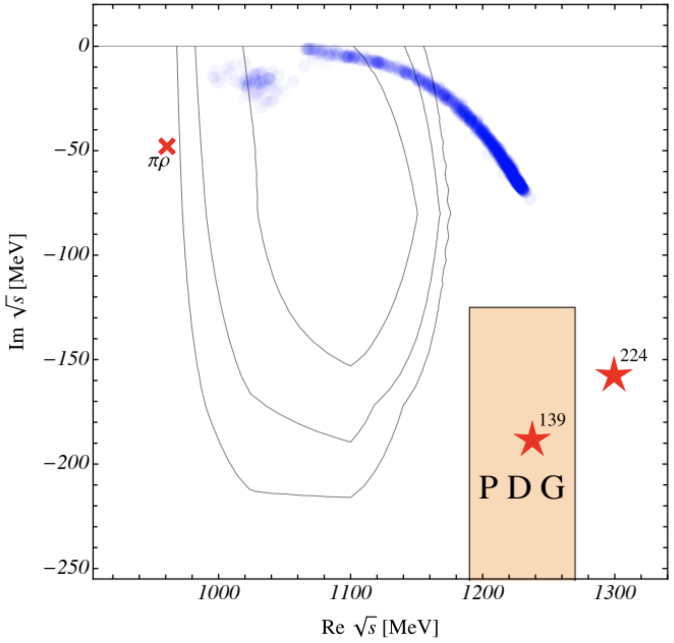




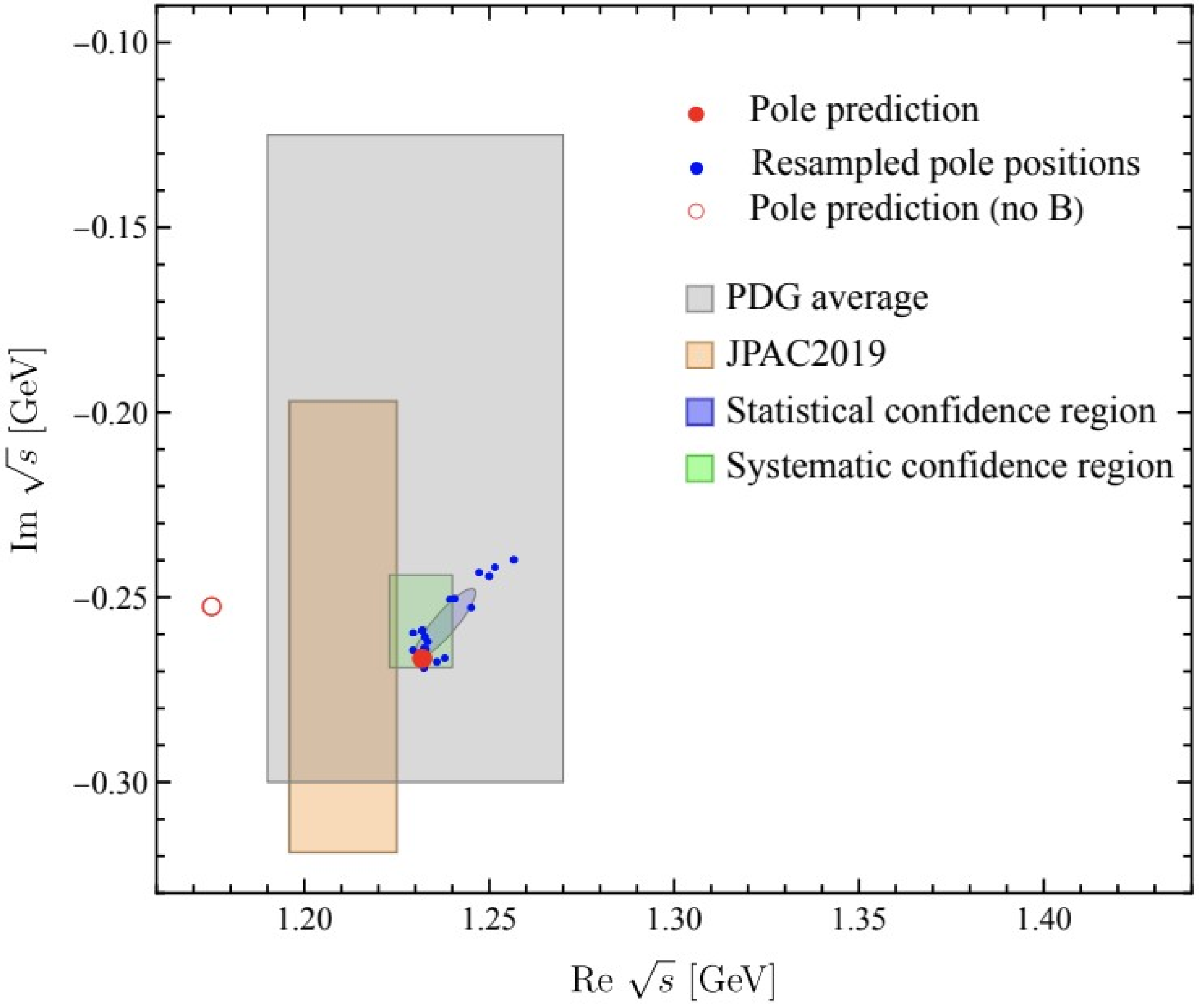
The real and imaginary parts of the couplings for S- and D- wave. The shaded areas give the uncertainties.

Maxim Mai, Andrei Alexandru, Ruair´ı Brett, Chris Culver, Michael D¨oring, Frank X. Lee, and Daniel Sadasivan, “Three-body dynamics of the a1(1260) resonance from lattice QCD,” PRL (2021),[2107.03973].

The pole position of the a_1(1260) (for a pion mass of 224 MeV)



Conclusion



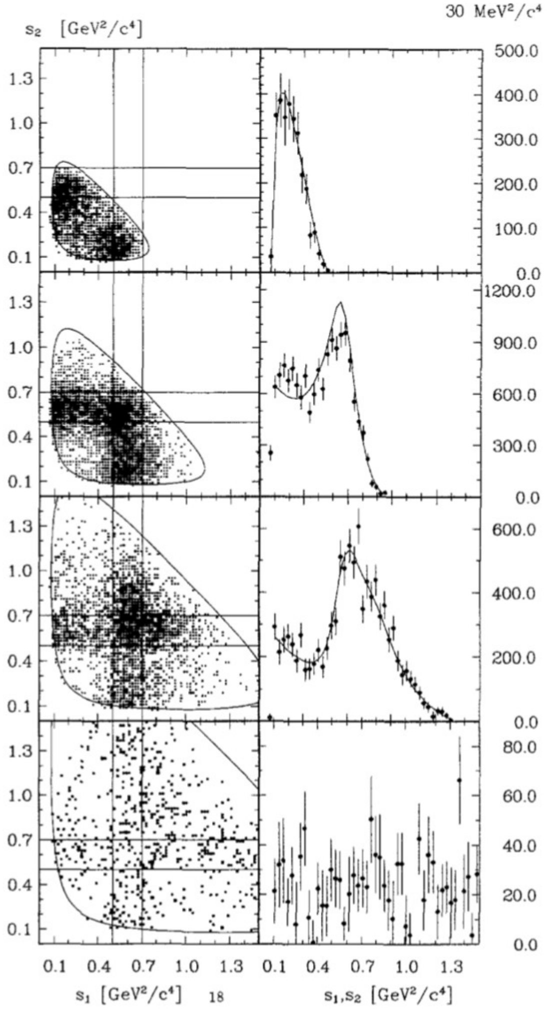
- We use unitarity and the Bethe-Salpeter Equation to develop a model for three pion interaction.

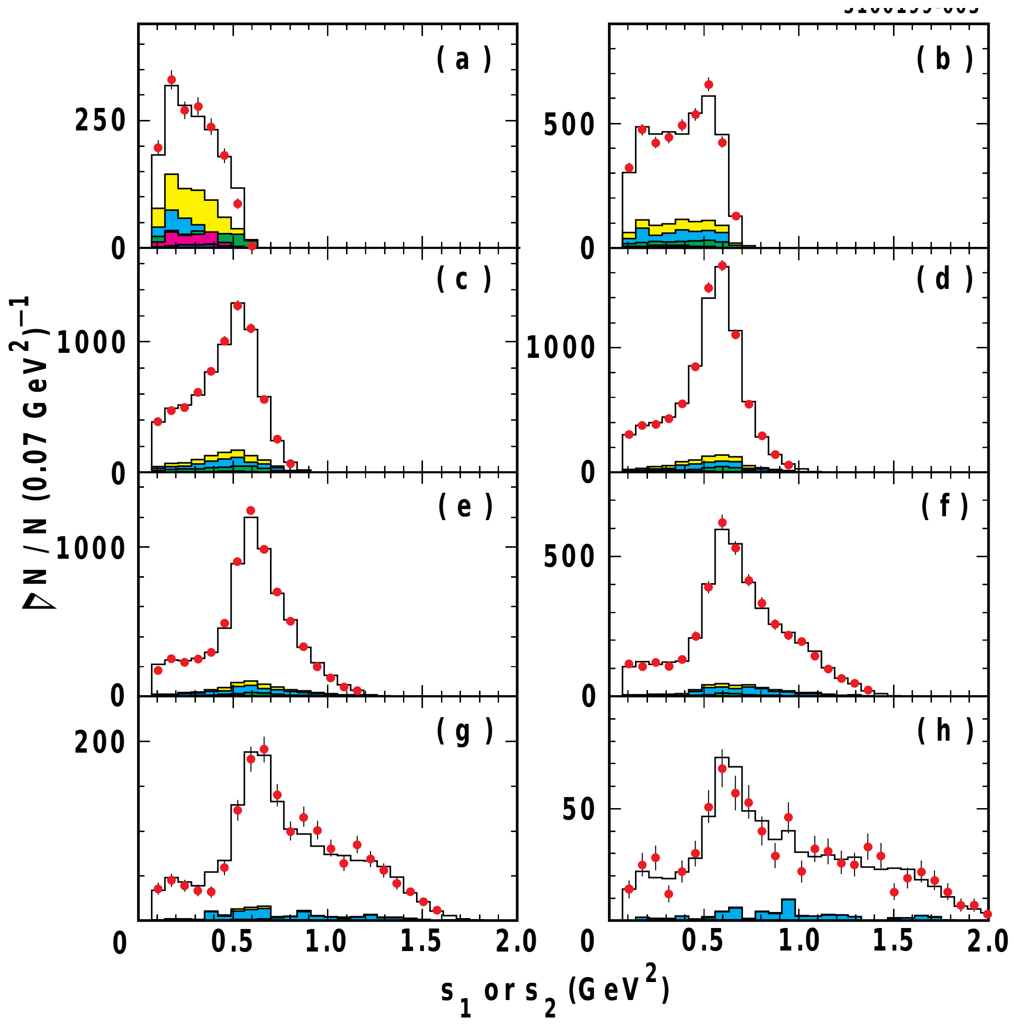
- This model can be fit to data from the ALEPH experiment.

- Using integration contours that respect the three-body analytic structure, we extract the pole position and couplings.

- We use the quantization condition to fit the ingredients of this model to energy eigenvalues from lattice QCD. This allows the calculation of the pole position without any fit to experimental data.

Dalitz Plot Data

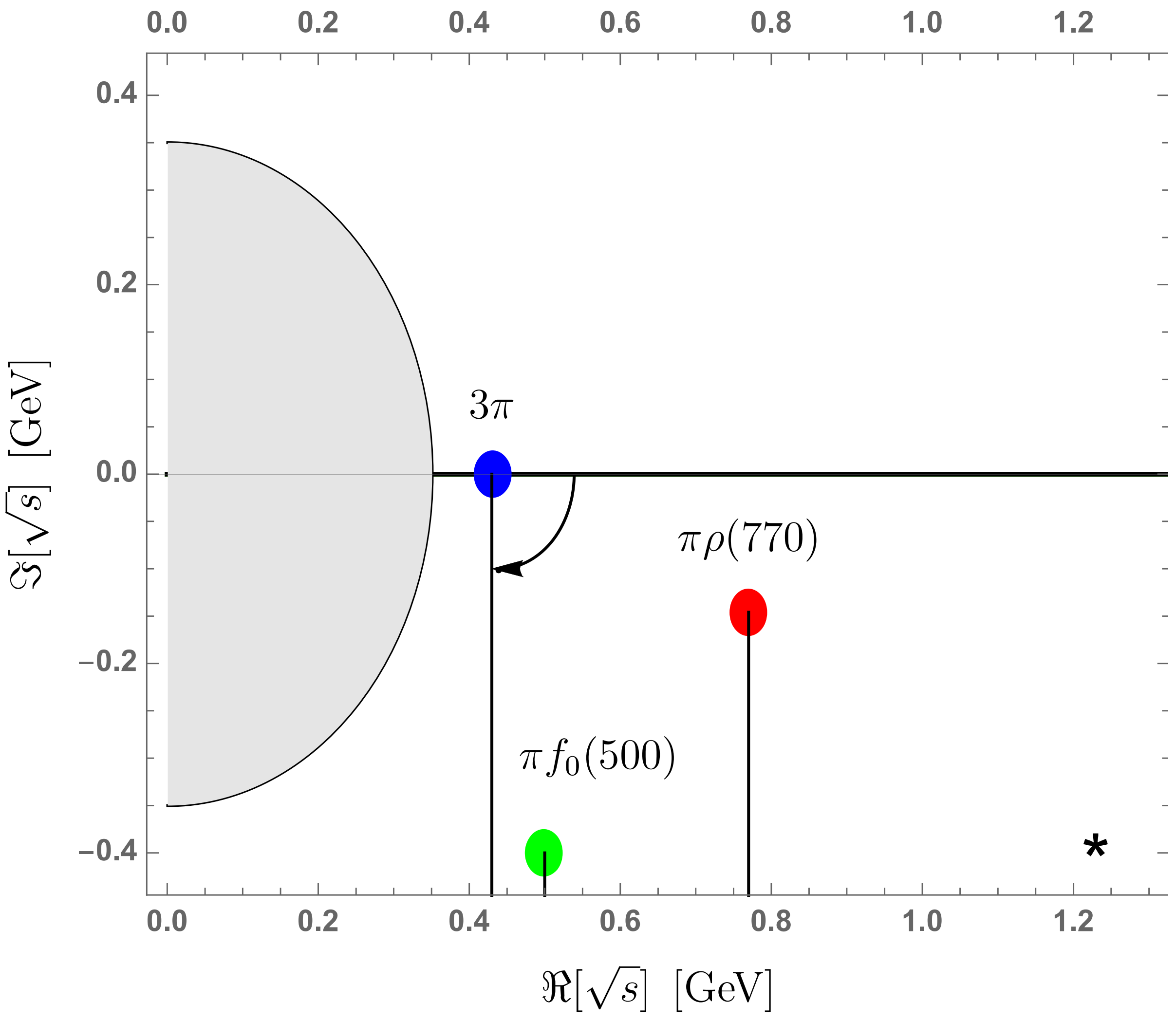




H. Albrecht et al. Z. Phys., C58, 1993

X D. M. Asner et al. Phys. Rev., D61:2000, hep-ex/9902022.

Three-Body Analytic Structure



Matching Conditions

The terms from the unitarity constraint, (left) and the Bethe-Salpeter Equation (right) which can be matched and then used to derive an analytic constraint on the Bethe-Salpeter Kernel.